MAGNETIC FLUX IN MESOSCOPIC CYLINDERS*

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This work is dedicated to Prof. Andrzej Fuliński on the occasion of his 70-th birthday

Our objective here is the study of the noise-assisted generation of magnetic flux in a collection of identical mesoscopic cylinders which are coupled via mutual inductances. With thermal (Johnson–Nyquist)-fluctuations acting at finite temperature, the system can be modeled in terms of a set of Langevin equations with a corresponding Fokker–Planck equation. In the limit of infinitely many constituents, the steady-state of the system is determined by a mean-field-like, nonlinear Fokker–Planck equation. The rich complexity of the generated average flux through each cylinder and its characteristic fluctuations are investigated as a function of various parameters such as the temperature, the coupling strength and an externally applied, uniform magnetic field.

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1. Introduction

Mesoscopic systems are in-between the micro- and the macro-world and consequently form a bridge between quantum and classical physics [1]. They exhibit a variety of novel and unusual phenomena of both quantum and classical origin. As such, they are of great interest not only from a fundamental research point of view, but also from the viewpoint of novel technological applications. An example of such a mesoscopic phenomenon is the persistent current in a metallic ring [2]. The existence of such currents has been experimentally confirmed by several groups [3].

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With this work we investigate a system of interacting three-dimensional mesoscopic cylinders which are coupled via mutual inductances. It has been shown before [4] that for two coplanar mesoscopic rings the self-inductance can suppress the persistent current; in contrast, the mutual inductances can cause an enhancement. Here, we consider a set of coaxial cylinders and analyze the case of the "thermodynamic limit" of an infinite number of interacting cylinders. The stationary state is then determined by a self-consistent state equation. Such a model presents an idealized archetype of *e.g.* long wires formed by "pieces" made of single-wall carbon nanotubes. In Sec. 2 we describe in detail the model and derive a set of Langevin equations for a flux threading each cylinder. A special case, *i.e.* a one-cylinder system, is briefly discussed in Sec. 3. A Fokker–Planck equation corresponding to the set of N Langevin equations is presented in Sec. 4. Addressing the thermodynamic limit of infinite many constituents we derive a steady-state equation from a nonlinear mean-field equation. In Sec. 5 we analyze the average flux and its fluctuations, both in the interacting and non-interacting mesoscopic cylinder systems, respectively.

2. Model description

In mesoscopic systems composed of a ring with toroidal or cylindrical geometry persistent currents can occur. They signify the phase coherence of the electrons, the so-called coherent electrons. In the ground state, at the temperature $T = 0$, the only electrons present in the system are the coherent ones. Their flow is persistent and non-dissipative. At non-zero temperature, $T > 0$, a partial set of these electrons become "normal" and their flow is dissipative. As a result, the amplitude of the persistent current decreases with temperature [5]. It has been confirmed experimentally [6] that mesoscopic rings connected to a current source exhibit a nonzero ohmic resistance. This implies that the flow of "normal" electrons can be modeled in terms of Ohm's law. The total current consists thus of a sum of the coherent current and an Ohmic current.

The mesoscopic cylinder considered herein is formed by the collection of N_{c} quasi one-dimensional rings (current channels) stacked along an axis. The coherent current is then a sum of contributions of single channels which can produce currents being either paramagnetic for an even number N_e of coherent electrons, or diamagnetic for an odd number of coherent electrons. The probability of finding a channel with an odd number of coherent electrons is denoted by P and the probability of finding a channel with an even number of coherent electrons is equal to $1 - P$. Next, consider a system of N identical mesoscopic cylinders placed concentrically and periodically in a uniform magnetic field B in the three-dimensional space. Because of the mutual inductance, the electric current in one cylinder will induce a magnetic flux in another cylinder. Therefore, the fluxes and the currents in the cylinders are coupled according to the expression [4, 7]

$$\phi_i = \sum_{k=1}^{N} \mathcal{M}_{ik} I_k + \phi_{\rm ext} \ , \tag{1}$$

where ϕ_i and I_i denote the flux and the current in the i-th cylinder, respectively. The flux $\phi_{\rm ext}$ is induced by an external uniform magnetic field B. The coupling coefficients $\mathcal{M}_{ik} = \mathcal{M}_{ki}$ (which form the matrix $\mathcal{M}$) are the mutual inductances for $i \neq k$ and self-inductances $\mathcal{L} = \mathcal{M}_{ii}$ for $i = k$ [7]. The current in the k-th cylinder is a sum

$$I_k = I_k^{\rm nor} + I_k^{\rm coh} \tag{2}$$

of the Ohmic (dissipative) current $I_k^{\rm nor}$ and the persistent current $I_k^{\rm coh}$. The Ohmic current $I_k^{\rm nor} = I_{\rm nor}(\phi_k)$ is determined by the Ohm's law and Lenz's rule, *i.e.*,

$$I_{\rm nor}(\phi_k) = -\frac{1}{R}\frac{d}{dt}\phi_k + \sqrt{\frac{2k_{\rm B}T}{R}}\,\mathit{\Gamma}_k(t) \ , \tag{3}$$

where R is a resistance of a single cylinder [8], $k_{\rm B}$ denotes the Boltzmann constant and $\mathit{\Gamma}_k(t)$ describes the thermal, Johnson–Nyquist fluctuations of the Ohmic current. This thermal noise is modeled by a set of independent Gaussian white noises of zero average, *i.e.*, $\langle \mathit{\Gamma}_k(t)\rangle = 0$ and δ-correlated function $\langle \mathit{\Gamma}_k(t)\mathit{\Gamma}_i(s)\rangle = \delta_{ki}\delta(t-s)$. The noise intensity $D_0 = \sqrt{2k_{\rm B}T/R}$ is chosen in accordance with the classical fluctuation–dissipation theorem [9]. The current of the coherent electrons $I_k^{\rm coh} = I_{\rm coh}(\phi_k, T)$ has been determined in Ref. [5] and reads

$$I_{\rm coh}(\phi_k, T) = \frac{N_{\rm c} I^*}{2}\Big[P g(\phi_k/\phi_0, T) + (1-P) g(\phi_k/\phi_0 + 1/2, T)\Big] \ , \tag{4}$$

where the flux quantum $\phi_0 := h/e$ is the ratio of the Planck constant h and the electron charge e. The characteristic current $I^* = h\,e\,N_e/(2 l_x^2 m_e)$, with N_e being the number of coherent electrons in a single current channel, l_x is the circumference of the cylinder and m_e is the mass of electron. Moreover, [5]

$$g(x, T) = \sum_{n=1}^{\infty} A_n(T) \sin(2n\pi x) \tag{5}$$

denotes the current in a channel with an even number of coherent electrons. The amplitudes read

$$A_n(T) = \frac{4T}{\pi T^*}\,\frac{\exp(-nT/T^*)}{1-\exp(-2nT/T^*)}\,\cos(n k_{\rm F} l_x) \ . \tag{6}$$

The characteristic temperature T^* is determined from the relation $k_{\mathrm{B}}T^* = \Delta_{\mathrm{F}}/2\pi^2$, where Δ_{F} marks the energy gap and k_{F} is the momentum at the Fermi surface.

Upon combining (3) and (4) into (1) we obtain the following set of stochastic equations:

$$\frac{1}{R}\sum_{k=1}^{N}\mathcal{M}_{ik}\frac{d\phi_k}{dt}=\phi_{\mathrm{ext}}-\phi_i+\sum_{k=1}^{N}\mathcal{M}_{ik}I_{\mathrm{coh}}(\phi_k,T)+\sqrt{\frac{2k_{\mathrm{B}}T}{R}}\sum_{k=1}^{N}\mathcal{M}_{ik}\Gamma_k(t)\,, \quad (7)$$

for $i = 1\ldots N$. Multiplying this system of equations by $\left(\mathcal{M}^{-1}\right)_{ni}$ and next summing over i one finds

$$\frac{1}{R}\frac{d\phi_n}{dt} = \sum_{i=1}^{N}(\mathcal{M}^{-1})_{ni}\,[\phi_{\mathrm{ext}} - \phi_i] + I_{\mathrm{coh}}(\phi_n,T) + \sqrt{\frac{2k_{\mathrm{B}}T}{R}}\;\Gamma_n(t)\;. \quad (8)$$

For the following let us introduce dimensionless variables. The dimensionless flux $x_n = \phi_n/\phi_0$ is given in units of the flux quantum. The dimensionless time reads $s = t/\tau_0$, where $\tau_0 = \mathcal{L}/R$ is the relaxation time of the averaged Ohmic current. The dimensionless Langevin equations (8) thus assume the form

$$\frac{dx_n}{ds} = -\,V'(x_n,T) - \sum_{i(\neq n)}^{N}\lambda_{ni}x_i + \sqrt{2D}\;\tilde{\Gamma}_n(s)\;, \quad (9)$$

where the prime denotes the derivative with respect to the first argument of the generalized potential $V(x_n,T)$, *i.e.* here with respect to x_n and the generalized potential is given by

$$V(x_n,T) = \tfrac{1}{2}\,a_n x_n^2 - b_n x_n - I_0\int^{x_n} f(y,T)dy\;. \quad (10)$$

The coupling constants are $\lambda_{ni} = \mathcal{L}(\mathcal{M}^{-1})_{ni}$, and the parameter $a_n = \mathcal{L}(\mathcal{M}^{-1})_{nn}$ corresponds to the n-th diagonal element of the inverse matrix $\mathcal{M}^{-1}$. The re-scaled, externally induced fluxes are $b_n = \gamma_n\phi_{\mathrm{ext}}/\phi_0$, where $\gamma_n = \mathcal{L}\sum_{i=1}^{N}(\mathcal{M}^{-1})_{ni}$. The re-scaled characteristic current is given by $I_0 = N_{\mathrm{c}}\,\mathcal{L}I^*/\phi_0$ and

$$f(y,T) = Pg(y,T) + (1-P)g(y+\tfrac{1}{2},T)\,. \quad (11)$$

The zero-mean re-scaled noise reads $\tilde{\Gamma}_n(s) = \sqrt{\tau_0}\;\Gamma_n(\tau_0 s)$ possessing the correlations $\langle\tilde{\Gamma}_n(s_1)\tilde{\Gamma}_m(s_2)\rangle = \delta_{nm}\delta(s_1 - s_2)$. Its overall intensity is determined by $D = k_{\mathrm{B}}T/2\varepsilon_0$, where $\varepsilon_0 = \phi_0^2/2\mathcal{L}$ [10].

3. The case of no coupling

For the noninteracting system, the flux dynamics threading a cylinder is described by the Langevin equation

$$\dot{x} = -V'(x,T) + \sqrt{2D}\,\tilde{\Gamma}(s)\,, \tag{12}$$

which is a particular case of (9) with a vanishing mutual inductance. The generalized potential

$$V(x,T) = \tfrac{1}{2}x^2 - \sigma x - I_0 \int^{x} f(y,T)dy, \qquad \sigma = \frac{\phi_{\rm ext}}{\phi_0}\,, \tag{13}$$

is in general multistable, see for details in Refs. [10] and [11]. The shape of the potential is very sensitive to the parameter P which enters implicitly the function $f(y,T)$ in (11). If $\sigma = 0$ and $P = 1$, the potential has — for sufficiently low temperature — maximum at $x = 0$, while for $P = 0$ it has a minimum. The global minimum (or minima) of the potential (13) can be interpreted as the self-sustaining fluxes (or currents) in the system. The additional but local minima correspond to meta-stable states and indicates the possibility of a so-called flux trapping [11]. For comparison with the coupled system, we assume that the flux trapping is absent in the system, *i.e.* the potential (13) is either monostable or bistable in dependence whether the temperature of the system is above or below some critical temperature $T_{\rm c}$ [11].

4. The coupled system

The coupled system of N cylinders is described by the set of N Langevin equations (9). The N-dimensional joint probability distribution $p(\{x_n\},s)$, which characterizes the dynamics of the fluxes $\{x_n\} = \{x_1, x_2, \ldots, x_N\}$ which thread the N cylinders fulfills a Fokker–Planck equation [9]. Its form follows from Eqs. (9), reading

$$\begin{aligned}\frac{\partial}{\partial s}p(\{x_n\},s) \;=\; & \sum_{n=1}^{N}\frac{\partial}{\partial x_n}\left[V'(x_n,T) + \sum_{i=1(\neq n)}^{N}\lambda_{ni}x_i\right]p(\{x_n\},s) \\ & + D\sum_{n=1}^{N}\frac{\partial^2}{\partial x_n^2}p(\{x_n\},s)\,. \end{aligned}\tag{14}$$

Due to the symmetry $\lambda_{ni} = \lambda_{in}$, the set of Eqs. (9) is a gradient system independent of the specific configuration of the cylinders. In consequence, the stationary solution of (14) is the Gibbs-like distribution,

$$
\begin{aligned}
p_{\rm st}(\{x_n\}) &\propto \exp\left[\frac{-W(\{x_n\})}{D}\right], \\
W(\{x_n\}) &= \sum_{n=1}^{N} V(x_n,T) + \sum_{n=1}^{N}\sum_{i=1(\neq n)}^{N} \lambda_{ni} x_n x_i \,.
\end{aligned} \tag{15}
$$

In the absence of the external magnetic field ($b_n = 0$), the mean flux in a *finite* chain of mesoscopic cylinders vanishes due to the symmetry of the potential $W(\{x_n\})$. A non-zero mean flux can occur only in the limit of infinitely many cylinders. We thus consider an infinite coaxial linear chain of interacting cylinders in the *mean field* approximation.

The reduced one-dimensional probability density $p(x_k, s)$ can be obtained from $p(\{x_n\}, s)$ by integrating it over all variables *except* x_k. Such an integration applied to (14) yields for the steady states the nonlinear equation

$$
\frac{\partial}{\partial x_k}\left[V'(x_k,T) + \sum_{i(\neq k)}^{N} \lambda_{ki}\langle x_i|x_k\rangle\right] p_{\rm s}(x_k) + D\frac{\partial^2}{\partial x_k^2}\, p_{\rm s}(x_k) = 0\,, \tag{16}
$$

where $p_{\rm s}(x_k)$ is the stationary, one-dimensional probability density and $\langle x_i|x_k\rangle = \int x_i p_{\rm s}(x_i|x_k)dx_i$ denotes a stationary conditional average of x_i with respect to the stationary conditional probability density $p_{\rm s}(x_i|x_k)$. The equation is not closed because $p_{\rm s}(x_i|x_k) = p_{\rm s}(x_i, x_k)/p_{\rm s}(x_k)$ is expressed by a two-dimensional probability density $p_{\rm s}(x_i, x_k)$. In turn, an equation for this quantity involves the three-dimensional probability density, and so on. In this way we obtain an hierarchy-chain of infinite coupled equations. To arrive at an equation which can be studied analytically, we have to introduce an approximate scheme. We proceed in the following way: The conditional average can be formally rewritten as $\langle x_i|x_k\rangle = \langle x_i\rangle + c_{ik}$, where c_{ik} describes the correlations between the i-th and the k-th cylinder. In the limit of a large numbers of cylinders, *i.e.* when $N \to \infty$, the system becomes statistically homogeneous so that the stationary average $\langle x_k\rangle = \langle x\rangle$ does not depend on the index k. Moreover, consistent with a mean-field description, we neglect correlations, *i.e.* we assume that $c_{ik} = 0$. Consequently, the stationary probability density $p_{\rm s}(x)$ of the representative $x = x_k$ satisfies a non-linear Fokker–Planck equation [12], *i.e.*,

$$
\frac{d}{dx}\left[U'(x,T) - \lambda\mu\right] p_{\rm s}(x) + D\frac{d^2}{dx^2}\, p_{\rm s}(x) = 0\,, \tag{17}
$$

with the "order parameter" reading

$$\mu \equiv \langle x \rangle = \int\limits_{-\infty}^{\infty} x\, p_{\mathrm{s}}(x)\, dx\,. \tag{18}$$

The effective, generalized potential is given by

$$U(x,T) = \tfrac{1}{2}\, ax^2 - \gamma\sigma x - I_0 \int\limits^{x} f(y,T) dy\,. \tag{19}$$

The parameter $a = \mathcal{L}(\mathcal{M}^{-1})_{kk}$ equals the (k-independent) diagonal element of the inverse matrix $\mathcal{M}^{-1}$, the parameter $b = \phi_{\mathrm{ext}}/\phi_0$ (*cf.* (13)) and $\gamma = \mathcal{L}\sum_i(\mathcal{M}^{-1})_{ki}$. The effective coupling constant $\lambda = -\sum_{i\neq k}\lambda_{ki} = -\mathcal{L}\sum_{i\neq k}(\mathcal{M}^{-1})_{ki}$. In the thermodynamic limit, all these quantities do not depend on the element indexes of the matrix $\mathcal{M}^{-1}$ and, therefore, the relation $\gamma = a - \lambda$ holds.

Both, the convergence and the approximate value of the above series are implied by the fast decay of the mutual inductance with increasing distance among cylinders [7]. The ratio λ is small and, usually, the absolute value of λ does not exceed 0.1, *i.e.* $-0.1 < \lambda < 0.1$ [7]. It is important to note that for the coaxial alignment considered in this work the effective coupling constant is positive, *i.e.*, $\lambda > 0$. Therefore, we expect a "ferromagnetic" state of the system characterized by a parallel alignment of the magnetic moments induced by currents flowing in the neighboring cylinders. For this ordering the currents in the neighboring cylinders flow in the same direction. This indicates the possibility of the symmetry breaking in the system when the external magnetic field $B = 0$.

The formal stationary solution of Eq. (17) reads

$$p_{\mathrm{s}}(x) = p_{\mathrm{s}}(x,\mu) = N_0(\mu)\exp\left[-\frac{U(x,T) - \lambda\mu x}{D}\right], \tag{20}$$

where the normalization constant $N_0(\mu)$ depends on the order parameter μ. Inserting this solution into (18) yields a self-consistent state equation of the form

$$\mu = F(\mu)\,, \tag{21}$$

where

$$F(\mu) = \frac{\int\limits_{-\infty}^{\infty} x\, \exp\left[-(U(x,T) - \lambda\mu x)/D(T)\right] dx}{\int\limits_{-\infty}^{\infty} \exp\left[-(U(x,T) - \lambda\mu x)/D(T)\right] dx}\,, \tag{22}$$

with the intensity of thermal fluctuations $D = D(T)$ being $\propto T$, see below Eq. (11).

5. Analysis of the nonlinear state equation

An analysis of the state equation for the case that $\phi_{\rm ext} = 0$, *i.e.* when the external magnetic field $B = 0$, has been presented in [13]. Therein, it has been shown that the so-called *flux state*, characterized by the non-vanishing mean magnetic flux $\mu = \langle x \rangle \neq 0$, can occur even if the external magnetic field $B = 0$. On the parameters plane $(T > 0, \lambda > 0)$ there is a monotonically increasing line, starting from the origin $(0, 0)$ below which the flux state appears, *i.e.* a finite, noise-induced flux emerges for sufficiently low temperatures and sufficiently strong coupling. Here, we focus on the case $B \neq 0$ with B being a uniform static magnetic field parallel to the axis of the coaxially formed cylinders. This means that the parameter σ in the potentials (13) and (19) becomes $\propto \phi_{\rm ext} \propto B$.

We next compare and contrast the properties of the coupled and the uncoupled system, respectively. Such an analysis can serve as a guideline for interpreting possible experimental results.

In the absence of mutual coupling the dynamics of the system is characterized by the generalized potential $V(x, T)$ given by (13). It depends strongly on both, the temperature T and the external flux σ which is imposed onto the system, see Fig. 1. Its shape changes from the symmetrical bistable form for $\sigma = 0$ to the asymmetrical, monostable form for $\sigma \in (0, 1/4)$. Next, for $\sigma = 1/4$ it assumes a symmetric monostable shape. For $\sigma \in (1/4, 1/2)$ it becomes again asymmetric and monostable. Finally, for $\sigma = 1/2$ it is symmetric bistable. This symmetry follows from (13) and can be recast as

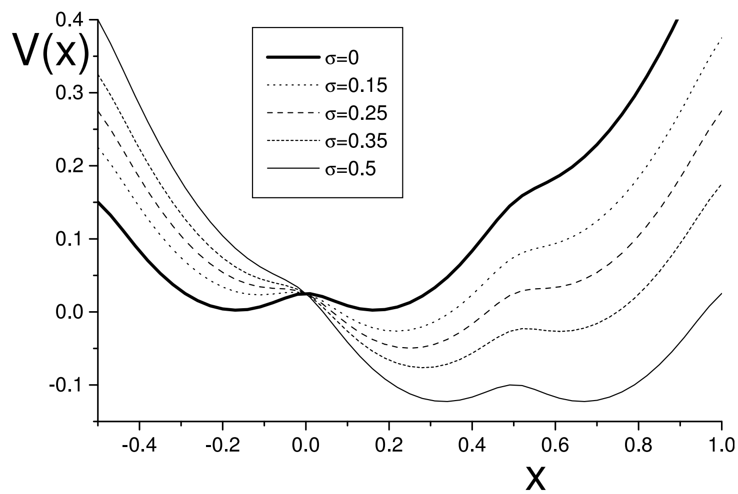

Fig. 1. The generalized potential (13) of the uncoupled system is depicted for several values of the external flux σ. The remaining values of the parameters are: $T/T^* = 0.5$, $D = 0.001T/T^*, I_0 = 1, P = 1/2$ in (11) and $k_{\rm F} l_x = 0.1$ in (6).

the relation $V(x, \sigma + 1/2) = V(x - 1/2, \sigma) - 1/8$. It indicates a kind of periodicity with a period $L = 1/2$.

In Fig. 2 we present the temperature dependence of the stationary mean magnetic flux μ for the case of coupled ($\lambda \neq 0$) and the uncoupled ($\lambda = 0$) cylinders. For uncoupled cylinders and in the absence of an external driving ($\sigma = 0$), the magnetic flux is $\mu = 0$, as expected. Switching on the external magnetic flux induces a non-zero value of the mean flux threading the cylinders. At high temperature, there is no coherent current and the system behaves asymptotically Gaussian. In this high temperature limit we have $\mu = \sigma$, both for the case of coupled and the uncoupled cylinders. At low temperatures, a small change of the externally induced magnetic flux σ results in a relatively large value of μ. This "amplification" of μ with respect to σ at small temperatures is steep but continuous, see in Fig. 3. This drastic increase is caused by the coherent current flowing in the system because in the low temperature limit the susceptibility of coherent electrons (represented by the flux-derivative of the current-flux characteristics) approaches infinity. These effects are present as well for the case of a coupled system (not depicted). In the whole temperature regime, the non-zero coupling between cylinders results in the increase of the mean flux μ. At low temperatures, the mean flux in the system approaches a finite value, indicating the possible collective behavior in the absence of an external driving, see in Ref. [13].

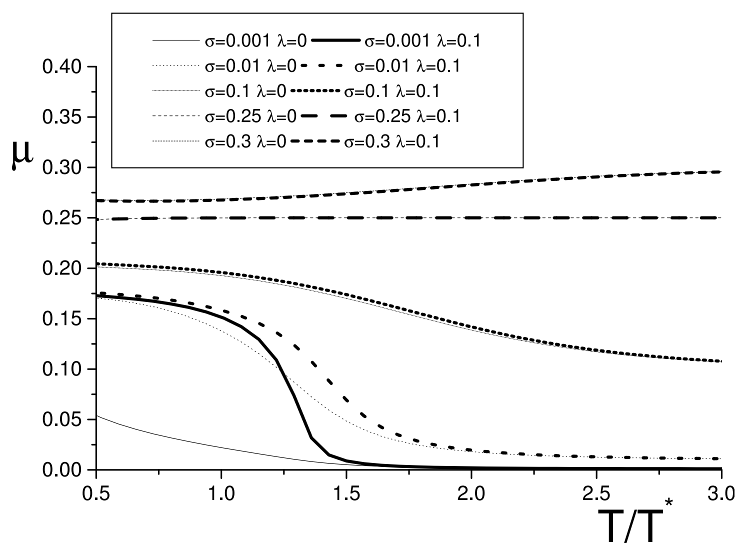

Fig. 2. The mean flux threading one cylinder for a coupled (set of nonvanishing coupling constants λ's) and an uncoupled ($\lambda = 0$) system for several values of the re-scaled externally induced flux σ, for $a = 1.02$ and $\gamma = a - \lambda$. The values of the remaining parameters are the same as in Fig. 1.

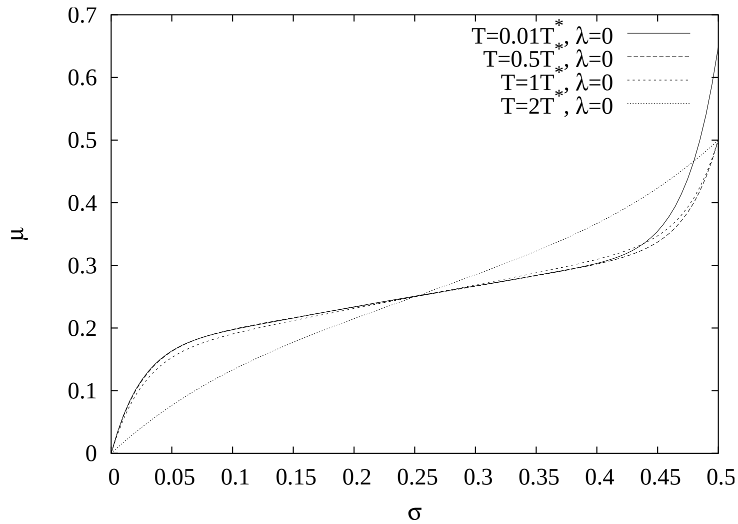

Fig. 3. The mean flux μ treading the uncoupled ($\lambda = 0$) cylinder at low temperatures is shown *vs.* the external magnetic flux σ. The remaining values of the parameters are as in Fig. 1.

Due to the intricate temperature dependence of the generalized potential caused by the coherent part (11), the system can exhibit an unusual behavior: We note that for $\sigma = 0.25$ (one half of the period $L = 1/2$) the mean flux μ essentially becomes independent of temperature, *i.e.* it depends very weakly on T. This value separates two regimes, *cf.* Fig. 2: for $\sigma < 0.25$ the mean flux is a decreasing function of the temperature while for $\sigma > 0.25$ it becomes an increasing function of temperature. This constitutes a temperature-induced effect in equilibrium which leads to an increase of the persistent current in the mesoscopic system. Similar effects are known to occur in mesoscopic systems, but there are typically caused by non-equilibrium sources of fluctuations [14]. Notably, this effect is preserved for the case of coupled cylinders with $\lambda > 0$.

Another measurable quantity of foremost interest and which is strongly affected by both, the external field σ and the non-zero coupling λ is the variance of the order parameter. This measure of the strength of fluctuations is depicted with Fig. 4. Its behavior at low and high temperature is generic: as temperature increases, fluctuations increase as well. At sufficiently high temperatures we observe — regardless of the coupling strength — the expected linear dependence, being typical for a Gaussian behavior. However, at moderate temperatures and small σ, there is a regime of the reduction of fluctuations: although temperature increases, fluctuations decline. It is a case when the potential changes its form from the bistable to

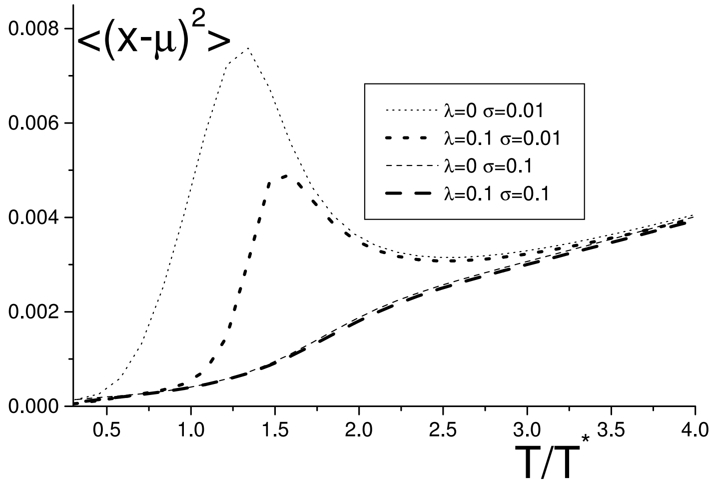

Fig. 4. The fluctuations of the order parameter are depicted for several values of the externally induced magnetic flux. The values of the remaining parameters have been chosen as in Fig. 1. In particular, the temperature is chosen to be below the characteristic temperature, *i.e.* $T/T^* = 0.5$.

monostable shape. In the presence of finite coupling, it corroborates the conjecture stated in [13] that the coupled system undergoes a noise-induced transition into an energetically favorable state, a finite flux state.

6. Conclusions

In conclusion, we have studied the behavior of a linear chain of interacting mesoscopic cylinders in a finite, uniform magnetic filed B. The influence of this externally applied magnetic field causes a rich and complex thermal noise assisted flux behavior, both for the case of uncoupled and coupled cylinders. Our study models the common experimental set-up when investigating persistent currents and flux in mesoscopic systems at low temperatures. The properties of the total current flowing in an individual cylinder follow readily from the results of the magnetic flux via the inversion of the relation in (1).

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