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QUANTUM STATISTICS AND MULTIPLE PARTICLE PRODUCTION* **

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Effects of quantum statistics are clearly seen in the final states of high-energy multiparticle production processes. These effects are being widely used to obtain information about the regions where the final state hadrons are produced. Here we briefly present and discuss the assumptions underlying most of these analyses.

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1. Introduction

Differences between quantum and classical (Boltzmann) statistics show up in a variety of systems. Here we will discuss systems, where the energy per particle is of the order of 100 MeV or more. In such systems, produced in high energy scattering processes, correlations due to quantum statistics are clearly seen in the data: Identical bosons seem to attract each other, identical fermions seem to repel each other. The quantitative description of these effects is interesting both for their own sake and in order to disentangle other more subtle correlations.

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There is still another motivation for this study, however, which is somewhat controversial, but very stimulating. Much work has been done on quantum statistics in multiple particle production processes. For reviews see *e.g.* [1] and [2]. According to most of this work, the study of interparticle correlations due to quantum statistics yields valuable information about interaction regions, *i.e.* about the regions where the hadrons are produced. The interaction regions are difficult to study, because they are both small and short-lived. Their typical sizes are of the order of fermis and also their life-times are of the order of fermis. A fermi in time is the time necessary to cross the distance of one fermi at the speed of light *i.e.* about 3×10^{-24} sek. Very few methods for studying interactions regions are available.

An added attraction is that the predictions from the recognized theories for the new data from the heavy ion collider RHIC turned out to be completely wrong (references can be traced *e.g.* from [3]). The name coined for this disaster is *the RHIC puzzle*. Consequently, both the general theoretical framework and the specific phenomenological assumptions have to be reanalyzed. In this paper we will present the basic assumptions of the most popular models.

In low energy scattering, *i.e.* when the center-of-mass kinetic energy of the two colliding particles is of the order of 1 GeV, when several particle are produced their distribution is roughly spherically symmetric. The first model to gain wide popularity, Fermi's model, used a slightly modified microcanonical distribution. For each multiplicity of particles, each final state with total energy and total momentum equal to their initial values was assumed to be equally probable. At higher energies spherical symmetry breaks down and the model does not work any more. At low energies, however, it seemed good until paper [4] got published.

In this paper the authors studied the opening angles *i.e.* the angles between the momenta for pairs of pions. From Coulomb interactions one would qualitatively expect that for pairs of like-sign charged pions the opening angles would be on the average smaller than for pairs of unlike-sign charged pions. Quantitatively, however, one can easily estimate that this effect is rather small. The authors expected, nevertheless, a similar effect, because it had been predicted that in the $\pi^+\pi^-$ system there is a resonance, now known as the ρ meson, while for like sign pairs no resonance had been expected. To their surprise, the experimental result was just the opposite. The opening angles for pairs of like-sign pions tended to be smaller. The result got explained in the seminal paper [5] as a result of the Bose–Einstein statistics of the identical pions. For many years the effect had been known as the GGLP effect. We will review the GGLP paper in the following section, but now let us discuss high energy scattering.

We will consider central heavy ion (*e.g.* gold–gold) scattering at center-of-mass energies of the order of 100 GeV per pair of colliding nucleons. In such collisions, in the center-of-mass system, due to Lorentz contraction, both nuclei take the form of thin pancakes. When the two pancakes fly through each other there are many nucleon–nucleon interactions, but at this energies the directions of flight of the nucleons change little and the pancakes survive. When they fly away from each other, many strings are formed and stretch connecting the color charges in one nucleon with the opposite color charges in the other. These string are a characteristic feature of quantum chromodynamics. When two opposite electric charges interact, the well-known field of forces extends over all space. In quantum chromodynamics the corresponding field is confined to a thin (diameter of the order of one fm) tube with the opposite color charges at the two ends. In a high energy central heavy ion collision many strings are produced and exist simultaneously. It is plausible that they merge and produce a well-defined, roughly cylindrical region with the two pancakes at its ends. This region is presumably first filled with quarks, antiquarks and gluons. Only later hadrons, mostly pions, emerge from it.

There is a number of questions one would like to ask. What is the size and shape of this region? What is its life-time and what is the duration of hadronization? Note that the life-time and the duration of hadronization are in general different. For instance, the region could live for 8 fm without emitting hadrons and then emit all the hadrons within 2 fm. If the content of the region can be considered a phase, it would be interesting to know its equation of state. This would be valuable information for cosmological models of the early stages of the Universe expansion and for models of the interiors of neutron stars — perhaps some of them have quark–gluon cores. Another set of questions is about the transition of this stuff into hadrons: is it a phase transition or a cross-over? If it is a phase transition, is it first order or continuous? If it is first order, what is the latent heat?

2. GGLP or HBT or BE correlations

Let us start this section with a remark about terminology. Starting from the seventies, the original acronym GGLP was being gradually replaced by the acronym HBT in honor of Hanbury-Brown and Twiss, who several years before the GGLP paper [5] had made a related discussion in astronomy [6]. They used successfully the Bose–Einstein correlations among photons to measure the diameters of distant stars. Now the name GGLP is hardly ever used. Kopylov and Podgoretskii explained [7] how, starting from a more general formulation, one can obtain the GGLP case by making a parameter tend to zero and the HBT case by making the same parameter tend to infinity. Thus, GGLP and HBT are two different limiting cases. In fact, some people replace HBT by the more neutral BEC standing for Bose–Einstein correlations.

In order to facilitate comparison with subsequent developments, we will present the GGLP results using a more general notation. GGLP start with an input single particle density operator — not to be confused with the actual single particle density operator for the system — in the form

$$\hat{\rho}_I = \int d^3x |\boldsymbol{x}\rangle \rho(\boldsymbol{x}) \langle \boldsymbol{x}|. \tag{1}$$

This corresponds to particles being produced incoherently from point sources. Function $\rho(\boldsymbol{x})$ gives the distribution of the sources in space. In this model the full information about the size and shape of the interaction region is given when function $\rho(\boldsymbol{x})$ is known. The corresponding input single particle density matrix reads

$$\rho_I(\boldsymbol{p};\boldsymbol{p}') = \int \rho(\boldsymbol{x}) e^{i\boldsymbol{q}\boldsymbol{x}}, \tag{2}$$

where $\boldsymbol{q} = \boldsymbol{p} - \boldsymbol{p}'$. Note that given the density matrix $\rho_I(\boldsymbol{p};\boldsymbol{p}')$ one can unambiguously obtain the distribution of sources $\rho(\boldsymbol{x})$ just by inverting the Fourier transformation. Unfortunately, this nice feature of the theory will be lost, when the theory is made more realistic.

Interpreting the input density matrix as a density matrix, one would obtain the single particle momentum distribution

$$\Omega_1(\boldsymbol{p}) = \rho_I(\boldsymbol{p};\boldsymbol{p}) = \int d^3x \rho(\boldsymbol{x}) = 1. \tag{3}$$

This is obviously unrealistic. *E.g.* very large momenta are forbidden by energy conservation. GGLP, however, interpreted $\Omega_1(\boldsymbol{p})$ as a weight for the states allowed by energy and momentum conservation. Thus, the result just means that the single particle distributions should be calculated from Fermi's model.

For pairs of identical particles, in general,

$$\Omega_2(\boldsymbol{p}_1,\boldsymbol{p}_2) \neq \rho_I(\boldsymbol{p}_1;\boldsymbol{p}_1)\rho_I(\boldsymbol{p}_2;\boldsymbol{p}_2), \tag{4}$$

because the right-hand side does not have the right symmetry with respect to the exchange of $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$. GGLP symmetrized it to get

$$\Omega_2(\boldsymbol{p}_1,\boldsymbol{p}_2) = \rho_I(\boldsymbol{p}_1;\boldsymbol{p}_1)\rho_I(\boldsymbol{p}_2;\boldsymbol{p}_2) + \rho_I(\boldsymbol{p}_1;\boldsymbol{p}_2)\rho_I(\boldsymbol{p}_2;\boldsymbol{p}_1). \tag{5}$$

For instance, substituting for the density distribution of sources

$$\rho(\boldsymbol{x}) = \frac{1}{\sqrt{2\pi R^2}^3} e^{-\boldsymbol{x}^2/2R^2} \tag{6}$$

one finds

$$\Omega_2(\boldsymbol{p}_1, \boldsymbol{p}_2) = 1 + e^{-R^2 \boldsymbol{q}^2}. \tag{7}$$

Again, as a distribution of momenta this is untenable, but interpreted as a weight it nicely reproduces the enhancement at small momentum differences seen in the data.

The GGLP paper has been very influential. It still is the most quoted paper in the field. It has, however, its weak points. On the technical side, it is easy to calculate the weights, but then the integration over momentum space is needed. In the usual case, when neither the nonrelativistic approximation, nor the ultrarelativistic approximation (all masses tending to zero) is justified, this is a cumbersome task. In practice, as far as I know, no one got with this approach beyond three identical particles, while in heavy ion collisions, at high energies hundreds of identical particles are being produced. Moreover, in order to calculate the momentum integral, one needs the exact numbers of all the other particles produced and this is usually not available. A reasonable approximate solution of this problem has been found and will be presented in the next section.

Also the physics behind the model is doubtful. Time does not appear explicitly. As easily checked, this corresponds to the assumption that all the identical particles are produced instantly and simultaneously at some time t_0. This is a most unlikely scenario. Just reflect at the question, in which reference frame this assumption should be satisfied. Moreover, classically one would like to find the probability distribution for a particle being produced at point $\boldsymbol{x}$ with momentum $\boldsymbol{p}$. In order to get consistency with quantum mechanics one has to compromise. In the GGLP model the particle is produced at point $\boldsymbol{x}$ and, in agreement with quantum mechanics, its momentum can be anything with equal probability. This is then modified by rejecting the states forbidden by energy and/or momentum conservation. It is much more likely, however, that the particles are produced as wave packets with some finite variances of position and momentum.

3. Two particle (reduced) correlation functions

Consider a two-particle correlation function

$$C_2(\boldsymbol{p}_1, \boldsymbol{p}_2) = \frac{\Omega_2(\boldsymbol{p}_1, \boldsymbol{p}_2)}{\Omega_{2\text{bckg}}(\boldsymbol{p}_1, \boldsymbol{p}_2)}. \tag{8}$$

Here Ω_2 is the experimental two-body momentum distribution and $\Omega_{2\text{bckg}}$ is the corresponding distribution with the Bose–Einstein correlations switched off. The latter cannot be obtained from the data without further assumptions, but the experimental groups have various methods for getting reasonable approximations to it. It would be more in agreement with the terminology used in statistics to put into the denominator the product of single particle distributions instead, but the correlation function defined here is more convenient to study separately the correlations due to quantum statistics. Assuming that this correlation function can be calculated from the GGLP approach without phase space integrations, one finds

$$\Omega_2(\boldsymbol{p}_1,\boldsymbol{p}_2) = \Omega_{2\mathrm{bckg}}(\boldsymbol{p}_1,\boldsymbol{p}_2)\left(1+\frac{|\rho(\boldsymbol{p}_1,\boldsymbol{p}_2)|^2}{\rho(\boldsymbol{p}_1,\boldsymbol{p}_1)\rho(\boldsymbol{p}_2,\boldsymbol{p}_2)}\right). \tag{9}$$

From now on we skip the subscript I, though ρ, strictly speaking, is not quite the single particle density matrix of the system. Formula (9), which has been proposed by Kopylov [8], is plausible and can be easily used for comparison of models with experiment. One measures Ω_2 and divides it by the estimated $\Omega_{2\mathrm{bckg}}$ to obtain the experimental C_2. This is compared with the C_2 calculated from the model. The procedure is approximate, but it avoids the integration over momentum space. Thus, the information how many and what kinds of particles have been produced is not needed.

For instance, for the Gaussian input density (6) the calculated correlation function is

$$C_2(\boldsymbol{p}_1,\boldsymbol{p}_2) = 1 + e^{-R^2\boldsymbol{q}^2}. \tag{10}$$

Comparing it with the data one obtains the root-mean-square radius of the interaction region R. Here by assumption this region is spherically symmetric. A natural generalization [9, 10] is to replace the product R^2q^2 in the exponent by $q_\mathrm{L}^2R_\mathrm{L}^2 + q_0^2R_0^2 + q_\mathrm{s}^2R_\mathrm{s}^2$, where q_L, q_0 and q_s are, respectively, the components of $\boldsymbol{q}$ along the beam axis and along some two axes perpendicular to it. Since the maximum length of the strings, achieved just before they break, increases with energy, one would expect for central collisions at high energies $R_\mathrm{L} \gg R_0 \approx R_\mathrm{s}$. Experimentally it is observed that R_L is comparable with R_0 and R_s. This has been explained as follows [18]. The correlations due to Bose–Einstein statistics are visible only for values of $|\boldsymbol{q}|$ of the order of R^{-1} or less. Therefore, they can be used only to measure the size of the region where particles with similar momenta are produced. This region, named by Sinyukov the homogeneity region, is in general smaller than the total interaction region.

The correction of the physical assumptions is much harder and will be discussed in the following sections. Essentially there are two strategies: the method of wave packets, with its close relative the method of covariant currents, and the methods inspired by the concept of the Wigner function. This leaves aside the approaches where a complete model is proposed, which can be used to calculate anything. In particular it may be used to calculate the correlations among the identical particles at small $|\boldsymbol{q}|$ whether or not they have something to do with some interaction region. For an example in this category *cf. e.g.* [11].

4. Wave packets and covariant currents

Let us replace the GGLP single particle input density operator by

$$\hat{\rho} = \int d^4x_{\rm s} \int d^4p_{\rm s} |\psi_{x_{\rm s}p_{\rm s}}\rangle \rho(x_{\rm s}, p_{\rm s}) \langle \psi_{x_{\rm s}p_{\rm s}}|, \tag{11}$$

which corresponds to the single particle input density matrix

$$\rho(\boldsymbol{p};\boldsymbol{p}') = \int d^4x_{\rm s} \int d^4p_{\rm s} \psi_{x_{\rm s}p_{\rm s}}(\boldsymbol{p}) \rho(x_{\rm s}, p_{\rm s}) \psi_{x_{\rm s}p_{\rm s}}(\boldsymbol{p}'). \tag{12}$$

Here the time dependence of the density matrix is not explicitly written. Function $\rho(x_{\rm s}, p_{\rm s})$ is the distribution of the space-time four-vectors and the energy-momentum four-vectors defining the sources. The distribution of particles, however, is consistent with quantum mechanics because of the wave functions $\psi_{x_{\rm s}p_{\rm s}}$. There are various ways of using this scenario.

Kopylov and Podgoretskii, who introduced it [12], assumed that the sources differ only by their positions in space–time. Thus

$$\psi_{x_{\rm s}p_{\rm s}}(\boldsymbol{p}) = e^{ipx_{\rm s}} \phi(\boldsymbol{p}), \tag{13}$$

where the fourth component of p, necessary to calculate the product $px_{\rm s}$, is given by $p_0 = \sqrt{\boldsymbol{p}^2 + m^2}$ and thus, is not an independent variable. The density matrix is

$$\rho(\boldsymbol{p};\boldsymbol{p}') = \phi(\boldsymbol{p})\phi^*(\boldsymbol{p}') \int d^4x_{\rm s} \int d^4p_{\rm s} \rho(x_{\rm s}, p_{\rm s}) e^{iqx_{\rm s}} \tag{14}$$

and

$$\Omega_1(\boldsymbol{p}) = |\phi(\boldsymbol{p})|^2. \tag{15}$$

This is the first success. The model can reproduce perfectly any single particle momentum distribution. On the other hand

$$C_2(\boldsymbol{p}_1, \boldsymbol{p}_2) = 1 + |\langle e^{iqx_{\rm s}} \rangle|^2, \tag{16}$$

where the averaging is over the distribution of sources $\rho(x_{\rm s}, p_{\rm s})$. Here functions $\phi(\boldsymbol{p})$ cancel. One way of calculating the distribution $\rho(p_{\rm s}, x_{\rm s})$ is to start with some initial distribution and then to propagate it, finding from some classical equations, *e.g.* Newton's or Boltzmann's, the functions $x_{\rm s}(t)$ and $p_{\rm s}(t)$.

Another variant, known as the method of covariant currents [13], is to put

$$\psi_{x_{\rm s}p_{\rm s}}(\boldsymbol{p}) = e^{ipx_{\rm s}} j\left(\frac{p_{\rm s}p}{m_{\rm s}}\right), \tag{17}$$

where $m_{\rm s}$ is the mass of the source, usually put equal to the particle mass. In the rest frame of its source each current reduces to the same function $j(p_0)$. In this approach the assumption of Kopylov and Podgoretskii that each source yields particles with the same momentum distribution in some common frame, *e.g.* in the center-of-mass frame, is replaced by the more plausible assumption that the momentum distribution for particles from any source looks the same in the rest frame of this source.

5. Wigner functions and their generalizations

In statistical physics there is a well-known method of including simultaneously positions and momenta. One uses the Wigner function related to the density matrix in the momentum representation by the formula [14, 15]

$$W(\boldsymbol{X},\boldsymbol{K}) = \int \frac{d^3q}{(2\pi)^3}\rho(\boldsymbol{K}+\tfrac{1}{2}\boldsymbol{q},\boldsymbol{K}-\tfrac{1}{2}\boldsymbol{q})e^{i\boldsymbol{q}\boldsymbol{X}}, \tag{18}$$

where

$$K = \tfrac{1}{2}(p+p'); \qquad X = \tfrac{1}{2}(x+x'). \tag{19}$$

Note that, for further use, K and X are defined as four-vectors, but in the definition of the Wigner function only three-vectors and a fixed time argument (not written explicitly) appear. The Fourier transformation can be inverted, so that there is a one to one relation between the density matrix and the corresponding Wigner function. The introduction of the Wigner function solves the problem when the production of all the particles is simultaneous at some time t. When the particles are produced in a time interval $[t_1, t_2]$, but so that the particles produced at different instants of time do not interfere, one can overage over time, with suitable weights, both sides of the equation. The time averaged density matrix, which one could hope to measure, is related to the time averaged Wigner function, but the important information about the time distribution of the production process is lost. When the particles produced at different instants of time interfere, even more information is lost. It is possible to define objects related by Fourier transformations to the various components contributing to the density matrix, but from the point of view of interpretation they are very different from Wigner functions [16].

Formally, one can write [14]

$$\rho(\boldsymbol{p};\boldsymbol{p}') = \int d^4 X e^{iqX} S(X,K), \tag{20}$$

where function $S(X,K)$ is known as the emission function. One of the difficulties is that the four-dimensional Fourier transformations cannot be inverted. The reason is that for fixed K, from $Kq = (p^2 - p'^2)/2 = 0$, one finds

$$q_0 = \frac{\boldsymbol{K}\boldsymbol{q}}{K_0}, \tag{21}$$

while in order to invert the Fourier transformation one needs $\rho(\boldsymbol{p};\boldsymbol{p}')$, at given K, for all values of $\boldsymbol{q}$ and q_0. In fact, this difficulty is general and appears as soon as we introduce time-dependent sources. Thus, there is an infinity of emission functions $S(X,K)$ which yield the same density matrix $\rho(\boldsymbol{p};\boldsymbol{p}')$.

For instance one could put

$$S(X,K) = \delta(X_0 - t)e^{-iq_0X_0}W(\boldsymbol{X},\boldsymbol{K};t). \tag{22}$$

This formula is correct in the sense that, as easily seen using the inverse of transformation (18), it yields the correct density matrix. It is, however, completely useless, because guessing this Wigner function is just as hard as guessing the density matrix in the momentum representation. Actually, this formula has a simple physical interpretation. Let us choose a moment of time t_0, when all the final hadrons are already present and do not interact any more. From this time on the density matrix for each hadron is

$$\rho(\boldsymbol{p},\boldsymbol{p}';t) = e^{-iq_0(t-t_0)}\rho(\boldsymbol{p},\boldsymbol{p}';t_0). \tag{23}$$

Thus, it is enough to know $\rho(\boldsymbol{p},\boldsymbol{p}';t_0)$ to predict all that happens later. What had happened earlier is irrelevant for this prediction. One can just as well assume that all the hadrons got created at $t = t_0$. Physically this is a stupid assumption, but formally it is good enough to predict the states of the system at later times.

One could ask what is the "reasonable" emission function corresponding to the realistic picture of particle emission. A formula for this function has been proposed by Shuryak [17]. Let us consider first the source labeled i. If it created particles in the pure state $A_i(\boldsymbol{p})$, the density matrix for the particles produced from this source would be $A_i(\boldsymbol{p})A_i^*(\boldsymbol{p}')$. Averaging over the parameters of the source, one obtains the density matrix for the particles produced by source i without the assumption that the state is pure.

Summing over the sources one obtains the overall single particle density matrix

$$\rho(\boldsymbol{p},\boldsymbol{p}') = \sum_i \langle A_i(\boldsymbol{p})A_i^*(\boldsymbol{p}')\rangle. \tag{24}$$

Expressing the amplitudes $A_i(\boldsymbol{p})$ in terms of their Fourier transforms $J_i(x)$ one can write for source i

$$\langle A_i(\boldsymbol{p})A_i^*(\boldsymbol{p}')\rangle = \int d^4X \int d^4Y e^{iqX+iKY} \langle J_i(X+\tfrac{1}{2}Y)J_i^*(X-\tfrac{1}{2}Y)\rangle, \tag{25}$$

where $Y = x - x'$. Summing over i and introducing the notation

$$\langle J(X+\tfrac{1}{2}Y)J^*(X-\tfrac{1}{2}Y)\rangle = \sum_i \langle J_i(X+\tfrac{1}{2}Y)J_i^*(X-\tfrac{1}{2}Y)\rangle \tag{26}$$

one finds

$$\rho(\boldsymbol{p},\boldsymbol{p}') = \int d^4X \int d^4Y e^{iqX+iKY} \langle J(X+\tfrac{1}{2}Y)J^*(X-\tfrac{1}{2}Y)\rangle. \tag{27}$$

Comparing this formula with the (ambiguous!) definition of the emission function (20) one finds

$$S(X,Y) = \int d^4Y e^{iKY} \langle J(X+\tfrac{1}{2}Y)J^*(X-\tfrac{1}{2}Y)\rangle. \tag{28}$$

The strategy is to guess the emission function, with some free parameters, using all the available information about the sources. Then, one calculates the density matrix and the correlation function C_2 in order to fix the parameters by comparison with experiment. The weak point is that, since there is an infinite variety of different emission functions which all give the same density matrix, a good fit to the data does not necessarily mean that the models being used, and consequently the parameters obtained, make sense.

6. Conclusions

The single particle momentum distribution gives the diagonal element of the single particle density matrix. Under some simplifying assumptions the correlations due to quantum statistics give information about the out of diagonal elements of this matrix. Even if we could find from experiment the complete single particle density matrix in the momentum representation, which is not quite the case, this is not enough to find unambiguously the space-time distribution of the sources. On the other hand, given a good model, it is possible to fit the experimental data related to the density matrix and thus find some parameters of the model.

The classical full description: for each particle find its momentum and the point in space-time where the particle has been produced, cannot be achieved respecting the rules of quantum mechanics. Two popular compromises are: use the positions and momenta of the sources instead of the positions and momenta of the particles or use X and $\boldsymbol{K}$ instead of x and $\boldsymbol{p}$. Given the general framework one needs some phenomenological assumptions to fill it. An example, admittedly not a very realistic one, is to assume that all the particles are produced simultaneously from a Gaussian distribution of point sources. In practice, of course, much more sophisticated phenomenology is being used.

For quantitative work one must include a number of complicating features, which have been ignored here for the sake of clarity and of economy of time. We give below a partial list. More information can be found in the reviews [1, 2].

- Many body symmetrization.
- Final state interactions.
- Production of resonances.
- Momentum–position correlations.
- Evolution of the interaction region before and during hadronization.

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