DIAGONAL MATRIX ELEMENTS OF THE EFFECTIVE HAMILTONIAN FOR $K^0 - \bar{K}^0$ SYSTEM IN ONE POLE APPROXIMATION

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We study the properties of time evolution of the K^0–$\bar{K}^0$ system in spectral formulation. Within the one-pole model we find the exact form of the diagonal matrix elements of the effective Hamiltonian for this system. It appears that, contrary to the Lee–Oehme–Yang (LOY) result, these exact diagonal matrix elements are different if the total system is CPT-invariant but CP-noninvariant.

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1. Introduction

This paper has been inspired by the results presented in [1] and [2]. Paper [1] analyses the problem of equality of particle and antiparticle masses, whereas [2] describes an exactly solvable model of the particle–antiparticle system — in this particular case $K^0 - \bar{K}^0$. The most important properties of antiparticles follow from the CPT symmetry. This symmetry, also known as the CPT theorem [3], determines the properties of the transition amplitudes under the action of the product of C, P and T transformations (charge conjugation, space inversion and time reversal, respectively). According to the CPT theorem, the transition amplitudes describing any physical process must be CPT-invariant. From this we conclude that the full Hamiltonian H of the system under consideration must be invariant under the product of the $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$ operators. Another conclusion which can be drawn here is that stable particles and their antiparticles must have the same mass. This property of the particle–antiparticle pair is true for stable particles and the same is usually assumed of unstable particles (*e.g.* K^0 and $\bar{K}^0$). Such an extension of a law true for stable particles to unstable particles is questioned in [1]. The reason for the widespread belief that this extension is correct is most probably the Lee, Oehme and Yang (LOY) approximation and the conclusions which follow from it — and more specifically, the properties of the effective Hamiltonian, $H_{\rm LOY}$, governing the time evolution in the subspace $\mathcal{H}_{\|}$. In our case $\mathcal{H}_{\|}$ is the subspace of the total Hilbert space of states $\mathcal{H}$, spanned by state vectors of K^0, $\bar{K}^0$ mesons.

Following the LOY approach, a nonhermitian Hamiltonian $H_{\|}$ is usually used to study the properties of the particle–antiparticle unstable system [4–9]

$$H_{\|} \equiv M - \frac{i}{2}\mathit{\Gamma}\,, \tag{1}$$

where

$$M = M^{+}\,, \quad \mathit{\Gamma} = \mathit{\Gamma}^{+} \tag{2}$$

are (2×2) matrices acting in $\mathcal{H}_{\|}$. The M-matrix is called the mass matrix and $\mathit{\Gamma}$ is the decay matrix. Lee, Oehme and Yang derived their approximate effective Hamiltonian $H_{\|} \equiv H_{\rm LOY}$ by adapting the one-dimensional Weisskopf–Wigner (WW) method to the two-dimensional case corresponding to the neutral kaon system. Almost all properties of this system can be described by solving the Schrödinger-like equation

$$i\frac{\partial}{\partial t}|\psi;t\rangle_{\|} = H_{\|}|\psi;t\rangle_{\|}\,, \qquad (t \geq t_0 > -\infty)\,, \tag{3}$$

(where we have used $\hbar = c = 1$). The initial conditions for Eq. (3) are

$$\|\;|\psi;t=t_0\rangle_{\|}\;\| = 1\,, \quad |\psi;t_0=0\rangle_{\|} = 0\,, \tag{4}$$

where $|\psi;t=t_0\rangle_{\|}$ belongs to $\mathcal{H}_{\|}$ ($\mathcal{H}_{\|} \subset \mathcal{H}$) and $\mathcal{H}_{\|}$ is spanned by orthonormal neutral kaons states: $|K^0\rangle \equiv |\mathbf{1}\rangle$, $|\bar{K}^0\rangle \equiv |\mathbf{2}\rangle$. Thus $\mathcal{H}_{\|} = P\mathcal{H}$, where

$$P \equiv |\mathbf{1}\rangle\langle\mathbf{1}| + |\mathbf{2}\rangle\langle\mathbf{2}\,|\,. \tag{5}$$

According to the standard result of the LOY approach, in a CPT invariant system, *i.e.* when

$$\Theta H \Theta^{-1} = H, \tag{6}$$

(where $\Theta = \text{CPT}$) we have

$$h_{11}^{\rm LOY} = h_{22}^{\rm LOY} \tag{7}$$

and

$$M_{11}^{\rm LOY} = M_{22}^{\rm LOY}, \tag{8}$$

where: $M_{jj}^{\mathrm{LOY}} = \Re(h_{jj}^{\mathrm{LOY}})$ and $\Re(z)$ denotes the real part of a complex number z (then $\Im(z)$ is the imaginary part of z), and $h_{jj}^{\mathrm{LOY}} = \langle \boldsymbol{j}|H_{\mathrm{LOY}}|\boldsymbol{j}\rangle$ $(j = 1, 2)$.

The universal properties of the two particles subsystem described by the H fulfilling the condition (6), may be investigated by the use of the matrix elements of the exact evolution operator for $\mathcal{H}_{||}$ instead of the approximate one used in the LOY theory. This exact evolution operator, $U_{\|}(t)$, can be written as follows: $U_{\|}(t) = PU(t)P$, where $U(t) \equiv e^{-itH}$ is the exact evolution operator acting in the total state space $\mathcal{H}$.

Assuming that the CPT symmetry is conserved in the system under considerations one finds that the matrix elements

$$A_{jk}(t) = \langle \boldsymbol{j}|U_{\|}(t)|\boldsymbol{k}\rangle \equiv \langle \boldsymbol{j}|U(t)|\boldsymbol{k}\rangle \quad (j,k = 1,2)\,, \tag{9}$$

of the exact $U_{\|}(t)$, obey

$$A_{11}(t) = A_{22}(t)\,. \tag{10}$$

General conclusions concerning the properties of the difference of the diagonal matrix elements $(h_{11} - h_{22})$ of the exact $H_{\|}$, (which can in general depend on time t [10]), where

$$h_{jk} = \langle \boldsymbol{j}|H_{\|}|\boldsymbol{k}\rangle \quad (j,k = 1,2) \tag{11}$$

can be drawn by analyzing the following expression derived in [1] for CPT invariant systems

$$h_{11}(t) - h_{22}(t) \equiv \frac{i}{\det \boldsymbol{A}(t)} \left(\frac{\partial A_{21}(t)}{\partial t} A_{12}(t) - \frac{\partial A_{12}(t)}{\partial t} A_{21}(t) \right). \tag{12}$$

In [1] it is shown that

$$h_{11}(t) - h_{22}(t) \neq 0 \tag{13}$$

when (6) holds and

$$[\mathcal{CP}, H] \neq 0 \tag{14}$$

that is in the exact quantum theory the difference $(h_{11}(t) - h_{22}(t))$ cannot be equal to zero with CPT conserved and CP violated. In Section 3 we will consider this relation in the context of an exactly solvable model. This problem is important because realistic calculations are carried out with the use of simplified and approximate models. Not all of them conform to the requirements of the exact (not approximate) quantum theory.

The aim of this paper is to calculate the difference of the diagonal matrix elements of the effective Hamiltonian, (12), in a CPT invariant and CP noninvariant system for the approximate model analyzed in [2], that is in the case of the one-pole model based on the Breit–Wigner ansatz, *i.e.* the same model as used in Lee, Oehme and Yang theory.

The paper is organized as follows. In Section 2 we review briefly the spectral formulation for the neutral kaon system and a model described in [2]: one pole approximation. Section 3 investigates the diagonal matrix elements of the effective Hamiltonian and their difference in the CPT invariant and CP noninvariant system in the case of the one-pole model. In Section 4 we present our conclusions and we estimate the numerical result of the investigated difference for the $K^0 - \bar{K}^0$ system. Appendix A contains the relevant integrals and derivatives used in Section 3. In Appendix B we give the exact formulae for expressions appearing in Section 3.

2. The model: one pole approximation

While describing the two and three pion decay we are mostly interested in the $|K_{\rm S}\rangle$ and $|K_{\rm L}\rangle$ superpositions of $|K^0\rangle$ and $|\bar{K}^0\rangle$. These states correspond to the physical $|K_{\rm S}\rangle$ and $|K_{\rm L}\rangle$ neutral kaon states [2,11,12]

$$|K_{\rm S}\rangle = p|K^0\rangle + q|\bar{K}^0\rangle\,, \quad |K_{\rm L}\rangle = p|K^0\rangle - q|\bar{K}^0\rangle\,. \tag{15}$$

We assume that these physical states are the initial physical states of the CPT-invariant system, *i.e.* at the instant of creation of neutral kaons. We have

$$\begin{aligned}\langle K_{\rm S}|K_{\rm S}\rangle &= \langle K_{\rm L}|K_{\rm L}\rangle \equiv |p|^2+|q|^2 = 1\,, &(16)\\ \langle K_{\rm S}|K_{\rm L}\rangle &= \langle K_{\rm L}|K_{\rm S}\rangle \equiv |p|^2-|q|^2 \stackrel{\rm def}{=} \Delta_K \neq 0\,. &(17)\end{aligned}$$

The time evolution of K^0 and $\bar{K}^0$ can be concisely presented in the following way:

$$\begin{aligned}|K_\alpha(t)\rangle &= e^{-iHt}|K_\alpha\rangle\\ &\equiv A_{K_\alpha K_\alpha}(t)|K_\alpha\rangle + A_{K_\alpha K_\beta}(t)|K_\beta\rangle + Qe^{-iHt}|K_\alpha\rangle\,,\end{aligned} \tag{18}$$

where $K_\alpha = K^0, \bar{K}^0$ and H is the full hermitian Hamiltonian and $Q = I - P$,

$$A_{K_\alpha K_\beta}(t) = \langle K_\alpha|e^{-iHt}|K_\beta\rangle \equiv \langle K_\alpha|K_\beta(t)\rangle\,. \tag{19}$$

Let us notice that amplitudes $A_{K_\alpha K_\beta}(t)$ for $K_\alpha, K_\beta = K^0, \bar{K}^0$ correspond to the previously defined amplitudes[1] $A_{jk}(t)$, where $j,k = 1,2$, (9).

[1] Amplitudes $A_{K^0K^0}(t)$, *etc.*, correspond to $P_{K^0K^0}(t)$, *etc.* respectively used in [2].

Consequently we may write

$$\begin{aligned}
A_{K^0K^0}(t) &\equiv \langle K^0|e^{-itH}|K^0\rangle = \langle \mathbf{1}|e^{-itH}|\mathbf{1}\rangle \equiv A_{11}(t)\,,\\
A_{K^0\bar{K}^0}(t) &\equiv \langle K^0|e^{-itH}|\bar{K}^0\rangle = \langle \mathbf{1}|e^{-itH}|\mathbf{2}\rangle \equiv A_{12}(t)\,,\\
A_{\bar{K}^0K^0}(t) &\equiv \langle \bar{K}^0|e^{-itH}|K^0\rangle = \langle \mathbf{2}|e^{-itH}|\mathbf{1}\rangle \equiv A_{21}(t)\,,\\
A_{\bar{K}^0\bar{K}^0}(t) &\equiv \langle \bar{K}^0|e^{-itH}|\bar{K}^0\rangle = \langle \mathbf{2}|e^{-itH}|\mathbf{2}\rangle \equiv A_{22}(t)\,.
\end{aligned} \tag{20}$$

Using the spectral formalism we can write unstable states $|\lambda\rangle$ as

$$|\lambda\rangle = \sum_q \omega_\lambda(q)|q\rangle \tag{21}$$

and then $|\lambda(t)\rangle$ as

$$|\lambda(t)\rangle \stackrel{\text{def}}{=} e^{-itH}|\lambda\rangle = \sum_q |q(t)\rangle\omega_\lambda(q)\,, \tag{22}$$

where $|q(t)\rangle = e^{-itH}|q\rangle$ and vectors $|q\rangle$ form a complete set of eigenvectors of the hermitian, quantum-mechanical Hamiltonian H and $\omega_\lambda(q) = \langle q|\lambda\rangle$. If the continuous eigenvalue is denoted by m, we can define the survival amplitude $A(t)$ (or the transition amplitude in the case of $K^0 \leftrightarrow \bar{K}^0$) in the following way:

$$A(t) = \int\limits_{\mathrm{Spec}(H)} dm\, e^{-imt}\rho(m)\,, \tag{23}$$

where the integral extends over the whole spectrum of the Hamiltonian and density $\rho(m)$ is defined as follows

$$\rho(m) = |\omega_\lambda(m)|^2\,, \tag{24}$$

where $\omega_\lambda(m) = \langle m|\lambda\rangle$.

The above formalism may be applied to $|K_{\rm S}\rangle$ and $|K_{\rm L}\rangle$ by introducing a hermitian Hamiltonian with a continuous spectrum of decay products labelled by α, β *etc.*,

$$H|\phi_\alpha(m)\rangle = m|\phi_\alpha(m)\rangle\,, \quad \langle\phi_\beta(m')|\phi_\alpha(m)\rangle = \delta_{\alpha\beta}\delta(m'-m)\,. \tag{25}$$

In accordance with formula (22) the unstable states $K_{\rm S}$ and $K_{\rm L}$ may now be written as a superposition of the eigenkets $|\phi_\alpha(m)\rangle$,

$$|K_{\rm S}\rangle = \int\limits_0^\infty dm \sum_\alpha \omega_{{\rm S},\alpha}(m)|\phi_\alpha(m)\rangle\,, \tag{26}$$

$$|K_{\rm L}\rangle = \int\limits_0^\infty dm \sum_\beta \omega_{{\rm L},\beta}(m)|\phi_\beta(m)\rangle\,. \tag{27}$$

Thus

$$|K_{\rm S}(t)\rangle \;=\; e^{-itH}|K_{\rm S}\rangle = \int\limits_0^\infty dm \;\sum_\alpha \omega_{{\rm S},\alpha}(m)e^{-itH}|\phi_\alpha(m)\rangle\,, \tag{28}$$

$$|K_{\rm L}(t)\rangle \;=\; e^{-itH}|K_{\rm L}\rangle = \int\limits_0^\infty dm \;\sum_\beta \omega_{{\rm L},\beta}(m)e^{-itH}|\phi_\beta(m)\rangle\,. \tag{29}$$

Using (28) and (29) we can write

$$\langle K_{\rm S}|K_{\rm S}(t)\rangle = \int\limits_0^\infty dm \;\sum_\alpha |\omega_{{\rm S},\alpha}(m)|^2 e^{-imt} \stackrel{\rm def}{=} A_{K_{\rm S}K_{\rm S}}(t)\,,$$

$$\langle K_{\rm L}|K_{\rm L}(t)\rangle = \int\limits_0^\infty dm \;\sum_\beta |\omega_{{\rm L},\beta}(m)|^2 e^{-imt} \stackrel{\rm def}{=} A_{K_{\rm L}K_{\rm L}}(t)\,,$$

$$\langle K_{\rm S}|K_{\rm L}(t)\rangle = \int\limits_0^\infty dm \;\sum_\Gamma \omega^*_{{\rm S},\gamma}(m)\omega_{{\rm L},\gamma}(m) e^{-imt} \stackrel{\rm def}{=} A_{K_{\rm S}K_{\rm L}}(t)\,,$$

$$\langle K_{\rm L}|K_{\rm S}(t)\rangle = \int\limits_0^\infty dm \;\sum_\sigma \omega^*_{{\rm L},\sigma}(m)\omega_{{\rm S},\sigma}(m) e^{-imt} \stackrel{\rm def}{=} A_{K_{\rm L}K_{\rm S}}(t)\,. \tag{30}$$

From (15) we can obtain

$$|K^0\rangle = \frac{1}{2p}(|K_{\rm S}\rangle + |K_{\rm L}\rangle) \tag{31}$$

and

$$|\bar{K}^0\rangle = \frac{1}{2q}(|K_{\rm S}\rangle - |K_{\rm L}\rangle)\,. \tag{32}$$

Now, using formulae (20), (28)–(30), we can express $A_{K^0K^0}(t)$ *etc.* in terms of quantities describing physical states, that is through the amplitudes $A_{K_{\rm S}K_{\rm S}}(t)$, $A_{K_{\rm L}K_{\rm S}}(t)$ *etc.* (see *e.g.* [2])

$$A_{K^0\bar{K}^0}(t) \equiv \frac{1}{4p^*q}\left[A_{K_{\rm S}K_{\rm S}}(t) - A_{K_{\rm L}K_{\rm L}}(t) - A_{K_{\rm S}K_{\rm L}}(t) + A_{K_{\rm L}K_{\rm S}}(t)\right], \tag{33}$$

$$A_{\bar{K}^0K^0}(t) \equiv \frac{1}{4pq^*}\left[A_{K_{\rm S}K_{\rm S}}(t) - A_{K_{\rm L}K_{\rm L}}(t) + A_{K_{\rm S}K_{\rm L}}(t) - A_{K_{\rm L}K_{\rm S}}(t)\right]\,. \tag{34}$$

One can also find that

$$A_{K_{\rm S}K_{\rm S}} + A_{K_{\rm L}K_{\rm L}} = 2\,(|p|^2 A_{K^0K^0}(t) + |q|^2 A_{\bar{K}^0\bar{K}^0}(t)\,)\,. \tag{35}$$

Assuming (6) and using (16), (35) we get

$$A_{K^0K^0}(t) = A_{\bar{K}^0\bar{K}^0}(t) \equiv \tfrac{1}{2}\left(A_{K_{\rm S}K_{\rm S}}(t) + A_{K_{\rm L}K_{\rm L}}(t)\right)\,. \tag{36}$$

It follows from (31), (32) and (33)–(36) that the probabilities $A_{K^0K^0}(t)$ *etc.* can be written in the following way:

$$\begin{aligned} A_{K^0K^0}(t) &= A_{\bar{K}^0\bar{K}^0}(t) = \int_0^\infty dm\; \rho_{K^0K^0}(m) e^{-imt} \\ &= \frac{1}{2}\int_0^\infty dm\; \sum_\alpha \left\{|\omega_{{\rm S},\alpha}(m)|^2 + |\omega_{{\rm L},\alpha}|^2(m)\right\} e^{-imt}\,, \end{aligned} \tag{37}$$

$$\begin{aligned} A_{K^0\bar{K}^0}(t) &= \int_0^\infty dm\; \rho_{K^0\bar{K}^0}(m) e^{-imt} \\ &= \frac{1}{4p^*q}\int_0^\infty dm\; \sum_\beta \Big\{|\omega_{{\rm S},\beta}(m)|^2 - |\omega_{{\rm L},\beta}(m)|^2\,, \\ &\quad -\omega^*_{{\rm S},\beta}(m)\omega_{{\rm L},\beta}(m) + \omega^*_{{\rm L},\beta}(m)\omega_{{\rm S},\beta}(m)\Big\} e^{-imt}\,, \end{aligned} \tag{38}$$

$$\begin{aligned} A_{\bar{K}^0K^0}(t) &= \int_0^\infty dm\; \rho_{\bar{K}^0K^0}(m) e^{-imt} \\ &= \frac{1}{4pq^*}\int_0^\infty dm\; \sum_\beta \Big\{|\omega_{{\rm S},\beta}(m)|^2 - |\omega_{{\rm L},\beta}(m)|^2 \\ &\quad +\omega^*_{{\rm S},\beta}(m)\omega_{{\rm L},\beta}(m) - \omega^*_{{\rm L},\beta}(m)\omega_{{\rm S},\beta}(m)\Big\} e^{-imt}\,. \end{aligned} \tag{39}$$

The Breit–Wigner ansatz [13]

$$\rho_{\rm BW}(m) = \frac{\Gamma}{2\pi}\frac{1}{(m-m_0)^2 + \frac{\Gamma^2}{4}} \equiv |\omega(m)|^2 \tag{40}$$

leads to the well known exponential decay law which follows from the survival amplitude

$$A_{\rm BW}(t) = \int\limits_{-\infty}^{\infty} dm\, e^{-imt} \rho_{\rm BW}(m) = e^{-im_0 t} e^{-\frac{1}{2}\Gamma|t|}. \tag{41}$$

(Note that the existence of the lower bound for the energy (mass) induces non-exponential corrections to the decay law and to the survival amplitude (41) — see [2].) It is reasonable to assume a suitable form for $\omega_{{\rm S},\beta}$ and $\omega_{{\rm L},\beta}$. More specifically, we use [2]

$$\omega_{{\rm S},\beta}(m) = \sqrt{\frac{\Gamma_{\rm S}}{2\pi}} \frac{A_{{\rm S},\beta}(K_{\rm S} \to \beta)}{m - m_{\rm S} + i\frac{\Gamma_{\rm S}}{2}}, \tag{42}$$

$$\omega_{{\rm L},\beta}(m) = \sqrt{\frac{\Gamma_{\rm L}}{2\pi}} \frac{A_{{\rm L},\beta}(K_{\rm L} \to \beta)}{m - m_{\rm L} + i\frac{\Gamma_{\rm L}}{2}}\,, \tag{43}$$

where $A_{{\rm S},\beta}$ and $A_{{\rm L},\beta}$ are the decay (transition) amplitudes. It is convenient to use the following definitions:

$$\gamma_{\rm S} \equiv \frac{\Gamma_{\rm S}}{2}, \quad \gamma_{\rm L} \equiv \frac{\Gamma_{\rm L}}{2}, \quad \Delta m \equiv m_{\rm L} - m_{\rm S}\,, \tag{44}$$

$$S \equiv \sum_\alpha |A_{{\rm S},\alpha}|^2, \quad L \equiv \sum_\alpha |A_{{\rm L},\alpha}|^2\,, \tag{45}$$

$$R \equiv \sum_\sigma \Re(A^*_{{\rm S},\sigma} A_{{\rm L},\sigma})\,, \quad I \equiv \sum_\sigma \Im(A^*_{{\rm S},\sigma} A_{{\rm L},\sigma})\,. \tag{46}$$

In the one-pole approximation (42), (43) $A_{K^0K^0}(t)$ can be conveniently written as

$$\begin{aligned} A_{K^0K^0}(t) &= A_{\bar{K}^0\bar{K}^0}(t) \\ &= -\frac{1}{2\pi}\left\{ e^{-im_{\rm S}t}\left(-\int\limits_0^{-m_{\rm S}/\gamma_{\rm S}} dy \frac{e^{-i\gamma_{\rm S}ty}}{y^2+1} + \int\limits_0^{\infty} dy \frac{e^{-i\gamma_{\rm S}ty}}{y^2+1}\right)\right. \\ &\qquad \left. + e^{-im_{\rm L}t}\left(-\int\limits_0^{-m_{\rm L}/\gamma_{\rm L}} dy \frac{e^{-i\gamma_{\rm L}ty}}{y^2+1} + \int\limits_0^{\infty} dy \frac{e^{-i\gamma_{\rm L}ty}}{y^2+1}\right)\right\}. \end{aligned} \tag{47}$$

Collecting only exponential terms in (47) one obtains an expression analogous to the WW approximation

$$A_{K^0K^0}(t) = A_{\bar{K}^0\bar{K}^0}(t) = \tfrac{1}{2}\left(e^{-im_{\rm S}t}e^{-\gamma_{\rm S}t} + e^{-im_{\rm L}t}e^{-\gamma_{\rm L}t}\right) + N_{K^0K^0}(t)\,. \tag{48}$$

Here $N_{K^0K^0}(t)$ denotes all non-oscillatory terms present in the integrals (47).

3. Diagonal matrix elements of the effective Hamiltonian

This section constitutes the main part of the paper. Using the decomposition of type (48) and the one-pole ansatz (42), (43), we find the difference (13), which is now formulated for the $K^0 - \bar{K}^0$ system. It can be written as follows

$$h_{11}(t) - h_{22}(t) = \frac{X(t)}{Y(t)}\,, \tag{49}$$

where

$$X(t) = i\left(\frac{\partial A_{\bar{K}^0 K^0}(t)}{\partial t} A_{K^0 \bar{K}^0}(t) - \frac{\partial A_{K^0 \bar{K}^0}(t)}{\partial t} A_{\bar{K}^0 K^0}(t)\right) \tag{50}$$

and

$$Y(t) = A_{K^0 K^0}(t) A_{\bar{K}^0 \bar{K}^0}(t) - A_{K^0 \bar{K}^0}(t) A_{\bar{K}^0 K^0}(t)\,. \tag{51}$$

To calculate (38), (39) we use the following relations [2]

$$\int\limits_0^\infty dm \sum_\alpha |\omega_{\mathrm{S},\alpha}(m)|^2 e^{-imt} = \frac{1}{\pi} e^{-im_{\mathrm{S}}t}\left[-J^{(0)}\left(\gamma_{\mathrm{S}}t, -\frac{m_{\mathrm{S}}}{\gamma_{\mathrm{S}}}\right) + K^{(0)}(\gamma_{\mathrm{S}}t)\right] \tag{52}$$

and

$$\begin{aligned}
&\int\limits_0^\infty dm \sum_\beta \Im\left(\omega_{\mathrm{S},\beta}(m)\, \varphi^*_{\mathrm{L},\beta}(m)\right)\ e^{-imt} \\
&= -\frac{\sqrt{\gamma_{\mathrm{S}}\gamma_{\mathrm{L}}}}{\pi} \int\limits_0^\infty dm\, \frac{a_1 m^2 + b_1 m + c_1}{[(m-m_{\mathrm{S}})^2 + \gamma_{\mathrm{S}}^2][(m-m_{\mathrm{L}})^2 + \gamma_{\mathrm{L}}^2]} e^{-imt} \\
&= -\frac{\sqrt{\gamma_{\mathrm{S}}\gamma_{\mathrm{L}}}}{\pi}\Bigg\{\frac{e^{-im_{\mathrm{S}}t}}{\gamma_{\mathrm{S}}}\left(D'_I\left(-J^{(0)}\left(\gamma_{\mathrm{S}}t, -\frac{m_{\mathrm{S}}}{\gamma_{\mathrm{S}}}\right)\right) + K^{(0)}(\gamma_{\mathrm{S}}t)\right. \\
&\quad \left. +\gamma_{\mathrm{S}} C_I\left(-J^{(1)}\left(\gamma_{\mathrm{S}}t, -\frac{m_{\mathrm{S}}}{\gamma_{\mathrm{S}}}\right)\right) + K^{(1)}(\gamma_{\mathrm{S}}t)\right) \\
&\quad +\frac{e^{-im_{\mathrm{L}}t}}{\gamma_{\mathrm{L}}}\left(F'_I\left(-J^{(0)}\left(\gamma_{\mathrm{L}}t, -\frac{m_{\mathrm{L}}}{\gamma_{\mathrm{L}}}\right)\right) + K^{(0)}(\gamma_{\mathrm{L}}t)\right. \\
&\quad \left. -\gamma_{\mathrm{L}} C_I\left(-J^{(1)}(\gamma_{\mathrm{L}}t, -\frac{m_{\mathrm{L}}}{\gamma_{\mathrm{L}}})\right) + K^{(1)}\left(\gamma_{\mathrm{L}}t\right)\right)\Bigg\},
\end{aligned} \tag{53}$$

where a_1, b_1, c_1 and C_I, D'_I, F'_I are defined in Appendix B and $J^{(0)}(a,\eta)$, $J^{(1)}(a,\eta)$, $K^{(0)}(a)$, $K^{(1)}(a)$ in Appendix A.

Using the above mentioned formulae from Appendixes A and B (without any additional simplifications and approximations) we get, for example

$$\begin{aligned} A_{K^0\bar{K}^0}(t) \;=\; & \frac{1+\pi}{8\pi p^* q}\Big\{ e^{-im_{\mathrm{S}}t} e^{-\gamma_{\mathrm{S}}t}[1+k_{\mathrm{S}}] \\ & -e^{-im_{\mathrm{L}}t} e^{-\gamma_{\mathrm{L}}t}[1-k_{\mathrm{L}}]\Big\} + N_{K^0\bar{K}^0}(t)\,, \end{aligned} \tag{54}$$

where

$$k_{\mathrm{S}} = \frac{\sqrt{\gamma_{\mathrm{S}}\gamma_{\mathrm{L}}}}{\gamma_{\mathrm{S}}}\left(-2\,i\,\gamma_{\mathrm{S}} C_I + D'_I - F'_I\right)\,, \tag{55}$$

$$k_{\mathrm{L}} = \frac{\sqrt{\gamma_{\mathrm{S}}\gamma_{\mathrm{L}}}}{\gamma_{\mathrm{L}}}\left(2\,i\,\gamma_{\mathrm{L}} C_I - D'_I + F'_I\right)\,, \tag{56}$$

and $N_{K^0\bar{K}^0}(t)$ is the non-oscillatory term containing the exponential integral function E_i and it has the following form:

$$\begin{aligned} N_{K^0\bar{K}^0}(t) \;=\; & \frac{1}{8\pi i p^* q}\Bigg\{ e^{-im_{\mathrm{S}}t} e^{-\gamma_{\mathrm{S}}t} E_i(\gamma_{\mathrm{S}}t + im_{\mathrm{S}}t)(1+\gamma_{\mathrm{S}}k_{\mathrm{S}}) \\ & + e^{-im_{\mathrm{L}}t} e^{-\gamma_{\mathrm{L}}t} E_i(\gamma_{\mathrm{L}}t + im_{\mathrm{L}}t)(-1+\gamma_{\mathrm{L}}k_{\mathrm{L}}) \\ & + e^{-im_{\mathrm{S}}t} e^{\gamma_{\mathrm{S}}t} E_i(-\gamma_{\mathrm{S}}t + im_{\mathrm{S}}t) \\ & \times \left(-1 + \sqrt{\gamma_{\mathrm{S}}\gamma_{\mathrm{L}}}\left[-2iC_I + \frac{1}{\gamma_{\mathrm{S}}}(-D'_I + F'_I)\right]\right) \\ & + e^{-im_{\mathrm{L}}t} e^{\gamma_{\mathrm{L}}t} E_i(-\gamma_{\mathrm{L}}t + im_{\mathrm{L}}t) \\ & \times \left(1 + \sqrt{\gamma_{\mathrm{S}}\gamma_{\mathrm{L}}}\left[2iC_I + \frac{1}{\gamma_{\mathrm{L}}}(D'_I - F'_I)\right]\right)\Bigg\}. \end{aligned} \tag{57}$$

Using the expression (A.9) for the derivative of E_i (Appendix A) we can find the derivatives which will be necessary for the following calculations, for example

$$\begin{aligned} \frac{\partial A_{K^0\bar{K}^0}(t)}{\partial t} \;=\; & \frac{1+\pi}{8\pi p^* q}\Big\{ e^{-im_{\mathrm{S}}t} e^{-\gamma_{\mathrm{S}}t}\left(-im_{\mathrm{S}} - \gamma_{\mathrm{S}}(1+k_{\mathrm{S}})\right) \\ & + e^{-im_{\mathrm{L}}t} e^{-\gamma_{\mathrm{L}}t}\left(im_{\mathrm{L}} - \gamma_{\mathrm{L}}(1+k_{\mathrm{L}})\right)\Big\} \\ & + \Delta N_{K^0\bar{K}^0}(t), \end{aligned} \tag{58}$$

where $\Delta N_{K^0\bar{K}^0}(t)$ is defined as follows:

$$\begin{aligned}\Delta N_{K^0\bar{K}^0}(t) \;=\; & \frac{1}{8\pi i p^* q}\Big\{ e^{-im_{\rm S}t}e^{-\gamma_{\rm S}t}E_i(\gamma_{\rm S}t+im_{\rm S}t)\left(-im_{\rm S}-\gamma_{\rm S}(1+k_{\rm S})\right) \\ & +e^{-im_{\rm L}t}e^{-\gamma_{\rm L}t}E_i(\gamma_{\rm L}t+im_{\rm L}t)\Big(im_{\rm L}-\gamma_{\rm L}(1+k_{\rm L})\Big) \\ & +e^{-im_{\rm S}t}e^{\gamma_{\rm S}t}E_i(-\gamma_{\rm S}t+im_{\rm S}t)\Big(im_{\rm S}-\gamma_{\rm S} \\ & +\sqrt{\gamma_{\rm S}\gamma_{\rm L}}(-2i\gamma_{\rm S}C_I-D_I'+F_I')\Big) \\ & +e^{-im_{\rm L}t}e^{\gamma_{\rm L}t}E_i(-\gamma_{\rm L}t+im_{\rm L}t)\Big(-im_{\rm L}-\gamma_{\rm L} \\ & +\sqrt{\gamma_{\rm S}\gamma_{\rm L}}(2i\gamma_{\rm L}C_I+D_I'-F_I')\Big)\Big\}. \end{aligned} \tag{59}$$

There are similar expressions for $A_{\bar{K}^0K^0}(t)$, $N_{\bar{K}^0K^0}(t)$, $\frac{\partial A_{\bar{K}^0K^0}(t)}{\partial t}$, $\Delta N_{\bar{K}^0K^0}(t)$.

The states $|K_{\rm L}\rangle$ and $|K_{\rm S}\rangle$ are superpositions of $|K^0\rangle$ and $|\bar{K}^0\rangle$ ((31), (32)). The lifetimes of the $|K_{\rm L}\rangle$ and $|K_{\rm S}\rangle$ particles may be denoted by $\tau_{\rm L}$ and $\tau_{\rm S}$, respectively, $\tau_{\rm L}=\frac{1}{\Gamma_{\rm L}}=(5.17\pm0.04)\times10^{-8}$ s being much longer than $\tau_{\rm S}=\frac{1}{\Gamma_{\rm S}}=(0.8935\pm0.0008)\times10^{-10}$ s [15].

Below we calculate the difference (49) for $t\sim\tau_{\rm L}$

$$h_{11}(t\sim\tau_{\rm L})-h_{22}(t\sim\tau_{\rm L})=\frac{X(t\sim\tau_{\rm L})}{Y(t\sim\tau_{\rm L})}. \tag{60}$$

When we consider only the long living states $|K_{\rm L}\rangle$ then we may drop all the terms containing $e^{-\gamma_{\rm S}t}|_{t\sim\tau_{\rm L}}$ as they are negligible in comparison with the elements involving the factor $e^{-\gamma_{\rm L}t}|_{t\sim\tau_{\rm L}}$. We also drop all the non-oscillatory terms $N_{K^0K^0}(t)$, $N_{\bar{K}^0K^0}(t)$, $N_{K^0\bar{K}^0}(t)$ (57) present in $A_{K^0K^0}(t)$ (47), $A_{\bar{K}^0K^0}(t)$ and $A_{K^0\bar{K}^0}(t)$ (54), because they are extremally small in the region of time $t\sim\tau_{\rm L}$ [2]. Similarly, because of the properties of the exponential integral function E_i, we can drop terms like $\Delta N_{\bar{K}^0K^0}$ in $\frac{\partial A_{\bar{K}^0K^0}}{\partial t}$ and $\Delta N_{K^0\bar{K}^0}$ (59) in $\frac{\partial A_{K^0\bar{K}^0}}{\partial t}$ (58). This conclusion follows from the asymptotic properties of the E_i function (A.8) and the fact that $\Delta N_{\bar{K}^0K^0}$, $\Delta N_{K^0\bar{K}^0}$ only contain expressions proportional to E_i.

We may now calculate the products

$$A_{K^0K^0}(t)\cdot A_{\bar{K}^0\bar{K}^0}(t)\,,\quad A_{K^0\bar{K}^0}(t)\cdot A_{\bar{K}^0K^0}(t)\,,$$
$$\frac{\partial A_{\bar{K}^0K^0}}{\partial t}(t)\cdot A_{K^0\bar{K}^0}(t)\,,\quad \frac{\partial A_{K^0\bar{K}^0}}{\partial t}(t)\cdot A_{\bar{K}^0K^0}(t)$$

that after the use of the above mentioned properties of $N_{K^0K^0}(t)$, $\Delta N_{K^0K^0}(t)$ and performing some algebraic transformations, lead to the following form of the difference (60):

$$h_{11}(t\sim\tau_{\rm L}) - h_{22}(t\sim\tau_{\rm L})) = \left(\frac{2\pi^2\sqrt{\gamma_{\rm S}\gamma_{\rm L}}}{\pi^2+2\pi+1}\right)\frac{Z}{W} \neq 0, \tag{61}$$

where

$$\begin{aligned} Z &= 4|p|^2|q|^2 - \frac{\pi^2+2\pi+1}{4\pi^2}\Big[1+\gamma_{\rm S}\Big(4\gamma_{\rm L}C_I^2 + \frac{1}{\gamma_{\rm L}}(-D_I'^2 - F_I'^2 + 4D_I'F_I') \\ &\quad +4iC_I(D_I'-F_I')\Big)\Big] \neq 0\,, \qquad (62)\\ W &= 2\Big(-C_I m_{\rm L} + D_I' - F_I'\Big) + i\Big[-4C_I\gamma_{\rm L} + \frac{m_{\rm L}}{\gamma_{\rm L}}(-D_I'+F_I')\Big] \neq 0\,. \qquad (63) \end{aligned}$$

4. Final remarks

Our results lead to the conclusion that in a CPT invariant and CP noninvariant system in the case of the exactly solvable one-pole model, the diagonal matrix elements do not have to be equal. In the general case the diagonal elements depend on time and their difference, for example at $t\sim\tau_{\rm L}$, is different from zero. This has been clearly demonstrated in the last section: Z and W in (61) are different from zero, so the difference $(h_{11}(t)-h_{22}(t))|_{t\sim\tau_{\rm L}} \neq 0$. From the above observation a conclusion of major importance can be drawn, namely that the measurement of the difference $(h_{11}(t)-h_{22}(t))$ should not be used for designing CPT invariance tests. This runs counter to the general conclusions following from the Lee, Oehme and Yang theory.

A detailed analysis of $h_{jk}(t)$, $(j,k=1,2)$ shows that the non-oscillatory elements $N_{\alpha,\beta}(t), \Delta N_{\alpha,\beta}(t)$ (where $\alpha,\beta = K^0, \overline{K}^0$) is the source of the non-zero difference $(h_{11}(t)-h_{22}(t))$ in the model considered. It is not difficult to verify that dropping all the terms of $N_{\alpha,\beta}(t), \Delta N_{\alpha,\beta}(t)$ type in the formula for $(h_{11}(t)-h_{22}(t))$ gives $(h_{11}^{\rm osc}(t)-h_{22}^{\rm osc}(t))=0$, where $h_{jj}^{\rm osc}(t)$, $(j=1,2)$, stands for $h_{jj}(t)$ without the non-oscillatory terms.

To obtain the numerical estimate the real and imaginary parts of $(h_{11}(t\sim\tau_{\rm L})-h_{22}(t\sim\tau_{\rm L}))$ it is necessary to put experimentally obtained values of $m_{\rm S}, m_{\rm L}, \gamma_{\rm S}, \gamma_{\rm L}$, *etc.*, into (61)–(63). According to the literature [5,7], if the total system is CPT-invariant but CP-noninvariant then we have (see, *e.g.*, [14])

$$p=\frac{1+\varepsilon}{\sqrt{2}}\,, \quad q=\frac{1-\varepsilon}{\sqrt{2}}\,, \tag{64}$$

and hence we get

$$\Delta_K = 2\,\Re\,(\varepsilon)\,, \tag{65}$$

where $|\varepsilon| \simeq 10^{-3}$ [5,7]. Putting experimental values [15]

$$\begin{aligned} \Delta m &= (3.489 \pm 0.008) \times 10^{-12}\ \mathrm{MeV}\,, && (66)\\ m_{\mathrm{S}} &\simeq m_{\mathrm{L}} \simeq m_{\mathrm{average}} = (497.648 \pm 0.022)\ \mathrm{MeV} && (67) \end{aligned}$$

and τ_{L}, τ_{S}, $\hbar$ into expressions (61)–(63) for the neutral kaon system we can obtain the following estimations

$$\Re\,(h_{11}(t \sim \tau_{\mathrm{L}}) - h_{22}(t \sim \tau_{\mathrm{L}})) \simeq -4.771 \times 10^{-18}\ \mathrm{MeV} \tag{68}$$

and

$$\Im\,(h_{11}(t \sim \tau_{\mathrm{L}}) - h_{22}(t \sim \tau_{\mathrm{L}})) \simeq 7.283 \times 10^{-16}\ \mathrm{MeV}\,. \tag{69}$$

So, the difference of the diagonal matrix elements of the effective Hamiltonian for the $K^0 - \bar{K}^0$ system in one pole approximation is different from zero. According to our evaluation

$$\frac{|\Re(h_{11}(t \sim \tau_{\mathrm{L}}) - h_{22}(t \sim \tau_{\mathrm{L}}))|}{m_{\mathrm{average}}} \equiv \frac{|m_{K^0} - m_{\bar{K}^0}|}{m_{\mathrm{average}}} \sim 10^{-21}\,. \tag{70}$$

Recent experiments give $\frac{|m_{K^0} - m_{\bar{K}^0}|}{m_{\mathrm{average}}} \leq 10^{-18}$ [15]. So our estimation (70) does not contradict the experimental results.

Deviations from the LOY result estimated in [2] have the order of magnitude $\frac{\gamma}{m}$. These estimations refer to amplitudes $A_{K_0\bar{K}_0}$ and $A_{\bar{K}_0K_0}$. However, these estimations could not be directly transformed into the calculation of the difference $(h_{11}(\tau) - h_{22}(\tau))$, because the difference depends not only on amplitudes of type $A_{K_0\bar{K}_0}$, but also on their derivatives (see relations (49)–(51)). There are products of type $\frac{\partial A_{\bar{K}^0K^0}(t)}{\partial t} A_{K^0\bar{K}^0}(t)$ in the numerator of the expression (49), whereas there are not any derivatives in the denominator of this expression. What is more, there is the difference of expressions of type $\frac{\partial A_{\bar{K}^0K^0}(t)}{\partial t} A_{K^0\bar{K}^0}(t)$ in the numerator of (49). So, it can hardly be expected, that the order of deviations from the LOY result of the relatively complicated expression (49) will be the same as the order of corrections to the LOY result of one of the following expressions: $A_{K_0\bar{K}_0}$ and $A_{\bar{K}_0K_0}$.

If estimation (70) is compared with a similar one obtained in [1,16–18] one can see that the numerical value of our estimation is much larger than the value of mentioned estimations. It is because the estimations given in the mentioned papers were obtained using a different method for the Lee–Fridrichs model [19].

The results $(h_{11}(t) - h_{22}(t)) \neq 0$ and (68), (69) and (70) seem to be very important as they have been obtained within the exactly solvable one-pole model based on the Breit–Wigner ansatz, *i.e.* the same model as used by Lee, Oehme and Yang.

As the final remark it should also be noted that the real parts of the diagonal matrix elements of the mass matrix $H_{\|}$, h_{11} and h_{22}, are considered in the literature as masses of unstable particles $|\mathbf{1}\rangle$, $|\mathbf{2}\rangle$ (*e.g.*, mesons K_0, $\bar{K}_0$). The interpretation of the diagonal matrix elements of $H_{\|}(t = 0) \equiv PHP$ is obvious (see [18]). They have the dimension of the energy (that is, the mass) and $h_{jj}(0) \equiv \langle \boldsymbol{j}|H|\boldsymbol{j}\rangle$, $(j = 1, 2)$. So their interpretation as masses of particle "1" and its antiparticle "2" at the instant $t = 0$ seems to be justified. Note that $H_{\|}$ has the following form ([10, 16–18])

$$H_{\|}(t) = PHP + V_{\|}(t)\,, \tag{71}$$

that is

$$H_{\|}(t) \equiv H_{\|}(0) + V_{\|}(t)\,. \tag{72}$$

The diagonal matrix elements of the operator $V_{\|}(t)$, *i.e.* $v_{jj}(t) = \langle j|V_{\|}(t)|j\rangle$, also have the dimension of the energy and in general they depend on time. So, the problem seems to be open: we can treat the matrix elements of the operator $V_{\|}(t)$ as a time-dependent correction to the energy or mass.

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Appendix A

This appendix contains the relevant integrals and derivatives used in Section 3.

Integrals $K^{(n)}(a)$ and $J^{(n)}(a, \eta)$ are defined as follows [2, 20]

$$K^{(n)}(a) \equiv \int\limits_0^\infty dx \frac{x^n}{x^2+1} e^{-iax}, \tag{A.1}$$

$$J^{(n)}(a, \eta) \equiv \int\limits_0^\eta dx \frac{x^n}{x^2+1} e^{-iax}. \tag{A.2}$$

If we assume $a \equiv (\gamma_{\mathrm{S/L}} t)$ and $\eta \equiv (-\frac{m_{\mathrm{S/L}}}{\gamma_{\mathrm{S/L}}})$, for $n = 0$ we get

$$\begin{aligned} K^{(0)}(\gamma_{\mathrm{S/L}} t) &= \int_0^\infty dy \frac{1}{y^2+1} e^{-i\gamma_{\mathrm{S/L}} t y} \\ &= \frac{\pi}{2} e^{-\gamma_{\mathrm{S/L}} t} - \frac{i}{2}\left[e^{-\gamma_{\mathrm{S/L}} t} E_i(\gamma_{\mathrm{S/L}} t) - e^{\gamma_{\mathrm{S/L}} t} E_i(-\gamma_{\mathrm{S/L}} t)\right], \end{aligned} \tag{A.3}$$

$$\begin{aligned} J^{(0)}(\gamma_{\mathrm{S/L}} t, -\frac{m_{\mathrm{S/L}}}{\gamma_{\mathrm{S/L}}}) &= \int_0^{-m_{\mathrm{S/L}}/\gamma_{\mathrm{S/L}}} dy \frac{1}{y^2+1} e^{-i\gamma_{\mathrm{S/L}} t y} \\ &= -\frac{1}{2i}\Bigg[-i\,\mathrm{sgn}\left(-\frac{m_{\mathrm{S/L}}}{\gamma_{\mathrm{S/L}}}\right) e^{-\gamma_{\mathrm{S/L}} t} \\ &\quad + e^{-\gamma_{\mathrm{S/L}} t} E_i\left(\gamma_{\mathrm{S/L}} t \left[1 - i\left(-\frac{m_{\mathrm{S}}}{\gamma_{\mathrm{S/L}}}\right)\right]\right) \\ &\quad - e^{\gamma_{\mathrm{S/L}} t} E_i\left(-\gamma_{\mathrm{S/L}} t \left[1 + i\left(-\frac{m_{\mathrm{S}}}{\gamma_{\mathrm{S/L}}}\right)\right]\right) \\ &\quad - e^{-\gamma_{\mathrm{S/L}} t} E_i\left(\gamma_{\mathrm{S/L}} t\right) \\ &\quad + e^{\gamma_{\mathrm{S/L}} t} E_i(-\gamma_{\mathrm{S/L}} t)\Bigg], \end{aligned} \tag{A.4}$$

where E_i is the exponential integral function and $\mathrm{sgn}\,(-\frac{m_{\mathrm{S/L}}}{\gamma_{\mathrm{S/L}}})$ stands for the sign of $(-\frac{m_{\mathrm{S/L}}}{\gamma_{\mathrm{S/L}}})$.

Any other integral of $K^{(n)}(a)$ or $J^{(n)}(a,\eta)$ for $n > 0$ can be obtained from (A.1) or (A.2) by differentiating (A.1) or (A.2) with respect to a and using the Fourier identity in (A.2) [2]. For $n = 1$ we have

$$\begin{aligned} K^{(1)}(\gamma_{\mathrm{S/L}} t) &= \int_0^\infty dy \frac{x}{y^2+1} e^{-i\gamma_{\mathrm{S/L}} t y} \\ &= -i\frac{\pi}{2} e^{-\gamma_{\mathrm{S/L}} t} - \frac{1}{2}\left[e^{-\gamma_{\mathrm{S/L}} t} E_i(\gamma_{\mathrm{S/L}} t) + e^{\gamma_{\mathrm{S/L}} t} E_i(-\gamma_{\mathrm{S/L}} t)\right], \end{aligned} \tag{A.5}$$

$$
\begin{aligned}
J^{(1)}(\gamma_{\mathrm{S/L}}t, -\frac{m_{\mathrm{S/L}}}{\gamma_{\mathrm{S/L}}}) &= \int\limits_0^{-m_{\mathrm{S/L}}/\gamma_{\mathrm{S/L}}} dy \frac{x}{y^2+1} e^{-i\gamma_{\mathrm{S/L}}ty} \\
&= -\frac{1}{2}\Bigg[i\,\mathrm{sgn}\left(-\frac{m_{\mathrm{S/L}}}{\gamma_{\mathrm{S/L}}}\right) e^{-\gamma_{\mathrm{S/L}}t} \\
&\quad -e^{-\gamma_{\mathrm{S/L}}t} E_i\left(\gamma_{\mathrm{S/L}}t\left[1-i\left(-\frac{m_{\mathrm{S}}}{\gamma_{\mathrm{S/L}}}\right)\right]\right) \\
&\quad -e^{\gamma_{\mathrm{S/L}}t} E_i\left(-\gamma_{\mathrm{S/L}}t\left[1+i\left(-\frac{m_{\mathrm{S}}}{\gamma_{\mathrm{S/L}}}\right)\right]\right) \\
&\quad +e^{-\gamma_{\mathrm{S/L}}t} E_i\left(\gamma_{\mathrm{S/L}}t\right) \\
&\quad +e^{\gamma_{\mathrm{S/L}}t} E_i(-\gamma_{\mathrm{S/L}}t)\Bigg]. \qquad \text{(A.6)}
\end{aligned}
$$

The exponential integral function E_i is defined in the following way [2, 20]:

$$
E_i(\pm xy) = \pm e^{\pm xy} \int\limits_0^{\infty} dt \frac{e^{-xt}}{y \mp t}\,, \quad \Re y > 0\,,\ x > 0\,. \qquad \text{(A.7)}
$$

We can use the very convenient asymptotic properties of E_i given in [21]

$$
\begin{aligned}
&E_i(0) = -\infty\,, \\
&E_i(\infty) = \infty\,, \\
&E_i(-\infty) = 0\,, \\
&E_i(i\infty) = i\pi\,, \\
&E_i(-i\infty) = -i\pi\,. \qquad \text{(A.8)}
\end{aligned}
$$

These properties of E_i have been used to obtain the final result (61)–(63).

In our calculations we have also used the formula for the derivative of E_i. Its final, general form is given below

$$
\frac{dE_i(\pm xy)}{dx} = \frac{1}{x} e^{\pm xy}\,. \qquad \text{(A.9)}
$$

Appendix B

In this Appendix we collect from [2] the coefficients a_1, b_1, c_1 and C_I, D_I', F_I' which were used in Section 3.

The calculations will be clearer if we write the sum of the product $\sum_\beta \omega^*_{\mathrm{S},\beta}\omega_{\mathrm{L},\beta}$ in the same way in which spectral functions defined by (42), (43) were used earlier in [2]

$$\left.\sum_\beta \omega^*_{\mathrm{S},\beta}\omega_{\mathrm{L},\beta}\right|_{\mathrm{BW}} = \frac{\sqrt{\gamma_\mathrm{S}\gamma_\mathrm{L}}}{\pi[(m-m_\mathrm{S})^2+\gamma_\mathrm{S}^2][(m-m_\mathrm{L})^2+\gamma_\mathrm{L}^2]} \times\{(a_R m^2 + b_R m + c_R) + i(a_I m^2 + b_I m + c_I)\}, \tag{B.1}$$

where

$$\begin{aligned} a_I &= I, \quad b_I = (\gamma_\mathrm{S}-\gamma_\mathrm{L})R - (m_\mathrm{S}+m_\mathrm{L})I\,, \\ c_I &= (\gamma_\mathrm{L} m_\mathrm{S} - \gamma_\mathrm{S} m_\mathrm{L})R + (m_\mathrm{S} m_\mathrm{L} + \gamma_\mathrm{S}\gamma_\mathrm{L})I \end{aligned} \tag{B.2}$$

similar formulae may be found for a_R, b_R, c_R, where

$$R = \frac{\Delta_K}{2\sqrt{\gamma_\mathrm{S}\gamma_\mathrm{L}}}(\gamma_\mathrm{S} S + \gamma_\mathrm{L} L)\,, \tag{B.3}$$

$$I = \frac{\Delta_K}{2\sqrt{\gamma_\mathrm{S}\gamma_\mathrm{L}}}\frac{\Delta m}{\gamma_\mathrm{S}-\gamma_\mathrm{L}}(\gamma_\mathrm{S} S - \gamma_\mathrm{L} L)\,, \tag{B.4}$$

and

$$\begin{aligned} S &= 1 + \frac{\gamma_\mathrm{S}}{\pi m_\mathrm{S}} + \mathcal{O}((\gamma_\mathrm{S}/m_\mathrm{S})^2)\,, \\ L &= 1 + \frac{\gamma_\mathrm{L}}{\pi m_\mathrm{L}} + \mathcal{O}((\gamma_\mathrm{L}/m_\mathrm{L})^2)\,. \end{aligned} \tag{B.5}$$

Equations (B.5) result from performing the following integration

$$\int_0^\infty dm \sum_\alpha |\omega_{\mathrm{S},\alpha}|^2 = \int_0^\infty dm \sum_\beta |\omega_{\mathrm{L},\beta}|^2 = 1. \tag{B.6}$$

This integral follows from the initial conditions defined by (15)–(17).

This expression is now factorized

$$\frac{a_I m^2 + b_I m + c_I}{[(m-m_\mathrm{S})^2+\gamma_\mathrm{S}^2][(m-m_\mathrm{L})^2+\gamma_\mathrm{L}^2]} = \frac{C_I m + D_I}{(m-m_\mathrm{S})^2+\gamma_\mathrm{S}^2} + \frac{E_I m + F_I}{(m-m_\mathrm{L})^2+\gamma_\mathrm{L}^2} \tag{B.7}$$

which leads, as usual, to a linear system of equations which allows us to calculate the coefficients C_I, D_I, E_I, F_I

$$\begin{aligned} E_I &= -C_I\,, \\ C_I \Delta m + D'_I + F'_I &= a_I\,, \\ C_I[(m_{\mathrm{L}}^2+\gamma_{\mathrm{L}}^2)-(m_{\mathrm{S}}^2+\gamma_{\mathrm{S}}^2)] - 2D'_I(m_{\mathrm{L}} - 2F'_I m_{\mathrm{S}} &= b_I\,, \\ D'_I(m_{\mathrm{L}}^2+\gamma_{\mathrm{L}}^2) + F'_I(m_{\mathrm{S}}^2+\gamma_{\mathrm{S}}^2) + C_I[m_{\mathrm{L}}(m_{\mathrm{S}}^2+\gamma_{\mathrm{S}}^2) - m_{\mathrm{S}}(m_{\mathrm{L}}^2+\gamma_{\mathrm{L}}^2)] &= c_I\,. \end{aligned} \tag{B.8}$$

In the last formula we have introduced new definitions

$$\begin{aligned} &E_I = -C_I\,, \\ &D'_I \equiv D_I + C_I m_{\mathrm{L}}\,, \quad F'_I \equiv F_I - C_I m_{\mathrm{L}}\,. \end{aligned} \tag{B.9}$$

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