Vol. 36 (2005) *ACTA PHYSICA POLONICA B* No 6

E_6 MULTIPLETS AND UNIFICATION IN EXTRA DIMENSIONS

Biswajoy Brahmachari†

Department of Physics, Vidyasagar Evening College
39, Sankar Ghosh Lane, Kolkata 700006, India

(Received February 24, 2005)

We study the effect of all matter multiplets contained in **27** representation of E_6 GUT on gauge coupling unification in extra dimensions. Extra members of **27** multiplets of all three generations have their 'zero modes' near $m_{\rm t}$ such that they can be directly probed. From TeV scale onwards extra dimensions open up, theory becomes $N = 2$ supersymmetric and gauge couplings unify or they do not, depending on how we distribute matter fields and gauge fields in bulk and brane. We find three such possible embeddings which will lead to perfect gauge coupling unification below 100 TeV region for one extra dimension and lower than that if number of extra dimensions is larger.

PACS numbers: 04.50.+h, 11.25.Mj, 12.10.Dm

Supersymmetry breaking is a hard problem. However, attempts have been made to link lightness of supersymmetry breaking scale to some large radius of compactifications in string theory [1]. Independently large extra dimensions of millimeter size have been invoked to stabilize gauge hierarchy problem as proposed in Ref. [2]. This way to tackle hierarchy problem is independent of the existence of low energy supersymmetry. Moreover, special features of open string theory [3] have also been used to try and bring down the fundamental string scale itself to TeV region [4, 5]. All these new results give good motivation for studying theories where extra dimensions show up much below 10^{18} GeV. One possible consequence of such large extra dimensions is that gauge coupling unification can happen in $M_X =$ few tens of TeV [6]. Then, even though our unified theory in extra dimensions is non-renormalizable, gauge coupling unification happens close enough to the low

† Address before August 2002: Theoretical Physics Division, Saha Institute of Nuclear Physics, Kolkata-64, India.

lying string scale in such a way that three gauge forces unify with gravity and side by side divergences of non-renormalizable effective gauge theory are properly handled by full string theory which takes over almost immediately.

Here we will study gauge coupling unification in E_6 Grand Unified Theory (GUT) [7] in extra dimensions. We know that fermions are unified in the **27** representation of E_6 which can be decomposed as

$$E_6 \supset \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$$

$$\begin{aligned} 27 \supset\ & \overbrace{(3,2,1/6)+(\overline{3},1,-2/3)+(1,1,1)}^{10\ \text{of SU(5)}}+\overbrace{(\overline{3},1,1/3)+(1,2,-1/2)}^{\overline{5}\ \text{of SU(5)}}+\overbrace{(1,1,0)}^{\text{RH neutrino}} \\ & \underbrace{+(\overline{3},1,1/3)+(3,1,-1/3)+(1,2,-1/2)+(1,2,1/2)+(1,1,0)}_{\text{exotic fermions}}. \end{aligned}$$

Because two extra singlets will not affect gauge coupling unification we will work with low energy multiplets that can be thought as $5+\overline{5}$ of SU(5). We know that in 4 dimensions introduction of complete SU(5) multiplets do not affect gauge coupling unification but the unified coupling can become non-perturbative before GUT scale is reached. However, as we will learn below, this is not necessarily the case in the presence of extra dimensions. Because this is a three generation analysis we can have full three copies of $5+\overline{5}$ hanging well below the GUT scale due to E_6 symmetry. That is $5+\overline{5}$ do not pair up and become as heavy as the GUT scale. Then we have four possible types of extra matter multiplets. We assume that their masses are near the top quark mass m_{t}. In extra dimensional models we use the terminology that their 'zero modes' are near m_{t}. At the scale $\mu_0 = 1$ TeV δ number of extra dimensions opens up and excited Kaluza–Klein states start to show. Under standard model gauge group we label extra matter superfields as

$$L_1=(1,2,-1/2)\ ;\ D_1=(\overline{3},1,1/3)\ ;\ L_2=(1,2,1/2)\ ;\ D_2=(3,1,-1/3). \tag{1}$$

Contribution of L_1 and L_2 to beta coefficients will be the same as usual lepton doublet whereas contribution of D_1 and D_2 will be same as down type antiquark. Gauge couplings evolve [6] with energy via the following equation where we have redefined $t=\frac{1}{2\pi}\ln(\Lambda)$, $t_0=\frac{1}{2\pi}\ln(\mu_0)$ and $\alpha=g^2/4\pi$. Here Λ is the cut-off scale where couplings are being evaluated.

$$\frac{d\alpha_i}{dt}=\left[(\beta_i-\tilde{\beta}_i)+\tilde{\beta}_i\ X_\delta\ e^{2\ \pi\ \delta\ (t-t_0)}\right]\ \alpha_i^2 \quad \text{and} \quad X_\delta=\frac{\pi^{\delta/2}}{\Gamma(1+\delta/2)}. \tag{2}$$

Γ is Euler Gamma function. Note that we must recover familiar Renormalization Group Equations (RGE) in the limit of either $\tilde{\beta}=0$ or $\delta=0$. We have used the usual notation [9] that βs are 4-dimensional coefficients whereas $\tilde{\beta}$s are higher dimensional coefficients. Expressions of $\tilde{\beta}$s are given in detail in Table I for $N=2$ supersymmetric standard model.

TABLE I

Contributions to $\tilde{\beta}$ coefficients in ($N = 2$) supersymmetric standard model. Extra E_6 multiplets L_1 and L_2 contribute the same as L and D_1 and D_2 contribute the same as $\overline{D}$.

fields	representation	$\tilde{\beta}_3$	$\tilde{\beta}_2$	$\tilde{\beta}_1$
H_1	$(1,2,\ \frac{1}{2})$	0	1	1
H_2	$(1,2,\ -\frac{1}{2})$	0	1	1
Q	$(3,2,\ -\frac{1}{6})$	η_Q 2	η_Q 3	$\frac{\eta_Q 1}{3}$
$\overline{D}$	$(\overline{3}, 1, \frac{1}{3})$	η_U 1	0	η_U $\frac{2}{3}$
$\overline{U}$	$(\overline{3}, 1, -\frac{2}{3})$	η_D 1	0	η_D $\frac{8}{3}$
L	$(1,2,\ -\frac{1}{2})$	0	η_L 1	η_L 1
$\overline{E}$	(1,1, 1)	0	0	η_E 2
gauge	(8,3, 0)	–6	–4	0

Let us quickly review the minimal scenario given by Dines Dudas and Gherghetta [6] which will set all our notations. There only gauge bosons and Higgs have bulk excitations whereas fermions remain at fixed points. Two Higgs doublets H_1 and H_2 are embedded in a single $N = 2$ Higgs superfield. All δ extra dimensions open up simultaneously at scale μ_0 which is a free parameter. Below the scale μ_0 we have $N = 1$ supersymmetry whereas above μ_0 we have $N = 2$ supersymmetry. In this case $\beta = (\frac{33}{5}, 1, -3)$ and $\tilde{\beta} = (\frac{3}{5}, -3, -6)$. Following Eq. (2) we write

$$\begin{aligned} \frac{d\alpha_Y}{dt} &= \left[6 + 3/5\ X_\delta e^{2\ \pi\ \delta\ (t-t_0)}\right]\ \alpha_Y^2\,, \\ \frac{d\alpha_2}{dt} &= \left[4 - 3\ X_\delta e^{2\ \pi\ \delta\ (t-t_0)}\right]\ \alpha_2^2\,, \\ \frac{d\alpha_3}{dt} &= \left[3 - 6\ X_\delta e^{2\ \pi\ \delta\ (t-t_0)}\right]\ \alpha_3^2\,. \end{aligned} \tag{3}$$

Using two loop renormalization group equations for the gauge couplings below μ_0 we can solve Eq. (3) numerically, the results are given in Fig. 1 CASE 1. Now let us examine the unification condition discussed by DDG by defining ratio

$$R_{ij} = \frac{\tilde{\beta}_i - \tilde{\beta}_j}{\beta_i - \beta_j}\,. \tag{4}$$

Then unification is achieved when we have

$$R_{12} = R_{13} = R_{23}\,. \tag{5}$$

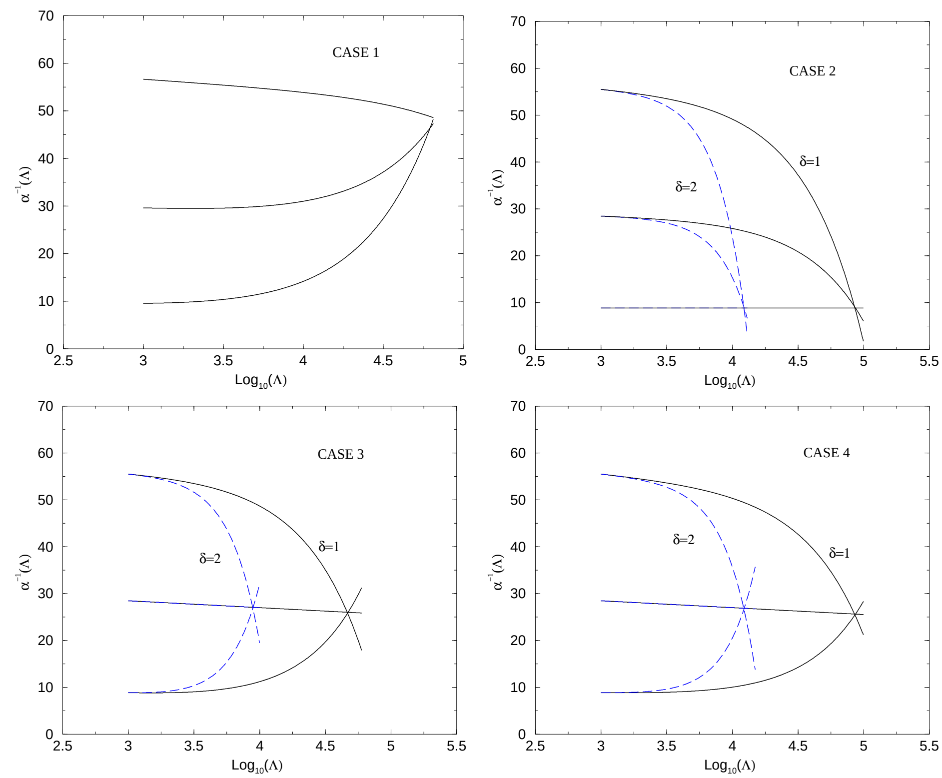

Fig. 1. CASE 1: Original DDG case with $\mu_0 \sim 1$ TeV where only gauge bosons and Higgs scalar has bulk excitations. Gauge couplings do not meet precisely. CASE 2: Only SU(2) and U(1) has bulk excitations. Three **27** generations at scale $m_{\rm t}$ of which leptons have bulk excitations with $\eta_L = 5, \eta_E = 1$ above μ_0. CASE 3: Only SU(3) and U(1) has bulk excitations. All three 27 multiplets have zero modes at scale $m_{\rm t}$, above μ_0 $\eta_E = 2, \eta_U = 2$ have bulk excitations. CASE 4: Only SU(3) and U(1) has bulk excitations. All three 27 multiplets have zero modes at scale $m_{\rm t}$, above μ_0 $\eta_D = 3, \eta_U = 2$ have bulk excitations.

For DDG case we have $\frac{R_{12}}{R_{13}} = 0.94$ and $\frac{R_{13}}{R_{23}} = 0.92$. Thus gauge coupling unification is only approximate. Let us examine scenario 3 discussed by Carone [8]. There are two $5 + \overline{5}$ of which leptons have bulk excitations (total 5 $N = 1$ pairs) and one generation of electron have bulk excitation (1 $N = 1$ pair). All fermions have zero modes at scale $\mathrm{O}(m_W)$. SU(3) gauge bosons stay on the boundary. Then we have $R_{12} = R_{13} = R_{23} = 1/2$ which gives perfect unification. We see that this case is very similar to our E_6 scenario except that we want to keep zero modes of all three generations of **27** near $m_{\rm t}$. This fixes β coefficients below μ_0 to be $\beta = (48/5, 4, 0)$.

However, adding one more $5 + \overline{5}$ near $m_{\rm t}$ will keep the difference $\beta_i - \beta_j$ unchanged which occurs in the denominator in Eq. (4). Thus couplings will still unify. Keeping only SU(2) and U(1) gauge bosons in the bulk we calculate unification condition using Eq. (5),

$$\eta_E = \frac{3\eta_L - 13}{2} \,. \tag{6}$$

For $\eta_L = 5, \eta_E = 1$ we get CASE 2

$$\beta = (48/5, 4, 0)\,, \quad \tilde{\beta} = (24/5, 2, 0)\,. \tag{7}$$

Then three generations of full **27** multiplets contribute to evolution of gauge couplings from the scale $m_{\rm t}$ onwards and changes coefficient β even though their effect does not show up in the difference $\beta_i - \beta_j$ which appears in the quantity R_{ij}.

Next let us consider the case when above μ_0 SU(3) and U(1) gauge bosons have bulk excitations but SU(2) do not. Again because bulk matter transform under bulk gauge group only $\overline{U}, \overline{D}$ and $\overline{E}$ can have bulk excitations. Then we calculate unification condition using Eq. (5)

$$\eta_D = \frac{14 - 2\eta_E - 5\eta_U}{3} \tag{8}$$

and two pairs of $\overline{E}$ and two pairs of $\overline{U}$ can have bulk excitations. Then below and above μ_0 the beta coefficients are

$$\beta = (48/5, 4, 0)\,, \quad \tilde{\beta} = (28/5, 0, -4)\,. \tag{9}$$

In this scenario we get,

$$R_{12} = R_{13} = R_{23} = 1\,. \tag{10}$$

The unification picture is given in Fig. 1 CASE 3. Second case is when three $\overline{D}$ pairs and one $\overline{U}$ pair have bulk excitations. In this case we get

$$\beta = (48/5, 4, 0)\,, \quad \tilde{\beta} = (14/5, 0, -2)\,. \tag{11}$$

In this scenario we get,

$$R_{12} = R_{13} = R_{23} = 1/2\,. \tag{12}$$

The unification picture is given in Fig. 1 CASE 4.

Even though strictly speaking Eq. (5) is valid at one loop, below μ_0 we have used two-loop running assuming all superpartners and extra E_6

matter near the scale $m_{\rm t}$. This does not affect gauge coupling unification appreciably as we see from Fig. 1. Also from Fig. 1 we see that for $\delta = 2$ unification scale is much lower than $\delta = 1$ case. We would like to stress that we have not introduced any new multiplet in *ad hoc* basis. We have only concentrated on multiplets contained within **27** representation of E_6 group. In next generation colliders 'zero modes' of these the E_6 exotic particles may be discovered [11]. Furthermore excited Kaluza–Klein modes of gauge bosons and matter fields will also be rigorously searched [12] in near future.

In the paper we have ignored heavy threshold corrections. This is because we have assumed that heavy GUT multiplets exist at or above the unification scale and they are degenerate in mass. This is in the spirit of an extended survival hypothesis in conventional unified models. However, in case some odd members of a heavy GUT multiplets have masses below the unification scale we will need to include heavy threshold corrections [10] to our results.

In a recently updated study Kang and Langacker have studied the discovery limits [13] of exotic E_6 multiplets in Fermilab Tevatron and CERN LHC. They conclude that multiplets as light as 200 GeV can be probed directly in Tevatron and as light as 1 TeV in LHC. A natural question in this context is: are we allowed to put exotic E_6 multiplets with masses near $m_{\rm t}$? In other words: are they excluded by direct and indirect experimental searches? The answer is that with present experimental accuracy exotic E_6 multiplets are allowed at around the scale $m_{\rm t}$ [14].

I would like to thank M.S. Berger for communications on Eq. (2).

REFERENCES

[1] I. Antoniadis, *Phys. Lett.* **B246**, 377 (1990); I. Antoniadis, C. Munoz, M. Quiros, *Nucl. Phys.* **B397**, 515 (1993); I. Antoniadis, K. Benakli, M. Quiros, *Phys. Lett.* **B331**, 313 (1994).

[2] N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, *Phys. Lett.* **B429**, 263 (1998).

[3] E. Witten, *Nucl. Phys.* **B471**, 135 (1996).

[4] J.D. Lykken, *Phys. Rev.* **D54**, 3693 (1996).

[5] C.P. Bachas, *J. High Energy Phys.* **9811**, 023, (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, *Phys. Lett.* **B436**, 257 (1998); G. Shiu, H.S-H. Tye, *Phys. Rev.* **D58**, 106007 (1998).

[6] K.R. Dienes, E. Dudas, T. Gherghetta, *Phys. Lett.* **B436**, 55 (1998); K.R. Dienes, E. Dudas, T. Gherghetta, *Nucl. Phys.* **B537**, 47 (1999); T. Kobayashi, J. Kubo, M. Mondragon, G. Zoupanos, *Nucl. Phys.* **B550**, 99 (1999).

[7] F. Gursey, P. Ramond, P. Sikivie, *Phys. Lett.* **B60** 177 (1976); F. Gursey, M. Serdaroglu, *Lett. Nuovo Cim.* **21**, 28 (1978).

[8] C.D. Carone, *Phys. Lett.* **B454**, 70 (1999).

[9] N. Borghini, Y. Gouverneur, M.H.G. Tytgat, *J. High Energy Phys.* **0111**, 024, (2001); G. Altarelli, F. Feruglio, *Phys. Lett.* **B511**, 257 (2001); H. Cheng, B.A. Dobrescu, C.T. Hill, *Nucl. Phys.* **B573**, 597 (2000); K. Huitu, T. Kobayashi, *Phys. Lett.* **B470**, 90 (1999); M. Masip, *Phys. Rev.* **D62**, 065011, (2000); D. Dumitru, S. Nandi, *Phys. Rev.* **D62**, 046006 (2000); E.G. Floratos, G.K. Leontaris, *Phys. Lett.* **B465**, 95 (1999); Guy F. de Teramond, *Phys. Rev.* **D60**, 095010 (1999); S.A. Abel, S.F. King, *Phys. Rev.* **D59**, 095010 (1999); D. Ghilencea, G.G. Ross, *Phys. Lett.* **B442**, 165, (1998); *Nucl. Phys.* **B569**, 391 (2000); A. Perez-Lorenzana, R.N. Mohapatra, *Nucl. Phys.* **B559**, 255 (1999); B. Brahmachari, *Phys. Rev.* **D65**, 067502 (2002); G. Bhattacharyya, S. Goswami, A. Raychaudhuri, *Phys. Rev.* **D66**, 033008 (2002); A.S. Dighe, A.S. Joshipura, *Phys. Rev.* **D64**, 073012 (2001); Z. Berezhiani, I. Gogoladze, A. Kobakhidze, *Phys. Lett.* **B522**, 107 (2001); Y. Kawamura, *Prog. Theor. Phys.* **105**, 691 (2001).

[10] R. Contino, L. Pilo, R. Rattazzi, E. Trincherini, *Nucl. Phys.* **B622**, 227 (2002); P.H. Chankowski, A. Falkowski, S. Pokorski, *J. High Energy Phys.* **0208**, 003 (2002); A. Hebecker, A. Westphal, *Ann. Phys.* **305**, 119 (2003); K. Choi, Ian-Woo Kim, talk given at Summer Institute 2003, Fuji-Yoshida, Japan, 12–19 Aug 2003, `hep-th/0312006`.

[11] V.D. Barger, E.W.N. Glover, K. Hikasa, W.Y. Keung, M.G. Olsson, C.J. Suchyta III, X.R. Tata, *Phys. Rev.* **D35**, 3366 (1987); K.I.Y. Bigi, Y.L. Dokshitzer, V.A. Khoze, J.H. Kuhn, P.M. Zerwas *Phys. Lett.* **B181**, 157 (1986); A.K. Ciftci, R. Ciftci, S. Sultansoy, *Phys. Rev.* **D65**, 055001 (2002).

[12] S. Gopalakrishna, M. Perelstein, J.D. Wells, `hep-ph/0110339`; T.G. Rizzo, *Phys. Rev.* **D64**, 095010 (2001); O.J.P. Eboli, M.B. Magro, P. Mathews, P.G. Mercadante, *Phys. Rev.* **D64**, 035005 (2001); I. Antoniadis, K. Benakli, *Int. J. Mod. Phys.* **A15**, 4237 (2000); S. Cullen, M. Perelstein, M.E. Peskin, *Phys. Rev.* **D62**, 055012 (2000); N. Arkani-Hamed, Y. Grossman, M. Schmaltz, *Phys. Rev.* **D61**, 115004 (2000); T.G. Rizzo, J.D. Wells, *Phys. Rev.* **D61**, 016007 (2000); D.A. Dicus, C.D. McMullen, S. Nandi, *Phys. Rev.* **D65**, 076007 (2002).

[13] J. Kang, P. Langacker, *Phys. Rev.* **D71**, 035014 (2005).

[14] J. Kang, P. Langacker, T. Li, *Phys. Rev.* **D71**, 015012 (2005); S. Hesselbach, F. Franke, H. Fraas, *Eur. Phys. J.* **C23**, 149 (2002); M. Drees, A. Yamada, *Phys. Rev.* **D53**, 1586 (1996); M. Drees, Published in Iwate Linear Colliders 1995, p. 587,*QCD* **161**, W579 (1995), `hep-ph/9509425`.