$a_0 \to \pi^0\eta$ DECAY IN QCD SUM RULES

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We investigate the $a_0 \to \pi^0\eta$ decay in the framework of QCD sum rules. We estimate the coupling constant $g_{a_0\pi\eta}$ which plays an essential role in the analysis of physical processes involving $a_0(980)$ meson. We also estimate the coupling constant $g_{a_0\pi\eta}$ by using the experimental limits of the decay width of the $a_0 \to \pi^0\eta$ decay and compare with our QCD sum rule result.

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The flavor SU(3) forms an approximate global symmetry of hadron spectrum according to which mesons are classified as bound states of a quark and antiquark ($\overline{q}q$) and they are placed in nonet representations of SU(3) group. However, whether light scalar mesons form a scalar nonet is still an open question. In the constituent quark model, $a_0(980)$ meson is represented by $\overline{q}q$ state [1], however, it is given by a four quark state $\overline{q}^2q^2$ [2] in the framework of MIT-bag model. Another possible structure of $a_0(980)$ meson is that of a $\overline{K}K$ molecule [3]. Understanding the nature and the quark substructure of the scalar mesons is still an open problem in hadron physics.

The scalar mesons play an important role in the hadronic decays. In the $V^0 \to \pi^0\eta\gamma$ decays, where V represents the lowest multiplet of vector mesons ρ, ω and ϕ, the $\pi^0\eta$ system is a scalar isovector state $I(J^{\rm PC}) = 1(0^{++})$. In particular, the analysis of the $V^0 \to \pi^0\eta\gamma$ decays requires the coupling constant $g_{a_0\pi\eta}$ [4].

At present, there is no quantitative theory which is based on a fundamental Lagrangian to calculate the properties of hadrons. Asymptotic freedom property of QCD allows perturbative calculations of strong interactions at short distances. However, in the long distance region perturbation theory fails and thus one has to confront the nonperturbative effects. The method of QCD sum rules developed by Shifman, Vainstain and Zakharov [5] is a

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very powerful and informative tool to be used in hadron phenomenology in the nonperturbative region. The decay channels of $a_0(980)$ meson can be analyzed in the context of QCD sum rules. In this work, we study $a_0 \to \pi^0\eta$ decay in the framework of three-point QCD sum rules and we obtain the coupling constant $g_{a_0\pi\eta}$.

In the QCD sum rule method, hadrons are represented by their interpolating quark currents. The correlation function of these currents is treated within the framework of the operator product expansion (OPE), where the short and long distance quark–gluon interactions are separated. In order to study the QCD sum rule for $a_0\pi\eta$-vertex, we consider the three-point correlation function

$$T_{\mu\nu}(p,p';q) = i\int d^4x\ d^4y\ e^{ip'\cdot y}e^{-ip\cdot x}\langle 0|T\{j_\mu^{\pi^0}(0)j^{a_0}(x)j_\nu^\eta(y)\}|0\rangle\,, \qquad (1)$$

where $j_\mu^{\pi^0}$, j^{a_0}, and j_ν^η are the interpolating currents for π^0, a_0 and η mesons, respectively. The interpolating currents in terms of quark fields are

$$\begin{aligned} j_\mu^{\pi^0} &= \frac{1}{2}(\overline{u}\gamma_\mu\gamma_5 u - \overline{d}\gamma_\mu\gamma_5 d)\,, \\ j_\nu^\eta &= \frac{1}{\sqrt{6}}(\overline{u}\gamma_\nu\gamma_5 u + \overline{d}\gamma_\nu\gamma_5 d) - \frac{2}{\sqrt{6}}\overline{s}\gamma_\nu\gamma_5 s\,, \\ j^{a_0} &= \frac{1}{2}(\overline{u}u - \overline{d}d)\,. \end{aligned} \qquad (2)$$

We choose the pseudovector current for the π^0 meson. We work in SU(2) flavor context with $m_u = m_d \equiv m_q$ and we work in the limit $m_q \to 0$.

The correlation function can be calculated phenomenologically in terms of hadron states. In order to construct the correlation function phenomenologically we consider the double dispersion relation satisfied by this function and we saturate this double dispersion relation for the $a_0 \to \pi\eta$ channel as

$$\begin{aligned} T_{\mu\nu}(p,p';q) \;=\; & \frac{\langle 0 \mid j^{a_0} \mid a_0(p)\rangle\langle a_0(p) \mid j_\mu^{\pi^0} \mid \eta(p')\rangle\langle \eta(p') \mid j_\nu^\eta \mid 0\rangle}{(p^2 - m_{a_0}^2)(p'^2 - m_\eta^2)} \\ & + \int_{s_0}^{\infty} ds \int_{s'_0}^{\infty} ds' \frac{\rho_{\mu\nu}^{\rm cont.}(s,s')}{(s-p^2)(s'-p'^2)} + \text{subtraction terms}\,, \end{aligned} \qquad (3)$$

where the hadronic spectral density $\rho^{\rm cont.}(s,s')$ includes the contributions of higher resonances and the continuum. The first part of Eq. (3) corresponds to low values of s and s' where the states are treated in the narrow-width approximation proportional to double δ-function. The current particle matrix element for η-meson is given by

$$\langle \eta(p') \mid j^\eta_\nu \mid 0\rangle = -i f_\eta p'_\nu\,, \tag{4}$$

where f_η is the decay constant of η-meson. The scalar current j^{a_0} is assumed to have a non-vanishing matrix element between the vacuum and $a_0(980)$ meson state

$$\langle 0 \mid j^{a_0} \mid a_0(p)\rangle = \lambda_{a_0}\,, \tag{5}$$

where λ_{a_0} is called the overlap amplitude. The overlap amplitude λ_{a_0} which was calculated in [6] is needed in the sum rule.

The matrix element of pseudovector current for π^0 is given as

$$\langle a_0(p) \mid j^{\pi^0}_\mu \mid \eta(p')\rangle = \frac{g_{a_0\pi\eta}}{m_{a_0}}\,(p-p')_\mu\,, \tag{6}$$

where $q = p - p'$. The correlation function in Eq. (1) can be decomposed as

$$T_{\mu\nu}(p,p';q) = T_1\; p_\mu p'_\nu + T_2\; p'_\mu p'_\nu + T_3\; p_\mu p_\nu + T_4\; p'_\mu p_\nu + T_5\; g_{\mu\nu}\,. \tag{7}$$

Since $\langle \eta(p') \mid j^\eta_\nu \mid 0\rangle \sim p'_\nu$, the physical part of the correlation function includes p'_ν structure. The corresponding structure in the theoretical part of correlation function is $p_\mu p'_\nu$. We are therefore interested in the invariant function T_1 in this work.

Using the Borel transformation

$$\mathcal{B}_{M^2}\left[\frac{1}{(s+Q^2)^k}\right] = \left[\frac{1}{(k-1)!}\right]\frac{e^{-s/M^2}}{M^{2(k-1)}}$$

with respect to $Q^2 = -p^2$ we then obtain the following result after double Borel transformation of T_1

$$\begin{aligned}\mathcal{B}_{M_1^2}\;\mathcal{B}_{M_2^2}\;T_1 \;=\; & -i\,\frac{g_{a_0\pi\eta}}{m_{a_0}}\,\lambda_{a_0} f_\eta\; e^{-m_{a_0}^2/M_1^2}\; e^{-m_\eta^2/M_2^2} \\ & + \int_{s_0}^{\infty} ds \int_{s'_0}^{\infty} ds'\,\rho^{\text{cont.}}\, e^{-s/M_1^2}\; e^{-s'/M_2^2}\,. \end{aligned} \tag{8}$$

The unknown subtraction term in the dispersion relation disappears after the Borel transformation.

We then calculate the perturbative and nonperturbative contributions to the three-point correlation function of $a_0(p) \to \pi(q)\eta(p')$ decay. In the

Euclidean region defined by $p^2 = -Q^2$, $p'^2 = -Q'^2$ where Q^2 and Q'^2 are large, the perturbative contribution can be approximated by the lowest order quark loop diagram shown in Fig. 1(a). Therefore we do not consider the diagram in Fig. 1(b). The contribution of the diagram in Fig. 1(a) can be written as

$$F_{1a} = N_c \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[D_F(k)\Gamma_1 D_F(k+p')\Gamma_2 D_F(k+p)\Gamma_3\right] , \tag{9}$$

where the quark propagator is $D_F(p_1) = i/(\not{p}_1 - m_q^2)$, Γ_i are the vertex functions of quark currents and $N_c = 3$ is the color factor.

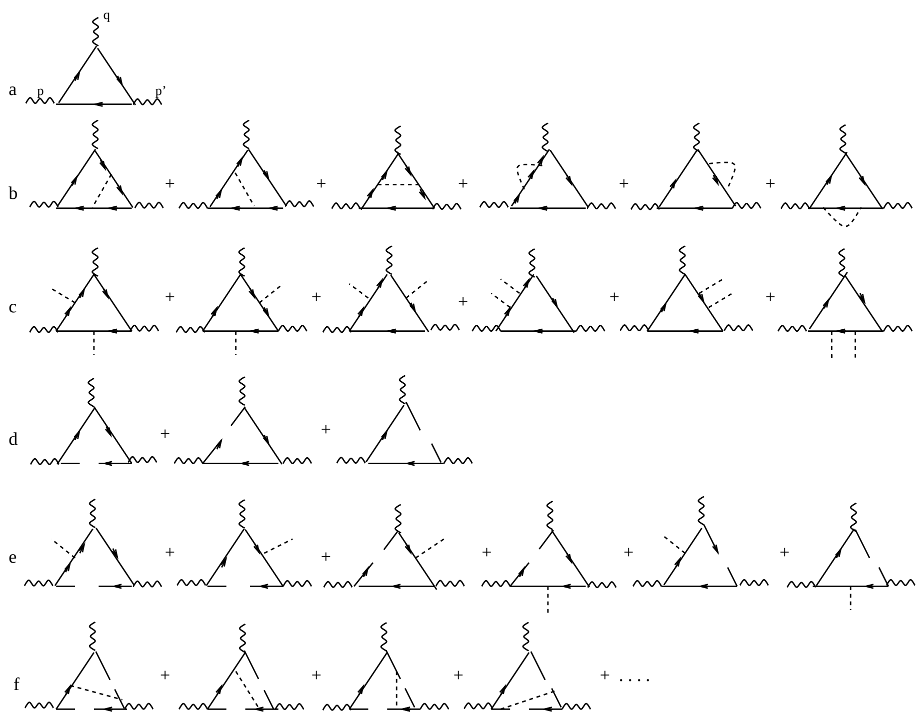

Fig. 1. Possible diagrams for three-point correlation function for $d \leq 6$. (a) lowest order bare-loop diagrams, (b) bare loop with a virtual gluon, (c) gluon condensate diagrams (d) quark condensate diagrams (e) quark condensate diagrams with one external field, and (f) diagrams obtained by simultaneous cutting of two quark lines in the diagrams (b).

Diagrams of power corrections shown in Fig. 1(c) and Fig. 1(f) do not give any contributions to $a_0(p) \to \pi(q)\eta(p')$ decay, because it is easily seen that the trace terms in the expressions corresponding to diagrams in Fig. 1(c) and Fig. 1(f) vanish. The relevant Feynman diagrams of power corrections for $a_0(p) \to \pi(q)\eta(p')$ decay are shown in Fig. 1(d) and Fig. 1(e). The last two diagrams in Fig. 1(d) and the last four diagrams in Fig. 1(e) vanish after double Borel transformation. We therefore do not consider them in the following. The remaining three diagrams contribute to $a_0(p) \to \pi(q)\eta(p')$ decay.

We perform the calculations of the power corrections in the fixed-point gauge, $x_\mu A^\mu = 0$. The general forms of the contributions corresponding to Feynman diagrams are derived with respect to their dimensions. For the first diagram in Fig. 1(d), there are four contributions with different dimensions for $d \leq 6$ as

$$
\begin{aligned}
F_{1d}(3d) &= N_c\langle\overline{\psi}\psi\rangle \, \frac{1}{4} \, \mathrm{Tr}\left[\mathit{\Gamma}_1 D_F(p')\mathit{\Gamma}_2 D_F(p)\mathit{\Gamma}_3\right] , \\
F_{1d}(4d) &= N_c \, \frac{m_q}{16} \, \langle\overline{\psi}\psi\rangle \, \frac{\partial}{\partial p_\lambda} \, \mathrm{Tr}\left[\mathit{\Gamma}_1 D_F(p')\mathit{\Gamma}_2 D_F(p)\mathit{\Gamma}_3\gamma_\lambda\right] , \\
F_{1d}(5d) &= -N_c \, \frac{m_q^2}{16} \, \langle\overline{\psi}\psi\rangle\frac{1}{2} \, \frac{\partial}{\partial p_\lambda} \, \frac{\partial}{\partial p_\lambda}\mathrm{Tr}\left[\mathit{\Gamma}_1 D_F(p')\mathit{\Gamma}_2 D_F(p)\mathit{\Gamma}_3\right] \\
&\quad -N_c \, \frac{1}{32} \, \langle\overline{\psi}gG^c_{\lambda\lambda'}(\lambda^c/2)\sigma_{\lambda\lambda'}\psi\rangle\frac{1}{2} \, \frac{\partial}{\partial p_\lambda} \, \frac{\partial}{\partial p_\lambda} \\
&\quad \times\mathrm{Tr}\left[\mathit{\Gamma}_1 D_F(p')\mathit{\Gamma}_2 D_F(p)\mathit{\Gamma}_3\right] \\
&\quad +N_c \, \frac{i}{96} \, \langle\overline{\psi}gG^c_{\lambda\lambda'}(\lambda^c/2)\sigma_{\lambda\lambda'}\psi\rangle\frac{1}{2} \, \frac{\partial}{\partial p_\lambda} \, \frac{\partial}{\partial p_{\lambda'}} \\
&\quad \times\mathrm{Tr}\left[\mathit{\Gamma}_1 D_F(p')\mathit{\Gamma}_2 D_F(p)\mathit{\Gamma}_3\sigma_{\lambda\lambda'}\right] , \\
F_{1d}(6d) &= N_c \, \frac{i}{2} \, \frac{\partial}{\partial p_\lambda} \, \frac{\partial}{\partial p_{\lambda'}} \, \frac{\partial}{\partial p_{\lambda''}} \\
&\quad \times\Big\{B_1 g_{\lambda'\lambda''} \, \mathrm{Tr}\left[\mathit{\Gamma}_1 D_F(p')\mathit{\Gamma}_2 D_F(p)\mathit{\Gamma}_3\gamma_\lambda\right] \\
&\quad +C_1 g_{\lambda\lambda''} \, \mathrm{Tr}\left[\mathit{\Gamma}_1 D_F(p')\mathit{\Gamma}_2 D_F(p)\mathit{\Gamma}_3\gamma_{\lambda'}\right] \\
&\quad +D_1 g_{\lambda'\lambda} \, \mathrm{Tr}\left[\mathit{\Gamma}_1 D_F(p')\mathit{\Gamma}_2 D_F(p)\mathit{\Gamma}_3\gamma_{\lambda''}\right]\Big\} ,
\end{aligned}
\tag{10}
$$

where B_1, C_1, and D_1 are constants which are given as $D_1 = B_1 = -\frac{ig^2\langle\overline{\psi}\psi\rangle^2}{(3^5\times 2^4)}$ and $C_1 = -5B_1$. In the notation of $F_{1d}(Nd)$, $1d$ shows the diagram (d) in Fig. 1 and (Nd) denotes the dimension N.

The first diagram in Fig. 1(e) has the $d = 5$ and $d = 6$ contributions that are given by

$$
\begin{aligned}
F_{1e}(5d)_1 &= N_c \, \frac{2g}{48\times 4} \, \langle\overline{\psi}G^c_{\lambda\rho}(\lambda^c/2)\sigma_{\lambda\rho}\psi\rangle \\
&\quad \times\frac{\partial}{\partial k_\lambda}\mathrm{Tr}\left.\left[\mathit{\Gamma}_1 D_F(p')\mathit{\Gamma}_2 D_F(p)\gamma_\rho D_F(p-k)\mathit{\Gamma}_3\sigma_{\lambda\rho}\right]\right|_{k=0} ,
\end{aligned}
$$

$$
\begin{aligned}
&F_{1e}(6d)_1 = -\frac{i}{3}\ N_c\ \frac{g^2}{3^3\times 2^4}\ \langle\overline{\psi}\psi\rangle^2\ \mathrm{Tr}[(\lambda^c/2)(\lambda^c/2)]\frac{\partial}{\partial k_\tau}\frac{\partial}{\partial k_\lambda}\\
&\times\mathrm{Tr}\left[\Gamma_1 D_F(p')\Gamma_2 D_F(p)\gamma_\rho D_F(p-k)\Gamma_3(\delta_{\tau\rho}\gamma_\lambda-\delta_{\tau\lambda}\gamma_\rho)\right]\Big|_{k=0}\\
&+\frac{i}{2}\ N_c\ \frac{g^2}{3^3\times 2^4}\ \langle\overline{\psi}\psi\rangle^2\ \mathrm{Tr}[(\lambda^c/2)(\lambda^c/2)]\frac{\partial}{\partial k_\tau}\frac{\partial}{\partial p_\lambda}\\
&\times\mathrm{Tr}\left[\Gamma_1 D_F(p')\Gamma_2 D_F(p)\gamma_\rho D_F(p-k)\Gamma_3(\delta_{\lambda\tau}\gamma_\rho-\delta_{\lambda\rho}\gamma_\tau-i\epsilon_{\lambda\tau\rho\xi}\gamma_5\gamma_\xi)\right]\Big|_{k=0}\,.
\end{aligned}
\tag{11}
$$

Similarly, we obtain the $d=5$ and $d=6$ contributions from the second diagram in Fig. 1(e) as

$$
\begin{aligned}
&F_{1e}(5d)_2 = -N_c\ \frac{2g}{48\times 4}\ \langle\overline{\psi}G^c_{\lambda\rho}(\lambda^c/2)\sigma_{\lambda\rho}\psi\rangle\\
&\times\frac{\partial}{\partial k_\lambda}\mathrm{Tr}\left[\Gamma_1 D_F(p')\gamma_\rho D_F(p'-k)\Gamma_2 D_F(p-k)\Gamma_3\sigma_{\lambda\rho}\right]_{k=0}\,,\\
&F_{1e}(6d)_2 = -\frac{i}{3}\ N_c\ \frac{g^2}{3^3\times 2^4}\ \langle\overline{\psi}\psi\rangle^2\ \mathrm{Tr}[(\lambda^c/2)(\lambda^c/2)]\frac{\partial}{\partial k_\tau}\frac{\partial}{\partial k_\lambda}\\
&\times\mathrm{Tr}\left[\Gamma_1 D_F(p')\gamma_\rho D_F(p'-k)\Gamma_2 D_F(p-k)\Gamma_3(\delta_{\tau\rho}\gamma_\lambda-\delta_{\tau\lambda}\gamma_\rho)\right]\Big|_{k=0}\\
&+\frac{i}{2}\ N_c\ \frac{g^2}{3^3\times 2^4}\ \langle\overline{\psi}\psi\rangle^2\ \mathrm{Tr}[(\lambda^c/2)(\lambda^c/2)]\frac{\partial}{\partial k_\tau}\frac{\partial}{\partial p_\lambda}\\
&\times\mathrm{Tr}\left[\Gamma_1 D_F(p')\gamma_\rho D_F(p'-k)\Gamma_2 D_F(p-k)\Gamma_3(\delta_{\lambda\tau}\gamma_\rho-\delta_{\lambda\rho}\gamma_\tau-i\epsilon_{\lambda\tau\rho\xi}\gamma_5\gamma_\xi)\right]\Big|_{k=0}\,.
\end{aligned}
\tag{12}
$$

For $a_0(p)\to\pi(q)\eta(p')$ decay the vertex functions are $\Gamma_1=-(i/\sqrt{6})\gamma_\nu\gamma_5$, $\Gamma_2=-(i/2)\gamma_\mu\gamma_5$, and $\Gamma_3=-(i/2)I$. Using these vertex functions in Eqs. (9)–(12) we then get the non-vanishing contributions from power corrections to the correlation function $T_{\mu\nu}(p,p';q)$ in the limit $m_q\to 0$ as

$$
\begin{aligned}
&T_{\mu\nu}(p,p';q) = F_{1d}(3d)+F_{1d}(5d)+F_{1e}(5d)_1+F_{1e}(5d)_2\\
&= -i\frac{1}{\sqrt{6}}\ \frac{3}{4}\ \langle\overline{\psi}\psi\rangle\ \frac{1}{p'^2p^2}\left[p_\mu p'_\nu+p_\nu p'_\mu-p\cdot p' g_{\mu\nu}\right]\\
&-i\frac{3}{16\sqrt{6}}\left\langle\overline{\psi}gG^n_{\lambda\lambda'}\left(\frac{\lambda^n}{2}\right)\sigma_{\lambda\lambda'}\psi\right\rangle\left(\frac{1}{p^6\,p'^6}\right)\\
&\Big\{p_\mu[2p^2p'^4p_\nu+(p'^4p^2-2p^2p'^2p\cdot p'+p^4p'^2)p'_\nu]\\
&+p^2(2p'^2p\cdot p'-p^2p'^2)(-p_\nu p'_\mu+p\cdot p' g_{\mu\nu})\\
&+p'^2p^2[p'^2p_\nu p'_\mu+2p^2p'_\mu p'_\nu-p'^2p\cdot p' g_{\mu\nu}-p^2p'^2g_{\mu\nu}]\Big\}
\end{aligned}
$$

$$+\frac{3i}{16\sqrt{6}} \left\langle \overline{\psi} g G^c_{\lambda\rho} \left(\frac{\lambda^c}{2}\right) \sigma_{\lambda\rho}\psi \right\rangle \frac{p_\mu p'_\nu + p_\nu p'_\mu - p\cdot p' g_{\mu\nu}}{p'^2 p^4}$$
$$+\frac{i}{16\sqrt{6}}\frac{1}{8} \left\langle \overline{\psi} g G^c_{\lambda\rho} \left(\frac{\lambda^c}{2}\right) \sigma_{\lambda\rho}\psi \right\rangle \left\{ \frac{8}{p^2 p'^6}[-p'^2 p_\nu p'_\mu + 3p'^2 p_\mu p'_\nu] \right.$$
$$+p'^2(p\cdot p')g_{\mu\nu} + \frac{8}{p^4 p'^4}[2p\cdot p' p_\nu p'_\mu + 2p^2 p'_\mu p'_\nu - 2p'^2 p_\mu p_\nu$$
$$\left. +2p\cdot p' p_\mu p'_\nu + (-2(p\cdot p')^2 + p^2)g_{\mu\nu}] \right\}, \quad (13)$$

where $\langle\overline{\psi} g G^c_{\lambda\rho}(\lambda^c/2)\sigma_{\lambda\rho}\psi\rangle = m_0^2\langle\overline{\psi}\psi\rangle$. The lowest order perturbative quark-loop diagram and the $d = 6$ contributions $F_2(6d)$, $F_3(6d)_1$ and $F_3(6d)_2$ do not make any contribution in the limit $m_q \to 0$.

The structure $p_\mu p'_\nu$ is chosen to compare the theoretical and phenomenological parts and to obtain the coupling constant $g_{a_0\pi\eta}$. We then find the theoretical part of the invariant function T_1 for $a_0(p) \to \pi(q)\eta(p')$ decay as

$$T_1 = -\frac{i}{\sqrt{6}} \langle\overline{\psi}\psi\rangle \frac{3}{4} \frac{1}{p'^2 p^2} + \frac{i}{\sqrt{6}} \langle\overline{\psi}\psi\rangle \frac{m_0^2}{4} \left[\frac{1}{p^4\, p'^2} + \frac{1}{p'^4 p^2} - \frac{q^2}{p^4 p'^4}\right]. \quad (14)$$

The first line in Eq. (14) is $d = 3$ and the second line $d = 5$ contributions. We perform double Borel transform with respect to the variables $Q^2 = -p^2$ and $Q'^2 = -p'^2$, using $\mathcal{B}_{M^2}(1/Q^2)^k = 1/[(k-1)!M^{2(k-1)}]$, we then obtain

$$\mathcal{B}_{M_1^2}\, \mathcal{B}_{M_2^2}\, T_1 = -\frac{i}{\sqrt{6}} \langle\overline{\psi}\psi\rangle \left\{\frac{3}{4} + \frac{m_0^2}{4} \left[\frac{1}{M_1^2} + \frac{1}{M_2^2} + \frac{q^2}{M_1^2 M_2^2}\right]\right\}, \quad (15)$$

where M_1^2 and M_2^2 are Borel masses corresponding to a_0 and η mesons, respectively, and $\langle\overline{\psi}\psi\rangle = \langle\overline{u}u\rangle + \langle\overline{d}d\rangle \approx 2\langle\overline{u}u\rangle$.

The η–η' mixing may have a considerable effect on the $g_{a_0\pi\eta}$ coupling constant. We note that the physical eigenstates $\mid\eta\rangle$ and $\mid\eta'\rangle$ can be expressed as the linear combinations of two states $\mid\eta_q\rangle$ and $\mid\eta_s\rangle$ as

$$\begin{aligned} \mid\eta\rangle &= \cos\phi \mid\eta_q\rangle - \sin\phi\mid\eta_s\rangle, \\ \mid\eta'\rangle &= \sin\phi\mid\eta_q\rangle + \cos\phi\mid\eta_s\rangle, \end{aligned} \quad (16)$$

where $\mid\eta_q\rangle = \frac{1}{\sqrt{2}}(\overline{u}u + \overline{d}d)$ and $\mid\eta_s\rangle = \overline{s}s$, and ϕ is the mixing angle [7]. The corresponding currents in terms of quark fields can be taken as the axial vector currents $j^q_{5\mu} = \frac{1}{\sqrt{2}}(\overline{u}\gamma_\mu\gamma_5 u + \overline{d}\gamma_\mu\gamma_5 d)$ and $j^s_{5\mu} = \overline{s}\gamma_\mu\gamma_5 s$.

After performing a double Borel transformation the higher resonance and continuum contributions are eliminated, and then matching the theoretical and physical parts given in Eq. (8) and (15), respectively, we then obtain the coupling constant $g_{a_0\pi\eta}$ including η–η' mixing effects as

$$g_{a_0\pi\eta} = \cos\phi\,\frac{2\langle\overline{u}u\rangle}{\sqrt{2}}\frac{m_{a_0}}{\lambda_{a_0}f_\eta}e^{m_{a_0}^2/M_1^2}e^{m_\eta^2/M_2^2}\left\{\frac{3}{4}+\frac{m_0^2}{4}\left[\frac{1}{M_1^2}+\frac{1}{M_2^2}+\frac{q^2}{M_1^2M_2^2}\right]\right\}. \tag{17}$$

For the numerical evaluation of the sum rule the values $m_0^2 = (0.82 \pm 0.02)$ GeV^2, $\langle\overline{u}u\rangle = (-0.014 \pm 0.002)$ GeV^3 [8], and $m_{a_0} = 0.98$ GeV, $m_\eta = 0.547$ GeV are used [9]. The overlap amplitude λ_{a_0} was determined by employing QCD sum rules method as $\lambda_{a_0} = (0.21 \pm 0.05)$ GeV^2 [6]. The η-meson decay constant is obtained as $f_\eta = (0.13 \pm 0.01)$ GeV using the experimental data given by Particle Data Group [9]. The recent value of the mixing angle is $\phi = (37.7 \pm 2.4)^\circ$ [7]. We note that in Eq. (17) q^2 is large and negative.

Fig. 2 shows that the stability region of the coupling constant $g_{a_0\pi\eta}$ is in the intervals $2 \leq M_1^2 \leq 5$ GeV^2 and $0.4 \leq M_2^2 \leq 0.5$ GeV^2 for $q^2 = -1$ GeV^2. The dependence of the coupling constant $g_{a_0\pi\eta}$ on the Borel parameters M_1^2 and M_2^2 are studied in Fig. 3. The limits of the stability

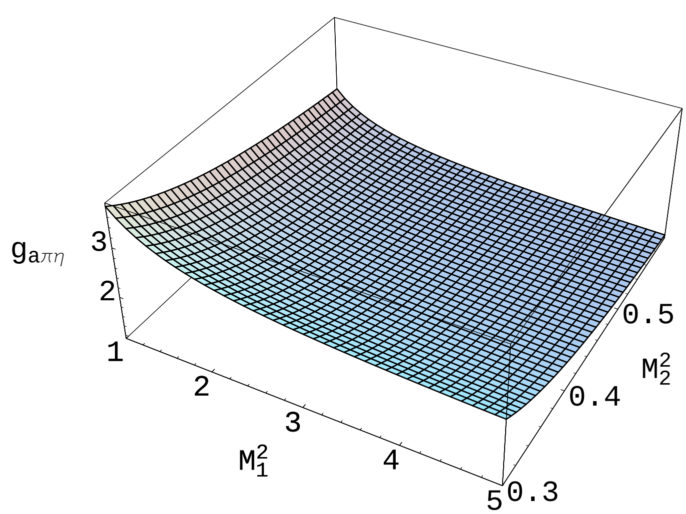

Fig. 2. The coupling constant $g_{a_0\pi\eta}$ as a function of the Borel parameters M_1^2 and M_2^2.

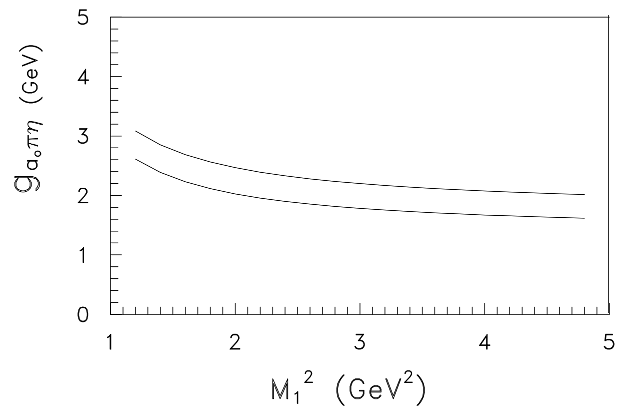

Fig. 3. The coupling constant $g_{a_0\pi\eta}$ as a function of the Borel parameter M_1^2 for different values of the Borel parameter M_2^2. The curves denote the limits of the stability region.

region is shown for $M_2^2 = 0.4$ GeV^2 and $M_2^2 = 0.5$ GeV^2 in Fig. 3 for $q^2 = -1$ GeV^2. We choose the middle value of M_1^2 as $M_1^2 = 3$ GeV^2 and we find the coupling constant $g_{a_0\pi\eta}$ as $g_{a_0\pi\eta} = (2.0 \pm 0.3)$ GeV. We also vary q^2 from -0.5 GeV^2 to -1.5 GeV^2, and we observe that our result is practically constant for this interval of q^2 values.

The coupling constant $g_{a_0\pi\eta}$ given in Eq. (6) is related to the decay width of the $a_0 \to \pi^0\eta$ as

$$\Gamma(a_0 \to \pi^0\eta) = \frac{g^2_{a_0\pi\eta}}{16\pi\ m_{a_0}} \sqrt{\left[1 - \frac{(m_\pi + m_\eta)^2}{m^2_{a_0}}\right]\left[1 - \frac{(m_\pi - m_\eta)^2}{m^2_{a_0}}\right]}\,. \quad (18)$$

If we use the experimental limits given by Particle Data group [9] as 50 MeV$<$ $\Gamma(a_0 \to \pi^0\eta) < 100$ MeV, we obtain the coupling constant $g_{a_0\pi\eta}$ as $1.96 < g_{a_0\pi\eta} < 2.78$ GeV. Our QCD sum rule result is consistent with this result. Furthermore, if we compare our value of the coupling constant $g_{a_0\pi\eta}$ with the value calculated using the light-cone QCD sum rule [10], our value is less than the limit values of the light-cone QCD sum rule result which is $2.6 \le g_{a_0\pi\eta} \le 3.4$ GeV.

The KLOE Collaboration estimated the coupling constant $g_{a_0K^+K^-}$ as $g_{a_0K^+K^-} = (2.3 \pm 0.7)$ GeV [11]. We see that the SU(3) relation $g_{a_0\pi\eta} = 0.85$ $g_{a_0K^+K^-}$ is satisfied within reasonable limits.

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