Vol. 36 (2005) *ACTA PHYSICA POLONICA B* No 6

A STUDY ON EVEN–EVEN Hf ISOTOPES USING THE CLUSTER MODEL

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(Received February 25, 2005)

In the present work the spectra of even–even Hf isotopes are studied through the selecting of core–cluster decomposition of the parent nucleus. The considered partition should give internal stability of the core–cluster combination. The modified Woods–Saxon and Coulomb potentials are used to reproduce the spectra of even–even Hf isotopes where the core-radius is taken as a free parameter. The theoretical calculations of the excitation energies and the transition probabilities $B(E2)$ of the ground state band are compared to the experimental data of the considered Hf isotopes. The obtained results reflect the ability of describing the pure rotational ground state band of even–even Hf isotopes through the core–cluster decomposition model.

PACS numbers: 21.60.Gx, 23.20.Js, 27.50.+e

1. Introduction

The cluster model of nuclear structure is well established for light nuclei [1–3]. There are now indications that one can apply the nuclear cluster model to the rare earth and actinide deformed nuclei [4–6]. The first question to be addressed, when proposing a cluster model description of a given nucleus, is the identity of the cluster and core to be used. Over 30 years ago, Ikeda *et al.* [7] suggested that for a nuclear molecule-like structure to appear, its excitation energy needs to be near or above the threshold energy for breakup into the constituent clusters and also below the top of the potential barrier. The basic assumption of this model is such that nuclei can be described accurately in terms of a system of two-component nuclei; each with its free state characteristics; interacting through a deep local potential. However, many binary decompositions satisfy this minimal requirement. Recently, Buck *et al.* [8] proposed that the choice must be done with reference to the binding energies of the cluster and core. Certainly, if the parent nucleus can be divided into a cluster and core, which are both doubly magic, this will be the most favored combination. However, this will not normally be possible, and in general a compromise to get the best balance of cluster and core individual binding energies will be needed. Therefore, one can search for the combination which yields the greatest overall deviation of summed experimental binding energies of the two bodies from the smoothly varying, underlying liquid drop value. In the present work we will study the spectroscopy of even–even Hf isotopes in the framework of core–cluster decomposition model. The methods of the cluster–core decomposition selection and the ground state excitation energy calculations are given in Section 2. A brief discussion of our obtained results in comparison to the experimental data is given in Section 3.

2. Method of calculations

In the present work the first step of calculations is to find the suitable core–cluster decomposition of the considered Hf isotopes. One can propose an alternative criterion for determining the appropriate cluster–core decomposition using more widely available information on experimental and theoretical liquid drop binding energies, which maximizes the expression [9]

$$D(1,2) = [B_A(Z_1,A_1) - B_M(Z_1,A_1)] + [B_A(Z_2,A_2) - B_M(Z_2,A_2)]\,, \quad (1)$$

where, B_A and B_M are the actual binding energy and the corresponding liquid drop value, respectively. This means that the parent Hf nucleus $(A_\mathrm{T}, Z_\mathrm{T})$ can be split into an even–even core (A_1, Z_1) and cluster (A_2, Z_2), where A_1, Z_1 are the mass number and the charge of the core nucleus and A_2, Z_2 are those for the cluster nucleus. The value of B_M is calculated according to the following formula [10]

$$B_M = a_V A - a_S A^{2/3} - \frac{a_\mathrm{C} Z^2}{A^{1/3}} - a_\mathrm{a}\frac{(A-2Z)^2}{A} + \delta\,, \quad (2)$$

where $a_V = 15.56$ MeV, $a_S = 17.23$ MeV, $a_\mathrm{C} = 0.7$ MeV, $a_\mathrm{a} = 23.285$ MeV and $\delta = 12/\sqrt{A}$ MeV.

For a given nucleus, when the the conditions $A_1 = A_\mathrm{T} - A_2$ and $Z_1 = Z_\mathrm{T} - Z_2$ are applied, D remains a function of two independent variables, the cluster mass and charge, (A_2, Z_2). A simpler form of D resulting from the observation that electric dipole transitions between low-laying bands of opposite parity in heavy nuclei are very weak. This implies that the total nuclear mass and charge should be distributed in the same proportions between core and cluster, resulting in the no-dipole constraint [9]

$$\frac{Z_1}{A_1} = \frac{Z_2}{A_2} = \frac{Z_\mathrm{T}}{A_\mathrm{T}}\,. \quad (3)$$

In the present calculations, for each cluster charge Z_2 the masses; A_2 and $A_2 + 2$; which come closest to satisfy this no-dipole constraint are such that

$$\frac{Z_2}{A_2} \geq \frac{Z_\mathrm{T}}{A_\mathrm{T}} \geq \frac{Z_2}{A_2 + 2}\,. \tag{4}$$

Therefore, we restrict our attention to two cluster masses for each cluster charge.

The spectra of the even–even Hf isotopes are studied by using the Bohr–Sommerfeld relation [11]

$$\int_{r1}^{r2} dr \sqrt{\frac{2\mu}{\hbar^2}[E - V_\mathrm{int}(r,R)]} = (2n+1)\frac{\pi}{2}\,, \tag{5}$$

where, r_1 and r_2 are the two inner most classical turning points and μ is the relative mass of the core–cluster decomposition. V is the interacting potential between the cluster and the core which depends on the relative distance r and the core radius R.

In this work, the deep local interacting potential between the core and the surface cluster is assumed as

$$V_\mathrm{int}(r,R) = V_N(r,R) + V_\mathrm{C}(r,R) + \eta^2 \frac{(l+0.5)^2}{2\mu r^2}\,, \tag{6}$$

where, V_N is the nuclear modified Woods–Saxon potential [12];

$$V_N(r,R) = -\frac{A_1 A_2}{A} V_0 \frac{F(r,R,x,a)}{F(0,R,x,a)}\,, \tag{7}$$

where

$$F(r,R,x,a) = \frac{x}{1+\exp[(r-R)/a]} + \frac{1-x}{1+\exp[(r-R)/(3a)]^3} \tag{8}$$

and V_C is the general Coulomb potential

$$\begin{aligned} V_\mathrm{C} &= \frac{C}{r}, & r \geq R \\ V_\mathrm{C} &= C\frac{3-(r/R)^2}{2R}\,, & r \leq R\,. \end{aligned} \tag{9}$$

The last term in equation (6) is the usual centrifugal term. The Pauli exclusion principle requires that all cluster nucleons must occupy states above the Fermi level of the core nucleus. One can enforce this constraint by restricting the cluster–core relative motion quantum numbers n and l as [8]

$$G = 2n + l = 4A_2 \,, \tag{10}$$

where G is the global quantum number and A_2 is the mass number of the cluster nucleus. In the present work, the Woods–Saxon potential parameters are taken as, $V_0 = 57.52$ MeV, $x = 0.334$ and $a = 0.74$ fm for all Hf isotopes, while the core radius parameter R is considered as a free parameter to reproduce the excitation energy of the considered even–even Hf isotopes.

The transition probability $B(E2: 2^+ \rightarrow 0^+)$ of a nucleus (A, Z) is given in terms of the reduced mass μ of its core–cluster relative motion as [13]

$$B(E2) = \frac{\mu^2 Z^2 r_0^4}{4\pi A^{2/3}} \,, \tag{11}$$

where r_0 depends on the type of experiment and can be assumed as a free parameter. In the present work, the value of this parameter is taken as; $r_0 = 1.543$ and $r_0 = 1.438$ fm in case of ^{12}C and ^{14}C clusters, respectively. These values of r_0 parameter were found by the least squares method comparing the experimental and model $B(E2)$ results.

3. Results and discussion

The potential parameters; V_0, x, a and R are adjusted firstly in case of^{164}Hf isotope by using the χ^2-test. But in case of the remnant isotopes, the core radius R is considered as a free parameter only and the other parameters are fixed.

Firstly, the charge of the cluster nucleus is determined through the drawing of the variable D (1,2) *versus* the cluster charge (Z_2) as shown in Fig. 1.

From this figure one can notice that there are two maxima at $Z_2 = 2$ and $Z_2 = 6$. In this work, either ^{12}C or ^{14}C is considered as a cluster nucleus in the following partition

$$^{A}_{72}\mathrm{Hf}_N \rightarrow ^{A_1}_{66} \mathrm{Dy}_{N_1} + ^{A_2}_{6} \mathrm{C}_{N_2} \tag{12}$$

which gives a satisfying agreement with the experimental energy states more than the α-cluster nucleus in the partition,

$$^{A}_{72}\mathrm{Hf}_N \rightarrow ^{A_1}_{70} \mathrm{Yb}_{N_1} + \alpha \,. \tag{13}$$

The spectra of Hf isotopes are calculated using equation (5) in case of both ^{12}C and ^{14}C cluster nuclei. In this equation, the core-radius parameter R is taken as a free parameter and its value is adjusted through the

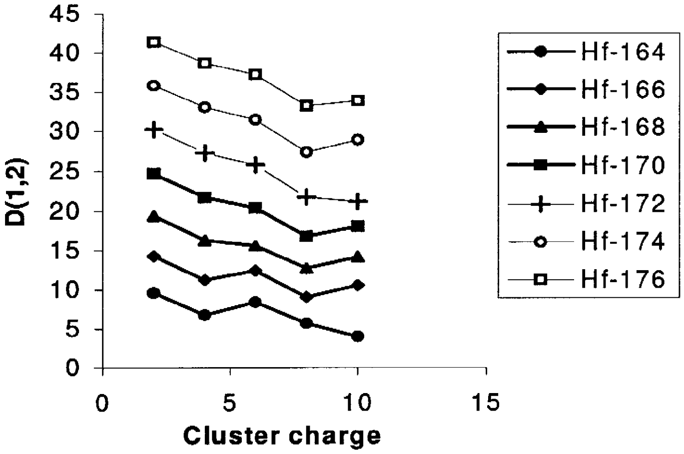

Fig. 1. Relation between $D(1,2)$ *versus* the cluster charge for all considered Hf isotopes.

χ^2-test. The core–cluster decomposition and both R parameter and χ^2 values, for each Hf isotope (from ^{164}Hf–^{178}Hf) are given in Table I. From this table it is clear that the ^{14}C-cluster results give a better agreement with the experiment than ^{12}C-cluster in case of ^{164}Hf and ^{166}Hf-isotopes. On the contrary, the ^{12}C-cluster results agree better with the experiment for heavier Hf-isotopes.

TABLE I

Values of μ(in a.m.u.) and core radius parameters (in fm) in case of ^{12}C and ^{14}C clusters for all considered Hf isotopes.

Isotope	$\mu(^{12}\mathrm{C})$	$R(^{12}\mathrm{C})$	χ^2	$\mu(^{14}\mathrm{C})$	$R(^{14}\mathrm{C})$	χ^2
^{164}Hf	11.12	5.955	0.309	12.810	5.511	0.138
^{166}Hf	11.13	5.947	0.386	12.821	5.504	0.178
^{168}Hf	11.14	5.943	0.047	12.834	5.498	0.149
^{170}Hf	11.15	5.936	0.109	12.848	5.491	0.366
^{172}Hf	11.16	5.929	0.065	12.862	5.483	0.344
^{174}Hf	11.17	5.923	0.143	12.875	5.477	0.334
^{176}Hf	11.18	5.916	0.109	12.889	5.470	0.325
^{178}Hf	11.19	5.909	0.043	12.90	5.463	0.213

The transition probabilities, $B(E2:2^+ \to 0^+)$ calculated for considered even–even Hf isotopes are compared in Table II to the experimental values [14]. The column denoted "T$_1$" corresponds to ^{12}C cluster whereas that denoted "T_2" corresponds to ^{14}C cluster. From this table one can notice that the calculations using either ^{12}C and ^{14}C-cluster are nearly the same. The theoretical values of $B(E2)$ decrease monotonically with increasing mass of Hf isotopes, while the experimental values increase on average with the mass of Hf isotopes. This might indicate that to reproduce the experimental values of $B(E2)$ one must introduce a slight change in either μ or r_0 values for each Hf isotope.

TABLE II

Calculations of transition probabilities (in e^2fm^4), $B(E2)$ in case of ^{12}C, T_1, and ^{14}C, T_2 in comparison to the experiment.

Isotope	$T_1 : B(E2)$	$T_2 : B(E2)$	Exp. $B(E2)$
^{164}Hf	9654	9653	
^{166}Hf	9595	9597	7000 ± 300
^{168}Hf	9536	9542	8400 ± 400
^{170}Hf	9478	9487	10000 ± 200
^{172}Hf	9421	9433	8200 ± 500
^{174}Hf	9365	9380	
^{176}Hf	9310	9327	10900 ± 300
^{178}Hf	9255	9275	9700 ± 200
χ^2	128.7	127.7	

In this model both cluster and core nuclei are spinless. The reliability of spectral information on Hf ground-state band is different for each isotope. The information for assigning the levels of these bands is given in Ref. [14]. One can see a strong evidence of anomalous behavior at different values for

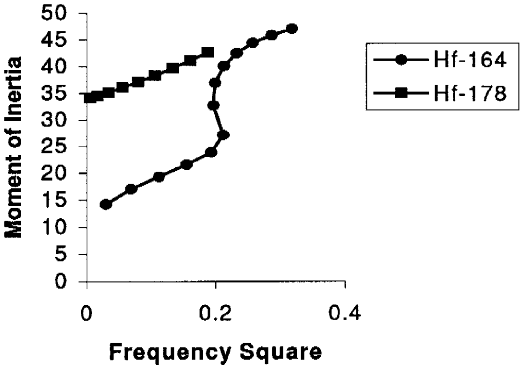

Fig. 2. The moment of inertia (in MeV) *versus* the square of frequency (in MeV^2).

each Hf isotope, as shown in Fig. 2. From this figure one can notice that this anomalous behavior is reduced with increasing the neutron number. In this work, we confine ourselves to $J^{\pi} \leq 14^{+}$, where the best agreement between theoretical and experimental ground-state bands occurred. Figs. 3–10 show the calculated and experimental excitation energy states *versus* the total spin of all considered Hf isotopes. From Figs. 3 and 4, one can see that the theoretical calculations using ^{14}C-cluster give a better agreement with the experimental data than those made by using ^{12}C-cluster in case of ^{164}Hf and ^{166}Hf-isotopes. The calculated ground state band excitation energies of the heavier Hf-isotopes give a more suitable agreement with the experimental data in case of ^{12}C-cluster than ^{14}C-cluster (see Figs. 4–10).

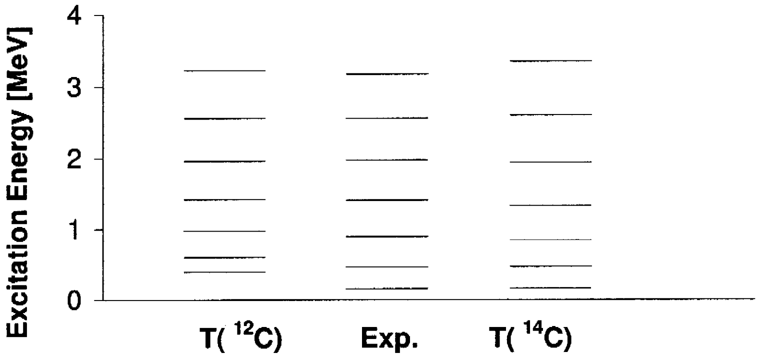

Fig. 3. The calculated ground state band excitation energy for ^{164}Hf-isotope *versus* the experimental data.

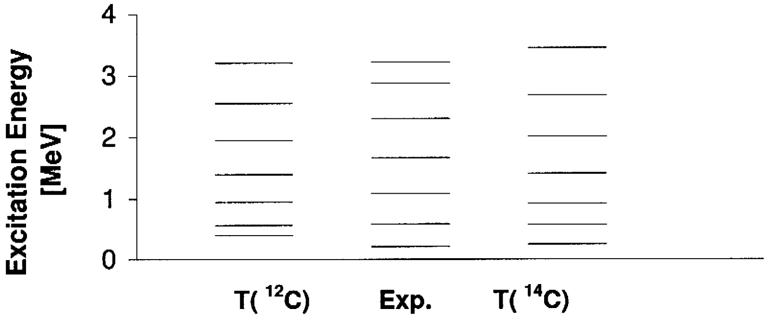

Fig. 4. The calculated ground state band excitation energy for ^{166}Hf-isotope *versus* the experimental data.

From this study one can conclude that the core–cluster decomposition model can reproduce the excitation energies of ground state band of heavy deformed nuclei up to $J^\pi = 14^+$. Also, the transition probabilities, $B(E2 : 2^+ \to 0^+)$, can be reasonably well reproduced by this model. Rather large experimental errors of $B(E2)$ values do not allow to decide whether fine tuning of r_0 parameter is necessary for individual Hf isotopes.

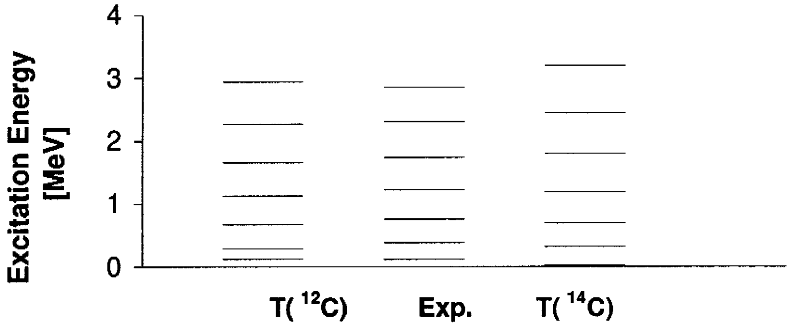

Fig. 5. The calculated ground state band excitation energy for ^{168}Hf-isotope *versus* the experimental data.

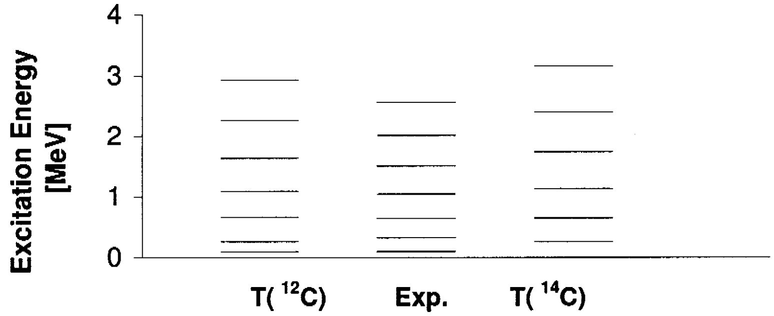

Fig. 6. The calculated ground state band excitation energy for ^{170}Hf-isotope *versus* the experimental data.

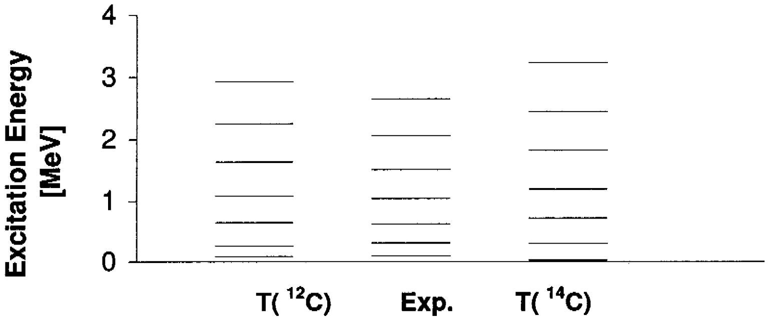

Fig. 7. The calculated ground state band excitation energy for ^{172}Hf-isotope *versus* the experimental data.

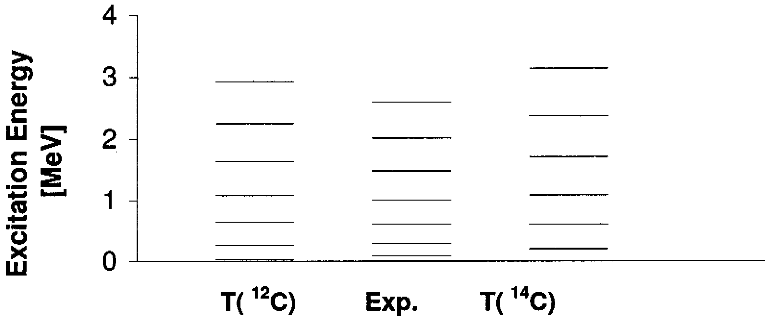

Fig. 8. The calculated ground state band excitation energy for ^{174}Hf-isotope *versus* the experimental data.

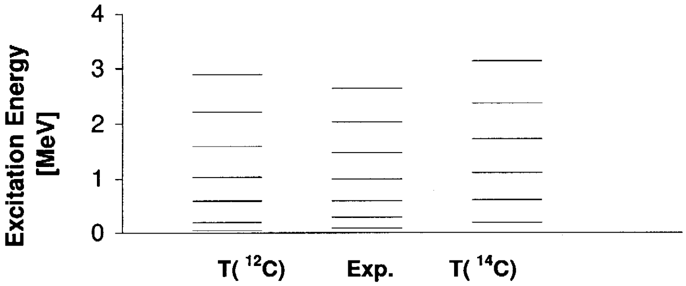

Fig. 9. The calculated ground state band excitation energy for ^{176}Hf-isotope *versus* the experimental data.

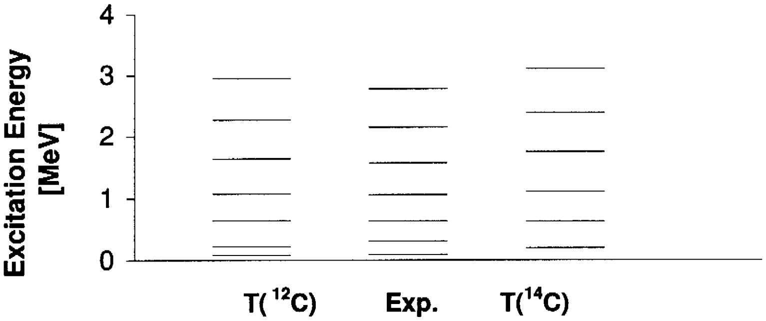

Fig. 10. The calculated ground state band excitation energy for ^{178}Hf-isotope *versus* the experimental data.

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