ON STATUS OF CONSTANT f IN THE CONFORMAL POINCARÉ GAUGE THEORY OF GRAVITATION

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The status of constant f is analysed in the framework of the Conformal Poincaré Gauge Theory of gravitation. It is shown, that for the Morris–Thorne wormholes and the Bianchi I cosmology this constant should have the different signs. A possible interpretation of the obtained results is discussed.

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It is known, that in framework of the Einstein's GR there is a number of difficulties. In particular, we point out the problem of description of various cosmological models and the impossibility of construction the Morris–Thorne wormhole (MTW). It is also known that within GR the Robertson–Walker Universe either possesses the initial singularity, or contains a "exotic" matter (physical vacuum or quintessence) or has initial event horizon [1]. Moreover, the anisotropic models (Bianchi I) in the framework of GR can be nonsingular only for the Kasner vacuum solution ($c = 1$)

$$ds^2 = -dt^2 + a_0 t^{q_1} dx^2 + b_0 t^{q_2} dy^2 + c_0 t^{q_3} dz^2 \tag{1}$$

in a degenerated case $q_1 = q_2 = 0$, $q3 = 1$ which is reduced to Minkovski Space–Time.

It is also known that vacuum MTW [2] has an event horizon on throat, and nonvacuum solutions describing them inevitably contain an "exotic" matter (with negative radial pressure).

As one of possible ways of exception of abovementioned problems a transition to the conformal Poincaré Gauge Theory of gravitation in the Einstein

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gauge and in the torsionless limit [3] which is based on equations ($C_{\alpha\beta\mu\nu}$ is the Weyl tensor, $\varkappa = 8\pi G/c^4$, f — an arbitrary parameter)

$$R^{\mu}{}_{\nu} - \frac{1}{2}\delta^{\mu}{}_{\nu}R + \varkappa f C^{\mu\alpha}{}_{\nu\beta}R^{\beta}{}_{\alpha} = \varkappa T^{\mu}{}_{\nu}\,, \tag{2a}$$

$$f C^{\mu\alpha}{}_{\nu\beta;\mu} = 0 \tag{2b}$$

can be considered.

The asymptotical solutions of system (2) near to singular points for the perfect fluid configurations have been investigated in work [4]. In [3] it has been shown, that for a static spherical symmetric line element

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + e^{\mu(r)}\left(d\Theta^2 + \sin^2\Theta d\phi^2\right) \tag{3}$$

in vacuum case and for a Robertson–Walker Universe the solutions of equations (2) coincide with corresponding GR solutions. Therefore, here we shall consider the system (2) for the MTW with a matter and for the anisotropic cosmological models (Bianchi I).

According to definition [2] the MTW is a topological handle in space-time which connects the distant regions of Universe. A line element for MTW corresponds to (3) with $\lambda = 0$. The functions $e^{\nu(r)}$ and $e^{\mu(r)}$ have symmetric minima at $r = 0$ (the MTW throat) and are everywhere positive. These functions are also monotonous to the left and to the right of the throat.

For the MTW line element the equations (2) are reduced to the following system (a prime denotes differentiation with respect to r)

$$\begin{aligned} R^{12}{}_{12} &= \frac{1}{4}e^{-\lambda}(2\mu'' + \mu'^2 - \lambda'\mu') \\ &= \frac{1}{6}\left(P_\perp - \frac{(2+y)\varepsilon + (1-y)P_r}{1+2y}\right) + W\,, \end{aligned} \tag{4a}$$

$$\begin{aligned} R^{23}{}_{23} &= \frac{1}{4}e^{-\lambda}\mu'^2 - e^{-\mu} \\ &= \frac{(1-y)(P_r - \varepsilon) - (1+2y)P_\perp}{3(1-4y)} - 2W\,, \end{aligned} \tag{4b}$$

$$\begin{aligned} R^{01}{}_{01} &= \frac{1}{4}e^{-\lambda}(2\nu'' + \nu'^2 - \lambda'\nu') \\ &= \frac{(1+2y)(\varepsilon - P_r) + 4(1-y)P_\perp}{6(1-4y)} - 2W\,, \end{aligned} \tag{4c}$$

$$\begin{aligned} R^{02}{}_{02} &= \frac{1}{4}e^{-\lambda}\mu'\nu' \\ &= \frac{1}{6}\left(P_\perp + \frac{(1-y)\varepsilon + (2+y)P_r}{1+2y}\right) + W\,, \end{aligned} \tag{4d}$$

here $W = r_0 e^{-\frac{3\mu}{2}}$, $y = \varkappa f W$, r_0 is an integration constant for equation (2b) (see [4,5]), $\varepsilon = -T^0{}_0$, $P_r = T^1{}_1$, $P_\perp = T^2{}_2 = T^3{}_3$ — the energy density, the radial pressure and the tangent pressure, respectively.

We research now the conditions of smooth junction for functions $\nu(r)$ and $\mu(r)$ with vacuum solution [6]

$$ds^2 = -(1-r_g e^{-\frac{\mu}{2}})dt^2 + \left[\frac{((e^\mu)')^2}{4e^\mu(1-r_g e^{-\frac{\mu}{2}})}\right] dr^2 + e^\mu(d\Theta^2 + \sin^2\Theta d\phi^2)\,, \quad (5)$$

here r_g is a gravitational radius of object, on the surface of configuration. In accordance with equation (2b) the Weyl tensor is everywhere continuous. For the line element (5) the nonzero components of this tensor is defined by expression

$$W = \frac{r_g e^{-\frac{3\mu}{2}}}{2}\,, \quad (6)$$

$$\left(W = -\frac{1}{2}C^{01}{}_{01} = C^{02}{}_{02} = C^{03}{}_{03} = C^{12}{}_{12} = C^{13}{}_{13} = -\frac{1}{2}C^{23}{}_{23}\right).$$

Then we have $r_0 = r_g/2 > 0$.

As the functions $\nu(r)$, $\mu(r)$ and their first derivatives $\nu'(r)$, $\mu'(r)$ are continuous on the surface of configurations also the components $R^{02}{}_{02}$ and $R^{23}{}_{23}$ are continuous. Using (4b), (4d) and (5) we obtain

$$r_g e^{-\frac{3\mu}{2}} = \frac{1}{3}\left(\frac{(1-y)(P_r-\varepsilon)-(1+2y)P_\perp}{1-4y}\right) - 2W\,, \quad (6a)$$

$$r_g e^{-\frac{3\mu}{2}} = \frac{1}{6}\left(P_\perp + \frac{(1-y)\varepsilon + (2+y)P_r}{1-4y}\right) + W\,. \quad (6b)$$

From requirements (6) and a boundary condition $P_r = 0$ we find

$$y(\varepsilon(1-y) + P_\perp(1+2y)) = 0\,. \quad (7)$$

The case $y = 0$ corresponds to $f = 0$ and is reduced to GR. For $y \neq 0$ we have

$$0 \leq \frac{P_\perp}{\varepsilon} = \frac{y-1}{2y+1} \leq 1 \quad (8)$$

(as $\varepsilon \geq P_\perp \geq 0$ for "usual" matter).

From (8) we find

$$y \leq -2 \quad (9a)$$

or

$$y \geq 1\,. \quad (9b)$$

According to definition of a MTW at throat the conditions $\mu' = 0$ and $\mu'' > 0$ should be fullfilled. Then from equations (4b) and (4d) we obtain

$$P_\perp = -\frac{1-y}{1+2y}\varepsilon + \frac{2+y}{1+2y}(12yW + e^{-\mu}(1-4y)) - 6W\,, \qquad (10a)$$

$$P_r = -12yW - e^{-\mu}(1-4y)\,. \qquad (10b)$$

Substituting the expressions (10) in (4a), we have an inequality

$$\frac{\varepsilon + P_r}{1+2y} < 0\,. \qquad (11)$$

As for "usual" matter $\varepsilon + P_r \geq 0$ then from (11) it follows that only the case (9a) can take place. Taking into consideration $y = \varkappa f W$ and $W > 0$ we come to a conclusion that for MTW within the framework of the theory based on system (2), the parameter f is negative

$$f < 0\,. \qquad (12)$$

We research now the equations (2) for a line element of cosmological model Bianchi I

$$ds^2 = -dt^2 + a^2(t)dx^2 + b^2(t)dy^2 + c^2(t)dz^2\,. \qquad (13)$$

The equations (2) are reduced for this case to the system (a dot denotes a differentiation with respect to t)

$$-\varkappa\varepsilon = G_0^0 + \varkappa f(AG_1^1 + BG_2^2 - (A+B)G_3^3)\,, \qquad (14a)$$

$$\varkappa P_1 = G_1^1 + \varkappa f(AG_0^0 - (A+B)G_2^2 + BG_3^3)\,, \qquad (14b)$$

$$\varkappa P_2 = G_2^2 + \varkappa f(BG_0^0 - (A+B)G_1^1 + AG_3^3)\,, \qquad (14c)$$

$$\varkappa P_3 = G_3^3 + \varkappa f(-(A+B)G_0^0 + BG_1^1 + AG_2^2)\,, \qquad (14d)$$

$$\dot{A} + (2A+B)h_2 + (A-B)h_3 = 0\,, \qquad (14e)$$

$$\dot{B} + (A+2B)h_1 + (B-A)h_3 = 0\,, \qquad (14f)$$

here $G_k^i = R_k^i - \frac{1}{2}R\delta_k^i$, $P_1 = T_1^1$, $P_2 = T_2^2$, $P_3 = T_3^3$, $A = C^{01}{}_{01}$, $B = C^{02}{}_{02}$, $h_1 = \frac{\dot{a}}{a}$, $h_2 = \frac{\dot{b}}{b}$, $h_3 = \frac{\dot{c}}{c}$.

We shall consider expression (14a) for initial point of cosmological evolution which corresponds to a minimum of metric functions: $\dot{a} = \dot{b} = \dot{c} = 0$, $\ddot{a} > 0$, $\ddot{b} > 0$, $\ddot{c} > 0$.

Then we obtain

$$\varepsilon_0 = \frac{f}{3}\left(\psi_1^2 + \psi_2^2 + \psi_3^2 - \psi_1\psi_2 - \psi_1\psi_3 - \psi_2\psi_3\right)\,, \qquad (15)$$

where $\psi_1 = \frac{\ddot{a}_0}{a_0}$, $\psi_2 = \frac{\ddot{b}_0}{b_0}$, $\psi_3 = \frac{\ddot{c}_0}{c_0}$. Expression (15) has a minimum $\psi_1 = \psi_2 = \psi_3 = 0$, corresponding to the vacuum solution.

Hence expression in brackets (15) at any ψ_1, ψ_2, ψ_3 will be non-negative.

Thus, the given problem can have the solutions describing a "usual" matter ($\varepsilon_0 \geq 0$) only at $f > 0$. In the result it is obtained, that the formulation of the considered problems corresponds to constants f with different signs. One of the reasons for such conclusion can be essential impossibility (as well as in GR) of correct investigation of MTW or Bianchi I cosmology within the framework of examined theory. As other interpretation of the given result it is possible to assume presence of dependence of constant f from properties of concrete objects. A similar situation takes place, as is known, in the framework of the Jordan–Brans–Dicke theory.

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