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CHIRAL DOUBLERS FOR MESONS, BARYONS AND PENTAQUARKS*

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The chiral doublers scenario is discussed from the basic point of view of the heavy–light quark symmetry. Simple estimation of the masses of charm and beauty mesons and baryons are presented. The case of heavy–light pentaquark is also reviewed.

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1. Introduction

The structure of the hadron spectroscopy is the simplest and the most fundamental feature of the strongly interacting matter. It is a base and historically a source of the hypothesis of quarks — the elementary ingredients of the Standard Model. At the same time predictions of the hadron masses and relations between them which are based on the first principles or well granted assumptions are very limited. It is therefore why all such predictions are of interest and of great value. One of the most important simplifications arises in the limit of the infinite heavy quark mass when the new spin–flavor symmetry appears in the Lagrangian of the Quantum Chromodynamics [1]. In this limit the heavy (h) and the light (l) degrees of freedom decouple from each other inside the hadron X:

$$|X\rangle = |h\rangle|l\rangle\,. \tag{1}$$

If at the same time one considers the opposite limit of the vanishing light quark masses we discover a new and powerful symmetry (HQS) describing the heavy–light quark systems:

$$\mathrm{SU}(2)_s \times \mathrm{SU}(2)_Q \times \mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R}, \tag{2}$$

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where the SU(2) groups describe the spin and flavor symmetry of the heavy degrees of freedom whereas the SU(3) are usual chiral symmetry of light part of the hadron. This symmetry applies to the systems which consist of the heavy $Q = c$, b and the light $q = u, d, s$ quarks. The corrections come from the expansion in powers of small light quark masses and inverse heavy quark masses. For such systems Hamiltonian H and total spin operator $\vec{J}$ divide into the heavy and the light part:

$$\begin{aligned} H &= H_h + H_l, \qquad H|X\rangle = (m_Q + \Lambda_l)|X\rangle\,, \\ \vec{J} &= \vec{J}_h + \vec{J}_l, \qquad \vec{J}|X\rangle = (\vec{j}_h + \vec{j}_l)|X\rangle\,, \end{aligned} \tag{3}$$

where $H_h|h\rangle = m_Q|h\rangle$ is essentially a mass of the heavy quark for the lowest states and $H_l|l\rangle = \Lambda_l|l\rangle$ is the light degrees of freedom contribution to the mass of the hadron. Similarly $\vec{j}_h$ is a spin of the heavy quark and $\vec{j}_l$ is an angular momentum of the light part. Additionally the HQS symmetry implies $[H, \vec{J}_Q] = 0 \Rightarrow\ [H, \vec{J}_l] = 0$, which means that the light and the heavy spins are good quantum numbers in the description of the hadron X. One of the consequences of the chiral symmetry is the existence of the chiral partners for the light hadrons of the same spin but of the opposite parity. One can then introduce the same idea to the heavy–light systems through the definition of the *chiral doublers*, *i.e.* hadrons which differ only in the parity P_l of the light degrees of freedom

$$|X\rangle = |h\rangle|l\rangle,\ \ |X'\rangle = |h\rangle|l'\rangle, \tag{4}$$

which were first introduced in [2] and independently later on in [3]. The parity operator $P = P_h \otimes P_l$ distinguishes between these states:

$$P|X\rangle = P_h|h\rangle P_l|l\rangle = \eta|X\rangle,\ \ P|X'\rangle = P_h|h\rangle P_l|l'\rangle = -\eta|X'\rangle, \tag{5}$$

where $P_l|l\rangle = -P_l|l'\rangle$ are chiraly related partners. The mass splitting between chiral doublers $\Delta m_{XX'} = M_{X'} - M_X$ vanishes in the chiral limit, however, due to the chiral symmetry breaking, it is non-zero. Nevertheless, the HQS symmetry implies that Δm is independent of

(i) the spin coupling between the heavy and light degrees of freedom,

(ii) the flavor of the heavy quark.

Actually at this basic level there is no need for any advanced calculations to make predictions and to compare the results with the experiment. The estimation of masses given below are crude, at the level of a few per cent, however, it is model independent based essentially on the HQS symmetry.

2. Mesons

2.1. D_s mesons

In the Fig. 1 (left panel) there is a meson pyramid of masses in the $\bar{c}s$ quark sector. The chiral doublers splitting $\Delta m_{0^-0^+} = 349$ MeV and $\Delta m_{1^-1^+} = 347$ MeV [4] in a nice agreement with the HQS symmetry point *(i)*. For the excited states $\Delta m_{1^+1^-} = 96$ MeV[1] which leads to the prediction that $\Delta m_{2^+2^-} = 96$ MeV or the mass of the $M_{2^-} = 2669$ MeV.

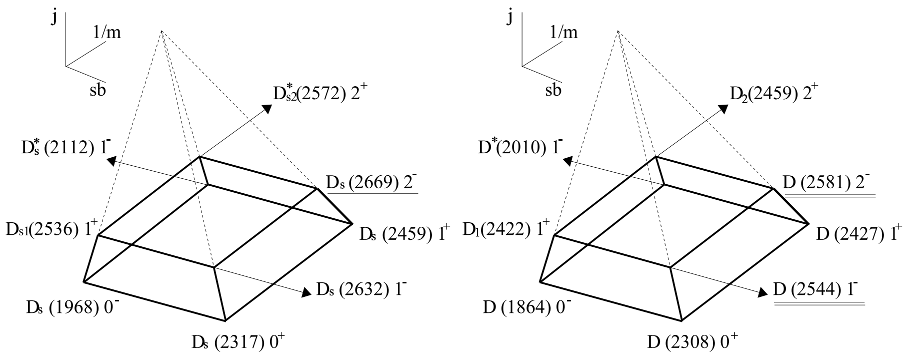

Fig. 1. The meson pyramid in the space of the heavy quark corrections $1/m$, the chiral symmetry breaking sb and the light spin excitations j. Usual $D_{(s)}$ mesons occupy the left-hand side wall of the pyramid and chiral doublers are on the right hand side. If the HQS symmetry was exact the pyramid would shrink to the line in j direction. The underlined mesons are predictions based on the HQS symmetry, doubly underlined require additional assumption (6).

2.2. D mesons

The meson pyramid of the $\bar{c}u(d)$ sector is given in the Fig. 1 (right panel). The mass splitting between the lowest chiral doublers is $\Delta m_{0^-0^+} \approx 444$ MeV and $\Delta m_{1^-1^+} \approx 417$ MeV [5]. The approximate equalities stress the fact that the measurement errors are of the order of 20 MeV much larger then in the case of D_s mesons. These numbers agree with each other within the errors

[1] The mass splitting follows from the Selex measurement [6]. This state, not confirmed so far by other experiments, gives the mass splitting which differs from the theoretical expectation $\Delta m_{1^+1^-} = 186$ MeV [7]. If the right value was closer to the theoretical expectation then one would have to appropriately rescale the predictions of the masses of the excited chiral doublers. Then one gets: $M_{D_s(1^-)} = 2721$ MeV [7], $M_{D_s(2^-)} = 2757$ MeV. The mass splitting for excited D mesons is $\Delta m_{1^+1^-} = 235$ MeV (based on (6)) and then $M_{D(1^-)} = 2657$ MeV, $M_{D(2^-)} = 2694$ MeV. For B mesons we have: $M_{B_s(1^-)} = 6171$ MeV, $M_{B_s(2^-)} = 6114$ MeV, $M_{B(1^-)} = 6072$ MeV and $M_{B(2^-)} = 6009$ MeV.

as is expected on the ground of HQS symmetry point *(i)*. However, one can also predict other meson masses at the price of the slightly lower accuracy. The doubler mass splitting is a function of the light degrees of freedom. Thus one cannot expect that the splitting of the excited D meson doublers is the same as for the excited D_s mesons. However, the ratio of the mass splittings between the lowest states doublers and the excited states doublers usually shows weaker dependence on the light degrees of freedom than the mass splitting itself. Then one can expect

$$\frac{M_{D_{s1}(1^+)} - M_{D_s(1^-)}}{M_{D_s(0^-)} - M_{D_s(0^+)}} \approx \frac{M_{D_1(1^+)} - M_{D(1^-)}}{M_{D(0^-)} - M_{D(0^+)}}\,, \tag{6}$$

thus the mass of the excited state is $M_{1^-} \approx 2544$ MeV which is 122 MeV above its chiral partner. Using point *(i)* of the symmetry HQS one also immediately finds that $M_{2^-} \approx 2581$ MeV.

2.3. B_s mesons

In the $\bar{b}s$ sector (Fig 2.) the only known mass is $M_{B_s} = 5369$ MeV [4] which in view of *(ii)* results in the chiral doubler mass $M_{0^+} = 5717$ MeV. However, one can extend the predictions for other mesons. The higher state 1^+ is due to the excitation of the light degrees of freedom. In the first approximation this should be independent of the heavy flavor. Then one has $M_{B_{s1}} - M_{B_s} \approx M_{D_{s1}} - M_{D_s} = 567$ MeV. Therefore one predicts $M_{B_{s1}} \approx$

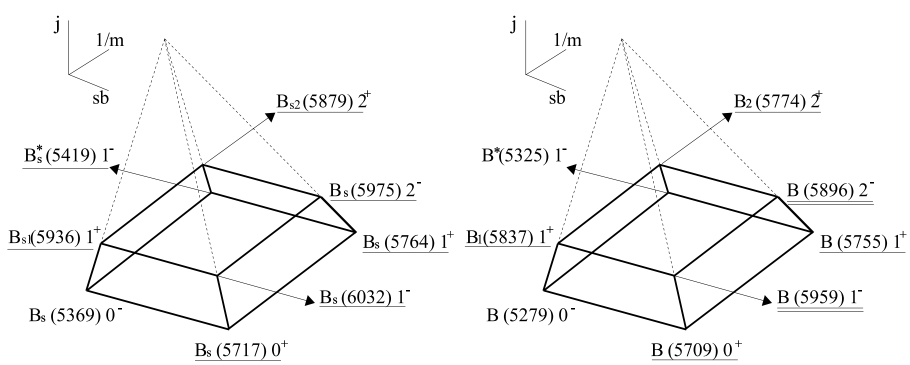

Fig. 2. The meson pyramid in the space of the heavy quark corrections $1/m$, the chiral symmetry breaking sb and the light spin excitations j. Usual $B_{(s)}$ mesons occupy the left hand side wall of the pyramid and chiral doublers are on the right hand side. If the HQS symmetry was exact the pyramid would shrink to the line in j direction. The underlined mesons are predictions based on the HQS symmetry, doubly underlined require additional assumption (6).

5936 MeV and its chiral doubler $M_{1^-} = 6032$ MeV (96 MeV above). The mass difference between 1^- and 0^- states is due to the hyperfine splitting suppressed by the mass of the heavy quark. This means that $M_{B_s^*} - M_{B_s} = \Lambda(m_s)/m_b$ where m_b is a mass of the heavy quark and Λ depends only on the light degrees of freedom. In such a situation one has the relation $M_{B_s^*} - M_{B_s} \approx (M_{D_s^*} - M_{D_s})m_c/m_b$. If one assumes $m_c/m_b \approx M_D/M_B = 0.35$ then $M_{B_s^*} = 5419$ MeV and its chiral doubler $M_{1^+} = 5764$ MeV. Finally, $M_{B_{s2}} - M_{B_s^*} \approx M_{D_{s2}} - M_{D_s^*} = 460$ MeV from which one has $M_{B_{s2}} = 5879$ MeV and its chiral doubler $M_{2^-} = 5975$ MeV.

2.4. B mesons

There are two experimentally known masses of mesons $M_B = 5279$ MeV and $M_{B^*} = 5325$ MeV [4] (Fig. 2). Their chiral doublers should have masses $M_{0^+} \approx 5709$ MeV and $M_{1^+} \approx 5755$ MeV (430 MeV above which is the mean value of the analogous mass splitting in the D meson sector). Performing the same analysis as in the case of B_s mesons one has $M_{B_1} - M_B \approx M_{D_1} - M_D = 558$ MeV so $M_{B_1} \approx 5837$ MeV. Its chiral doubler $M_{1^-} \approx 5959$ MeV is 122 MeV above which one expects on the ground of the HQS symmetry point *(ii)*. Similarly $M_{B_2} - M_{B^*} \approx M_{D_2} - M_{D^*} = 449$ MeV so $M_{B_2} \approx 5774$ MeV and its chiral doubler $M_{2^-} \approx 5896$ MeV.

3. Baryons

In the Fig. 3 (left panel) the hadron spectroscopy of the lowest heavy baryons cqq, $(q = u, d)$ is given. The mass splitting between chiral doublers for the lowest energy state is $\Delta_{(1/2)^+(1/2)^-} = 308$ MeV. Based on this number and the HQS symmetry one can find the masses of chiral doublers of Σ_c baryons $M_{1/2^-} = 2763$ MeV and $M_{3/2^-} = 2828$ MeV. The case of csq system is given on the Fig. 3 (right panel). The chiral doublers mass splitting for the lowest state is $\Delta_{(1/2)^+(1/2)^-} = 320$ MeV. However $\Delta_{(3/2)^+(3/2)^-} = 170$ MeV. This is in disagreement with our basic picture. The state $\Xi(2815)$ with quantum number assignment $(3/2)^-$ is particularly mysterious if one compares the hyperfine splitting. Mass difference $\Delta_{(3/2)^+(1/2)^+} = 175$ MeV whereas $\Delta_{(3/2)^-(1/2)^-} = 25$ MeV. This discrepancy is very difficult to understand. One rather suspects that the $\Xi(2815)$ is not a chiral doubler of $\Xi(2645)$. Using the value 320 MeV one predicts the masses of chiral doublers $M_{(1/2)^-} \approx 2894$ MeV and $M_{(3/2)^-} \approx 2965$ MeV.

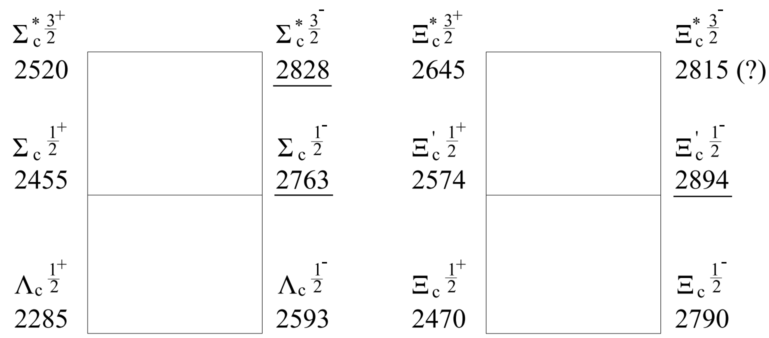

Fig. 3. The baryons in the space of the heavy quark corrections $1/m$ (vertical direction) and the chiral symmetry breaking (horizontal direction). Usual baryons are on the left and chiral doublers on the right hand side. If the HQS symmetry was exact these planes would shrink to the points. The underlined baryons are predictions based on the HQS symmetry.

4. Pentaquarks

If the heavy–light pentaquarks exist then their chiral doublers exist as well. The prediction of mass splitting between them requires model calculations because the experimental data are not available. The HQS symmetry alone relates the different states. If a numerical value is known for one of such states it is also known for others. But, of course, the symmetry alone is not enough to find numerical values if none of the numbers are known. To find the relation between doublers mass splittings for mesons, baryons and pentaquarks we need the uniform platform for calculation. The most natural choice is a chiral soliton model which is the extension of the chiral effective Lagrangian that includes also baryons [8]. In the case of the heavy–light systems the chiral soliton model was adapted in papers [9, 10]. However, in those papers the chiral doublers were not treated properly. Only recently the full effective Lagrangian with both chiral copies and the chiral shift terms was considered [11]

$$\begin{aligned} L = & - i\mathrm{Tr}\,(\bar{H}v^\mu D_\mu H) + g_H \mathrm{Tr}\,(H\gamma^\mu\gamma_5 A_\mu \bar{H}) + m_H \mathrm{Tr}\,(\bar{H}H) \\ & - i\mathrm{Tr}\,(\bar{G}v^\mu D_\mu G) + g_G \mathrm{Tr}\,(G\gamma^\mu\gamma_5 A_\mu \bar{G}) + m_G \mathrm{Tr}\,(\bar{G}G) \\ & + g_{GH} \mathrm{Tr}\,(\gamma_5 \bar{G} H \gamma^\mu A_\mu) + (\mathrm{H.c.})\,. \end{aligned} \tag{7}$$

The fields H and G are usual heavy mesons multiplets $(0^-, 1^-)$ and $(0^+, 1^+)$:

$$H = \frac{1 + v^\mu\gamma_\mu}{2}(\gamma_5 D - \gamma^\mu D^*_\mu), \quad G = \frac{1 + v^\mu\gamma_\mu}{2}(\gamma_5 \tilde{D} - \gamma^\mu \tilde{D}^*_\mu). \tag{8}$$

The last term in (7) describes the interaction between chiral doublers via light axial current. The axial current A_μ reads

$$A_\mu = \frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger)\,, \tag{9}$$

where $\xi^2 = U = \exp(i\vec{\pi}\cdot\vec{r})$ and v_μ is the four-velocity of the heavy quark. In this case the pion field is taken as the Skyrme hedgehog ansatz $\vec{\pi} = F(r)\hat{n}$. The parity copies G, H differ in the sign of the constituent mass contribution $m_G = -m_H \approx -\Sigma$, where Σ denotes the one-loop heavy meson self-energy [2, 3, 7, 12]. The result is a split between the heavy–light mesons of opposite parity (chiral doublers).

First of all one can easily verify that the interaction term g_{GH} vanishes in the infinite heavy quark limit. Then using standard approach one can find mass formulae for isoscalar baryon M, pentaquark M_5 and their chiral doublers $\tilde{M}$, $\tilde{M}_5$ [10, 11]

$$\begin{aligned} M &= M_{\rm sol}+m_D - \frac{3}{2}g_H F'(0) + \frac{3}{8}\frac{1}{I_1}, \quad \tilde{M}=M_{\rm sol}+m_{\tilde{D}} - \frac{3}{2}g_G F'(0)+\frac{3}{8}\frac{1}{I_1}\,, \\ M_5 &= M_{\rm sol}+m_D - \frac{1}{2}g_H F'(0) + \frac{3}{8}\frac{1}{I_1}, \quad \tilde{M}=M_{\rm sol}+m_{\tilde{D}} - \frac{1}{2}g_G F'(0)+\frac{3}{8}\frac{1}{I_1}\,, \end{aligned} \tag{10}$$

where $M_{\rm sol}$ is the $O(N_c)$ classical mass of the Skyrmion, $m_D = (3M_{D^*} + M_D)/4$, $m_{\tilde{D}} = (3M_{\tilde{D}^*} + M_{\tilde{D}})/4$ are masses of the multiplets $(0^-, 1^-)$ and $(0^+, 1^+)$ averaged over heavy-spin. Combining above formulae one finds

$$\Delta_B = \Delta_M + \frac{3}{2}F'(0)g_G\delta_G\,, \quad \Delta_5 = \Delta_M + \frac{1}{2}F'(0)g_G\delta_G\,, \tag{11}$$

where $\Delta_B, \Delta_M, \Delta_5$ are mass splittings between chiral doublers for baryons, mesons and pentaquarks and $\delta_g = 1 - g_G/g_H$ measures the difference between the axial couplings for both copies. Still one can eliminate the poorly known factors from (11) arriving at the relation

$$\Delta_5 = \frac{\Delta_B + 2\Delta_M}{3}. \tag{12}$$

Using the mass splitting between Λ_c baryons $\Delta_B = 308$ MeV (Ξ_c gives 320 MeV) and $\Delta_M \approx 430$ MeV one estimates $\Delta_P \approx 390$ MeV.

The case of the pentaquarks is more speculative in comparison to usual hadrons which is, however, not a surprise as the whole subject is under debate and the sole existence of these particles is still questionable.

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