DETECTING SUBTLE EFFECTS OF PERSISTENCE IN THE STOCK MARKET DYNAMICS*

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The conventional formal tool to detect effects of the financial persistence is in terms of the Hurst exponent. A typical corresponding result is that its value comes out close to 0.5, as characteristic for geometric Brownian motion, with at most small departures from this value in either direction depending on the market and on the time scales involved. We study the high frequency price changes on the American and on the German stock markets. For both corresponding indices, the Dow Jones and the DAX respectively, the Hurst exponent analysis results in values close to 0.5. However, by decomposing the market dynamics into pairs of steps such that an elementary move up (down) is followed by another move up (down) and explicitly counting the resulting conditional probabilities we find values typically close to 60%. This effect of persistence is particularly visible on the short time scales ranging from 1 up to 3 minutes, decreasing gradually to 50% and even significantly below this value on the larger time scales. We also detect some asymmetry in persistence related to the moves up and down, respectively. This indicates a subtle nature of the financial persistence whose characteristics escape detection within the conventional Hurst exponent formalism.

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1. Introduction

The financial dynamics results in fluctuations whose nature is, as pointed out by Bachelier [1] already in 1900, of the Brownian character. By now we know that it is much more complex and fascinating than just the ordinary Brownian motion. Already the distribution of stock market returns

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is far from being Gaussian and at the short time scales large fluctuations develop heavy power law asymptotics with an exponent $\alpha = 3$ [2], well outside the Levy stable regime [3], however. The autocorrelation function of returns drops down very quickly and after a few minutes it reaches the noise level. At the same time however the volatility autocorrelation function decays very slowly with time [3], largely according to the power law, and remains positive for many months. On a more advanced level of global quantification, the financial dynamics appears to be describable in terms of multifractality both in the transaction-to-transaction price increments and in the inter-trade waiting times [4]. This indicates a hierarchically convoluted self-similar organization of the market dynamics. One related issue is an effect of persistence. Its commonly adopted measure — the Hurst exponent — is the mode of each multifractal spectrum. The Hurst exponent, however, is a global measure while the effects of persistence may in principle depend on the market phase. Below we address this issue using the high-frequency records (years 1998–99) of the two from among the world leading stock market indices, the Dow Jones Industrial Average (DJIA) for the United States and the Deutsche Aktienindex (DAX) for Germany.

2. Conventional methods

There exist two commonly accepted and best-known methods to evaluate the long-range dependences in the statistical series. The older one is the so-called rescaled range or R/S analysis [5]. This method originates from previous hydrological analysis of Hurst [6] and allows to calculate the self-similarity parameter H. A drawback of this method however is that it may spuriously detect some apparent long-range correlations that result from non-stationarity. A method that avoids such artifacts is the Detrended Fluctuation Analysis (DFA) [7]. In this method one divides a time series $g(t_i)$ of length N $(i = 1, \ldots N)$ into M disjoint segments ν of length n and calculates the signal profile

$$Y_\nu(i) = \sum_{k=1}^{i} (g(k) - \langle g \rangle), \qquad i = 1, \ldots, N\,, \tag{1}$$

where $\langle \ldots \rangle$ denotes the mean. For each segment ν the local trend is then estimated by least-squares fitting the straight line (in general a polynomial) $\tilde{Y}_\nu(i)$ and the corresponding variance

$$F^2(\nu, n) = \frac{1}{n} \sum_{j=1}^{n} \{Y[(\nu-1)n + j] - \tilde{Y}(j)\}\,. \tag{2}$$

Finally, one calculates the mean value of the root mean square fluctuations over all the segments ν:

$$\bar{F}(n) = \frac{1}{M} \sum_{\nu=1}^{M} F(\nu, n) \,. \tag{3}$$

The power-law scaling of the form

$$\bar{F}(n) \sim n^H \tag{4}$$

indicates self-similarity and is considered to provide a measure of persistence. If the process is white noise then $H = 0.5$. If the process is persistent then $H > 0.5$; if it is anti-persistent then $H < 0.5$.

The above procedure applied to the returns

$$g(t) = \ln P(t + \Delta t) - \ln P(t) \,, \tag{5}$$

where $P(t)$ represents the price time series, results in numbers as listed in the last column of Table I for Δt ranging from 1 min up to 30 min. In addition to the DAX and the DJIA this table includes also what for brevity we here call Nasdaq30 and what for the purpose of this work is constructed as a simple sum of the prices of 30 high-capitalization companies belonging to the Nasdaq Composite basket. As one can see from Table I, typically the so-calculated Hurst exponents H point to a trace of anti-persistence but in fact they do not deviate much from 0.5, especially that an error involved in estimation equals about 0.4% for $\Delta t = 1$ min and increases up to 1.5% for $\Delta t = 30$ min due to an effective shortening of the series.

Still this result does not eliminate a possibility that there exist some more local effect of persistence that simply average out when estimated from the longer time intervals. In fact, some proposals to calculate the local counterparts of H, based on variants of DFA, are already present in the literature [8,9] and point to such effects indeed. The accuracy of the related methods is however not yet well established. Furthermore, observations and experience prompt a possibility that the financial persistence may happen to occur asymmetrically, *i.e.*, a move up may be followed by another move up more often than a move down by another move down, or vice versa. Such effects may carry a very valuable information about the dynamics but remain indistinguishable within the conventional methods and unexplored so far. In order therefore to explore a possibility and character of such effects we return to the very definition of persistence.

TABLE I

Several combinations of the conditional probabilities $p_{\alpha,\beta}$ as defined by Eq. (6) for the DAX, DJIA and for the basket of the largest Nasdaq **(NQ30)** companies returns on a sequence of different time scales Δt ranging from 1 up to 30 min. The last column lists the corresponding Hurst exponents. The high-frequency price changes analyzed here cover the time period from 01.12.1997 until 31.12.1999.

Index/Scale	$\mathbf{P_{11}}$	$\mathbf{P_{1-1}}$	$\mathbf{P_{10}}$	$\mathbf{P_{-1-1}}$	$\mathbf{P_{-11}}$	$\mathbf{P_{-10}}$	**Hurst exp.**
DAX/1min	0.567	0.427	0.005	0.562	0.431	0.005	0.493
DAX/2min	0.568	0.428	0.002	0.561	0.435	0.003	0.496
DAX/3min	0.554	0.444	0.0002	0.548	0.449	0.0014	0.497
DAX/4min	0.539	0.459	0.001	0.529	0.469	0.001	0.498
DAX/5min	0.528	0.470	0.0006	0.514	0.484	0.0008	0.5
DAX/10min	0.493	0.507	0.0002	0.476	0.522	0.0006	0.498
DAX/15min	0.483	0.515	0.0004	0.460	0.538	0.0009	0.498
DAX/30min	0.483	0.516	0.0002	0.459	0.540	0.0002	0.495
DJIA/1min	0.558	0.399	0.042	0.558	0.398	0.043	0.502
DJIA/2min	0.555	0.416	0.027	0.561	0.413	0.026	0.499
DJIA/3min	0.526	0.452	0.021	0.531	0.449	0.019	0.498
DJIA/4min	0.504	0.479	0.016	0.504	0.478	0.016	0.498
DJIA/5min	0.498	0.487	0.013	0.497	0.488	0.014	0.495
DJIA/10min	0.497	0.491	0.011	0.498	0.493	0.008	0.491
DJIA/15min	0.502	0.491	0.006	0.487	0.504	0.007	0.491
DJIA/30min	0.506	0.487	0.0066	0.472	0.521	0.0068	0.491
NQ30/1min	0.539	0.454	0.006	0.54	0.455	0.005	0.5
NQ30/2min	0.547	0.449	0.003	0.546	0.45	0.003	0.499
NQ30/3min	0.539	0.458	0.003	0.529	0.468	0.003	0.501
NQ30/4min	0.532	0.464	0.002	0.518	0.478	0.002	0.499
NQ30/5min	0.53	0.467	0.002	0.515	0.481	0.002	0.501
NQ30/10min	0.538	0.46	0.001	0.511	0.487	0.0006	0.502
NQ30/15min	0.526	0.472	0.001	0.497	0.5	0.001	0.5

3. Measuring persistence by conditional probabilities

Given a time series of price returns $g(t_i)$, where t_i denotes the consecutive equidistant moments of time, to each i we assign $+1$ if $g(t_i)$ is positive (price goes up), -1 if it is negative (price goes down) and 0 if it happens to be 0 (price remains unchanged). We then explicitly count the number $N_{\alpha,\beta}$ of all the neighboring pairs $\{g(t_i), g(t_{i+1})\}$ of the type $\alpha, \beta = \{-1, 0, +1\}$ for fixed values of α and β and do so for all the nine combinations of different α and β. Finally, we calculate

$$p_{\alpha,\beta} = N_{\alpha,\beta} / \sum_{\beta'=-1,0,+1} N_{\alpha,\beta'}, \tag{6}$$

which corresponds to a conditional probability that a return of the type β is preceded by a return of the type α. This procedure can, of course, be performed on any time scale $\Delta t = t_{i+1} - t_i$.

Six combinations of $p_{\alpha,\beta}$ corresponding to $\alpha = \pm 1$ and to all the three possible values of β are listed in Table I for several values of Δt starting from 1 up to 30 min.

Quite interestingly — and somewhat unexpectedly in view of the corresponding values of the Hurst exponents (last column in Table I) that are very close to 0.5 like for the white noise — both the DAX and the DJIA show significant effects of persistence on the small time scales. A move up (down) is followed by another move up (down) significantly more often than by a move in opposite direction. For Δt larger than 5–10 min we observe a crossover: the fluctuations become anti-persistent and, what is particularly interesting, this effect is visibly asymmetric towards moves down as a systematically observed relation $p_{-1,-1}$ and $p_{+1,+1}$ indicates. For the basket of the Nasdaq stocks this crossover also takes place, though on the somewhat larger time scales. Quite interestingly — and somewhat unexpectedly in view of the corresponding values of the Hurst exponents (last column in Table I) that are very close to 0.5 like for the white noise — both the DAX and the DJIA show significant effects of persistence on the small time scales. A move up (down) is followed by another move up (down) significantly more often than by a move in opposite direction. For Δt larger than 5–10 min we observe a crossover: the fluctuations become anti-persistent and, what is particularly interesting, this effect is visibly asymmetric towards moves down as a systematically observed relation $p_{-1,-1}$ and $p_{+1,+1}$ indicates. For the basket of the Nasdaq stocks this crossover also takes place, though on the somewhat larger time scales.

The time period (01.12.1997–31.12.1999) of the stock market variability studied here displays a richness of phases. In the first half of this period it for instance includes a spectacular draw up (DAX more than 50%) followed by an even faster draw down to the original level. Previous study [10] based on the correlation matrix formalism provides a serious indication that the dynamics of long-term stock market increases is more competitive and less collective (as far as correlations among the individual stocks forming an index is concerned) than during long-term decreases.

It is thus interesting to inspect if and how our indicators of persistence correlate with the different phases of the market dynamics. Two principal such coefficients (rectangles), $p_{+1,+1}$ and $p_{-1,-1}$, calculated for $\Delta t = 1$ min over one trading day time intervals for all the consecutive days covering our 01.12.1997–31.12.1999 time period, versus the corresponding DAX changes are shown in Fig. 1. The correlation is visible indeed.

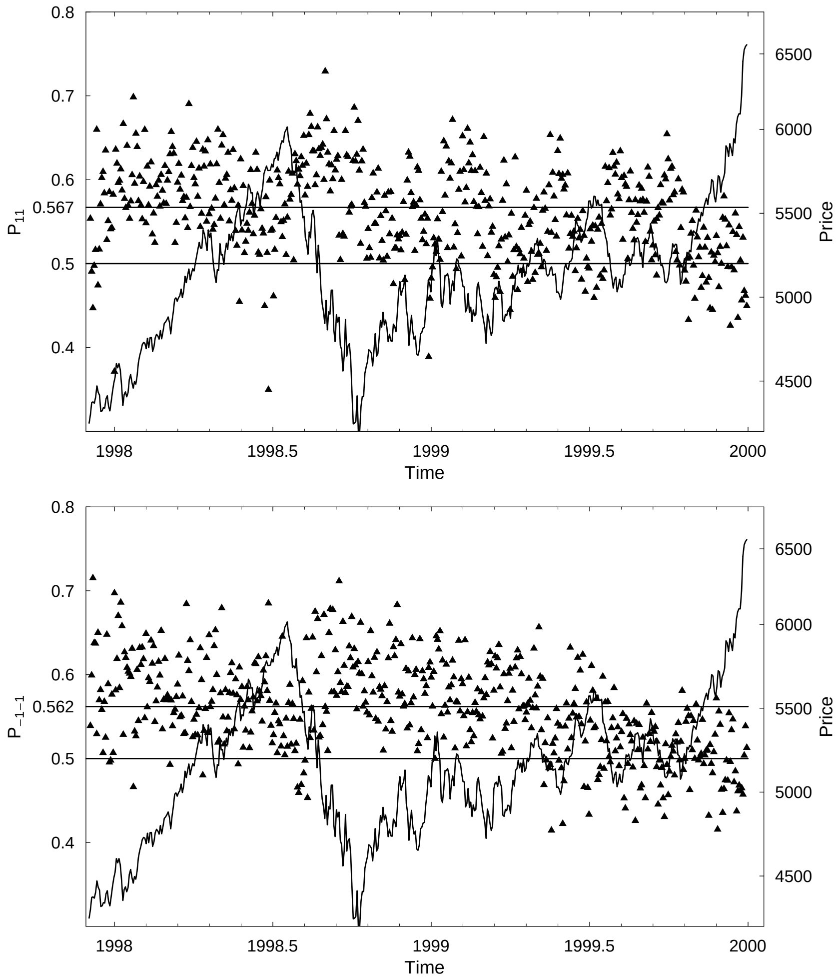


Fig. 1. The DAX time dependence over the period 01.12.1997–31.12.1999. The black triangles (▲) in panels (a) and (b) correspond to $p_{+1,+1}$ and $p_{-1,-1}$, respectively, calculated separately for the consecutive one trading day time intervals from $\Delta t = 1$ min returns.

The long-term increases systematically lead to a decrease of persistence both in $p_{+1,+1}$ and in $p_{-1,-1}$. A sharp increase of the DAX in the end of the period here analyzed pulls these two coefficients even below 0.5. On the contrary, the decreases are seen to be lifting our persistency coefficients up to as high as ~ 0.7. Even more, changes of the trend in $p_{+1,+1}$ and $p_{-1,-1}$ are somewhat shifted in phase relative to each other which is another manifestation of asymmetry in persistence. That all such effects may reflect a general logic of the stock market dynamics can be seen also from Fig. 2 which displays the same quantities for the DJIA in the same period of time and qualitatively analogous correlations can be postulated. More focus on this last issue is given in Figs. 3 and 4 and the corresponding Tables II and III.

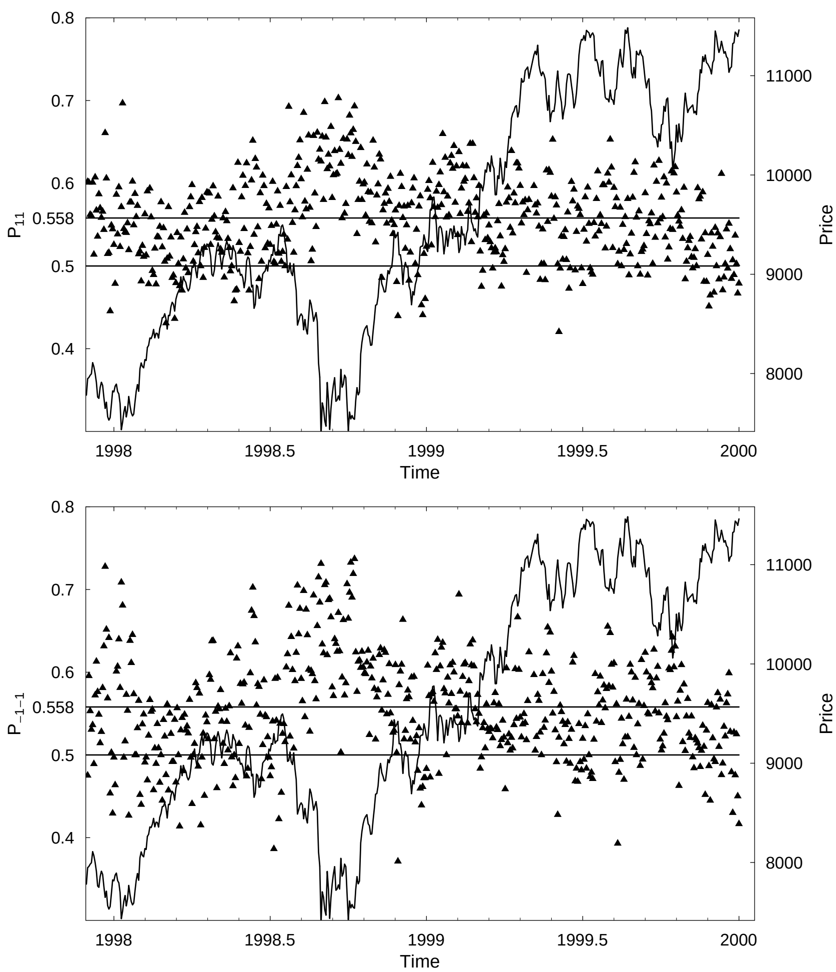


Fig. 2. Same as Fig. 1 but for the Dow Jones Industrial Average (DJIA).

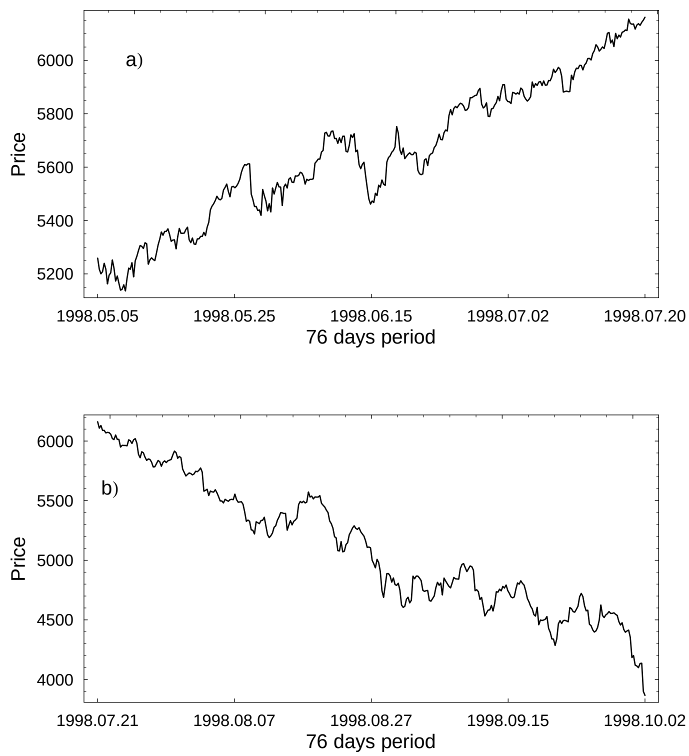


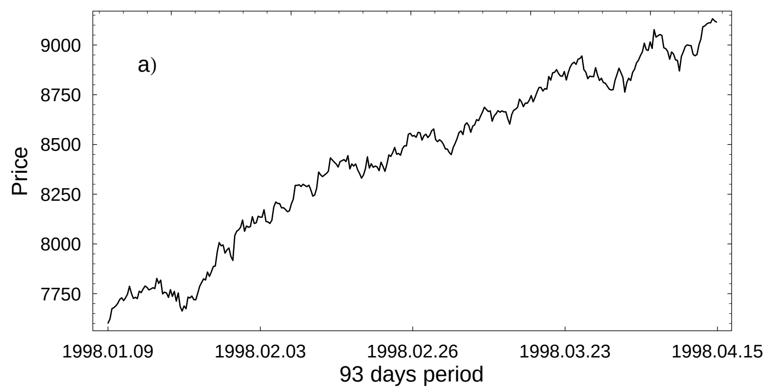
Fig. 3. DAX during its long-term global increase in the period 05.05.1998–20.07.1998 (a) and during its long-term global decrease in the period 21.07.1998–02.10.1998 (b). These both periods correspond to the same number (76) of trading days.

TABLE II

Conditional probabilities $p_{\alpha,\beta}$ and the Hurst exponents for the DAX $\Delta t = 1$ min changes corresponding to the time periods as in Fig. 3(a) (DAX/increase) and as in Fig. 3(b) (DAX/decrease), respectively.

Data	$\mathbf{P_{11}}$	$\mathbf{P_{1-1}}$	$\mathbf{P_{10}}$	$\mathbf{P_{-1-1}}$	$\mathbf{P_{-11}}$	$\mathbf{P_{-10}}$	**Hurst exp.**
DAX(increase)	0.562	0.433	0.004	0.57	0.424	0.006	0.491
DAX(decrease)	0.615	0.382	0.003	0.585	0.412	0.002	0.504

Two sizable periods of the global market increases and decreases, respectively, both for the DAX and for the DJIA, are here extracted and the corresponding conditional probability coefficients $p_{\alpha,\beta}$ calculated for those periods separately. Again one sees that the related fluctuations are persis-



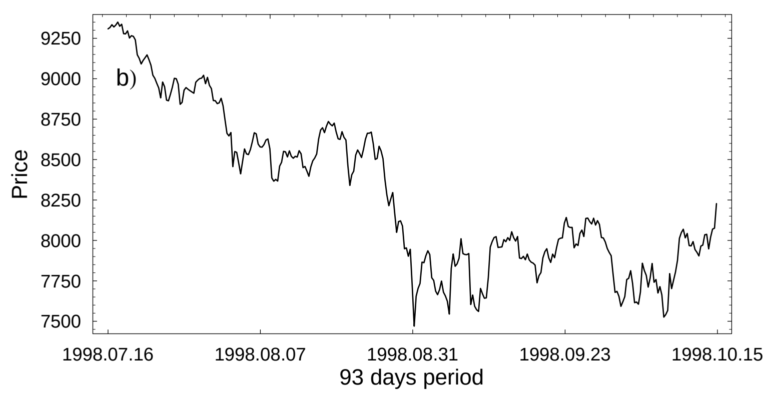


Fig. 4. DJIA during its long-term global increase in the period 09.01.1998–15.04.1998 (a) and during its long-term global decrease in the period 16.07.1998–15.10.1998 (b). These both periods correspond to the same number (93) of trading days.

TABLE III

Conditional probabilities $p_{\alpha,\beta}$ and the Hurst exponents for the DJIA $\Delta t = 1$ min changes corresponding to the time periods as in Fig. 4(a) (DJIA /increase) and as in Fig. 4(b) (DJIA/decrease), respectively.

Data	$\mathbf{P_{11}}$	$\mathbf{P_{1-1}}$	$\mathbf{P_{10}}$	$\mathbf{P_{-1-1}}$	$\mathbf{P_{-11}}$	$\mathbf{P_{-10}}$	**Hurst exp.**
DJIA(increase)	0.534	0.415	0.051	0.523	0.424	0.052	0.495
DJIA(decrease)	0.609	0.363	0.028	0.629	0.342	0.029	0.505

tent and that these persistency effects are even stronger during decreases. Furthermore, even during the same market phase (either global increase or decrease) the asymmetry in persistence between the moves up and down may occur.

4. Conclusion

The above observations are intriguing and of course demand a much more systematic study as they carry a potential to shed more light on mechanism of the stock market dynamics. The present study however already indicates direction concerning this specific issue. There definitely exist higher order correlations in the financial dynamics that escape detection within the conventional methods. In this connection the wavelet based formalism, due to its ability to focus on local effects, seems to offer a promising frame to develop consistent related methodology, such that a link to multifractality perhaps can also be traced. The wavelet based formalism can also be generalized to account for the asymmetry in persistence — an effect identified above. Finally and ultimately, one needs to develop a realistic theoretical model of the financial dynamics such that also the above effects can be incorporated. A variant of the generalized Weierstrass random walk as developed by Kutner [11] may appear an appropriate solution especially that the Weierstrass-type functions may generate log-periodicity — a kind of correlations that underlay the financial dynamics [12, 13].

REFERENCES

[1] L. Bachelier, *Ann. Sci. de l'Ecole Norm. Sup.* **III-17**, 21 (1900).
[2] P. Gopikrishnan, V. Plerou, L.A. Nunes Amaral, M. Mayer, H.E. Stanley, *Phys. Rev.* **E60**, 5305 (1999).
[3] S. Drożdż, J. Kwapień, F. Grümmer, F. Ruf, J. Speth, *Acta Phys. Pol. B* **34**, 4293 (2003).
[4] P. Oświęcimka, J. Kwapień, S. Drożdż, *Physica A* **347**, 626 (2005).
[5] B.B. Mandelbrot, J.R. Wallis, *Water Resources Res.* **5**, 967 (1969).
[6] H.E. Hurst, *Trans. Am. Soc. Civil Engineers* **116**, 770 (1951).
[7] C.-K. Peng, S.V. Buldyrev, A.L. Goldberger, S. Havlin, M. Simons, H.E. Stanley, *Phys. Rev.* **E47**, 3730 (1993).
[8] D. Grech, Z. Mazur, *Physica A* **336**, 133 (2004).
[9] A. Carbone, G. Castelli, H.E. Stanley, *Physica A* **344**, 267 (2004).
[10] S. Drożdż, F. Grümmer, A.Z. Górski, F. Ruf, J. Speth, *Physica A* **287**, 440 (2000).
[11] R. Kutner, *Physica A* **264**, 84 (1999); *Physica A* **314**, 786 (2002).
[12] S. Drożdż, F. Ruf, J. Speth, M. Wójcik, *Eur. Phys. J.* **B10**, 589 (1999).
[13] M. Bartolozzi, S. Drożdż, D.B. Leinweber, J. Speth, A.W. Thomas, `cond-mat/0501513`, *Inter. J. Mod. Phys.* **C** in print.