THE CONVERGENCE OF EUROPEAN BUSINESS CYCLES 1980–2004*

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The degree of convergence of the business cycles of the economies of the European Union is a key policy issue. In particular, a substantial degree of convergence is needed if the European Central Bank is to be capable of setting a monetary policy which is appropriate to the stage of the cycle of the Euro zone economies. I consider the annual rates of real GDP growth on a quarterly basis in the main economies of the EU (France, Germany, Italy, UK, Spain, Belgium and the Netherlands) over the period 1980Q1–2004Q4. An important empirical question is the degree to which the correlations between these growth rates contain true information rather than noise. The technique of random matrix theory is able to answer this question, and has been applied successfully in the physics journals to financial markets data. I find that the correlations between the growth rates of most of the core EU economies contain substantial amounts of true information, and exhibit considerable stability over time. Even in the late 1970s and early 1980s, these economies moved together closely over the course of the business cycle. There was a slight loosening at the time of German re-unification, but the economies have moved back into close synchronisation. The same result holds when Spain is added to the group of core EU countries. However, the problems of the German economy which arose from the early 1990s onwards has led to Germany becoming increasingly less synchronised with the rest of the core EU. Further, the results obtained with a data set of the converged EU core plus the UK show no real convergence between the UK and this group of economies.

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1. Introduction

Most of the countries of the European Union (EU) participated in the formation of the new currency, the Euro, on 1 January 1999. The EU's second largest economy, the UK, remains outside and retains sterling as its currency, and Denmark and Sweden also retain their own currencies.

A key feature of a monetary union such as the Euro is that monetary policy is common to all member states. The structure of interest rates is effectively identical throughout the union. There may be small differences from state to state, but these are decidedly second-order.

It is therefore desirable that the economies of the member states of a monetary union should follow similar business cycles. The level of interest rates appropriate in an economy which is experiencing a boom is unlikely to be so for an economy which is in recession.

This paper examines the extent to which the business cycles of the main EU economies have been in synchronisation over the 1980–2004 period, and how this has altered over this period. We examine the performance of the EU 'core', the large economies of France, Italy and Germany plus Belgium and the Netherlands, which were founder members of both the EU itself and of the Euro, and the core plus the large economy of Spain, which did not join the EU until 1982 but which was a founder member of the Euro. This is contrasted with the core plus the UK, which whilst a member of the EU since 1973 has not joined the Euro and has been consistently the least supportive of ideas of further European integration.

We use the technique of random matrix theory (Mehta 1991) to analyse the correlations between the growth rates of the economies over time. Section 2 discusses the relevance of this theory, and Section 3 sets out the empirical results.

2. Random matrix theory

Quarterly data exists for most of the EU economies over the past twenty years or so for the level of real output in the economy (GDP). We can therefore calculate annual growth rates quarter-by-quarter. The correlations between these growth rates for the various economies will inform us about the extent to which their business cycles are in synchronisation.

In other words, the degree of synchronisation of the business cycles may be quantified by calculation of the correlation matrix of the matrix of observations formed from the time series of GDP growth for each economy.

If $\boldsymbol{M}$ is an $N \times T$ rectangular matrix (T observations of the GDP growth of the N economies) and $\boldsymbol{M}^T$ is its transpose, the correlation matrix $\boldsymbol{C}$ as defined below is an $N \times N$ square matrix

$$\boldsymbol{C} = \frac{1}{T}\boldsymbol{M}\boldsymbol{M}^T .$$

However, due to the finite size of N (which corresponds to the number of economies) and T (which is the number of observations of GDP) then a reliable determination of the correlation matrix may prove to be problematic. The structure of the correlation matrix may be dominated by noise rather than by true information.

In order to assess the degree to which an empirical correlation matrix is noise dominated we can compare the eigenspectra properties of the empirical matrix with the theoretical eigenspectra properties of a random matrix. Undertaking this analysis will identify those eigenstates of the empirical matrix who contain genuine information content. The remaining eigenstates will be noise dominated and hence unstable over time. This technique has been applied by many researchers to financial market data (for example, (Mantegna *et al.* 1999), (Laloux *et al.* 1999), (Plerou *et al.* 1999), (Plerou 2000), (Bouchaud *et al.* 2000), (Drozdz *et al.* 2001)).

For a scaled random matrix $\boldsymbol{X}$ of dimension $N \times T$, (*i.e.* where all the elements of the matrix are drawn at random and then the matrix is scaled so that each column has mean zero and variance one), then the distribution of the eigenvalues of the correlation matrix of $\boldsymbol{X}$ is known in the limit $T, N \to \infty$ with $Q = T/N \geq 1$ fixed (Sengupta *et al.* 1999). The density of the eigenvalues of the correlation matrix, λ, is given by:

$$\rho(\lambda) = \frac{Q}{2\pi}\frac{\sqrt{(\lambda_{\max} - \lambda)(\lambda - \lambda_{\min})}}{\lambda} \quad \text{for } \lambda \in [\lambda_{\min}, \lambda_{\max}]$$

and zero otherwise, where $\lambda_{\max} = \sigma^2(1+1/\sqrt{Q})^2$ and $\lambda_{\min} = \sigma^2(1-1/\sqrt{Q})^2$ (in this case $\sigma^2 = 1$ by construction).

The eigenvalue distribution of the correlation matrices of matrices of actual data can be compared to this distribution and thus, in theory, if the distribution of eigenvalues of an empirically formed matrix differs from the above distribution, then that matrix will not have random elements. In other words, there will be structure present in the correlation matrix.

To analyse the structure of eigenvectors lying outside of the noisy subspace band the Inverse Participation Ratio (IPR) may be calculated. The IPR is commonly utilised in localisation theory to quantify the contribution of the different components of an eigenvector to the magnitude of that eigenvector (thus determining if an eigenstate is localised or extended) (Plerou *et al.* 1999).

Component i of an eigenvector ν_i^α corresponds to the contribution of time series i to that eigenvector. That is to say, in this context, it corresponds to the contribution of economy i to eigenvector α. In order to quantify this we define the IPR for eigenvector α to be

$$I^\alpha = \sum_{i=1}^{N} (\nu_i^\alpha)^4 \ .$$

Hence an eigenvector with identical components $\nu_i^\alpha = 1/\sqrt{N}$ will have $I^\alpha = 1/N$ and an eigenvector with one non-zero component will have $I^\alpha = 1$. Therefore the inverse participation ratio is the reciprocal of the number of eigenvector components significantly different from zero (*i.e.* the number of economies contributing to that eigenvector).

3. The data and the results

Quarterly levels of real GDP over the period 1980Q1–2004Q4 are available from the OECD database for the largest EU economies, France, Germany, Italy, Spain, UK, Belgium and the Netherlands. The first three plus the Benelux[1] countries are widely regarded as forming the EU 'core', being the founder members of the (then) European Economic Community.

We analyse the correlation matrix of real GDP growth rates for various permutations of these economies.

For the 'core' EU economies, the theoretical range of the eigenvalues for a random matrix of the relevant order is between 0.61 and 1.50. The eigenvalues of the empirical correlation matrix of annual growth rates of the EU core over the 1980Q1–2004Q4 period are 2.93, 1.12, 0.33, and 0.24. For the seven economies as a whole, the eigenvalues are in the range 0.23 to 3.81. These results indicate the presence of a large amount of true information in the correlation matrix.

In terms of those eigenvalues which lie outside the noisy sub-space band the most important from a macroeconomic perspective is the largest eigenvalue. The application of these techniques to equities traded in financial markets have demonstrated that this eigenmode corresponds to the 'market' eigenmode (*e.g.* Gopikrishnan *et al.*, 2000). In this context the largest eigenvalue will inform us as to the degree to which the movements of the EU economies are correlated.

The contribution which each of the core economies makes to eigenvector 1 can be seen from calculating the IPR. The components are in fact (0.10, 0.50, 0.47, 0.52, 0.49), which gives a calculated value of the IPR of 0.247, indicating that four out of the five economies are contributing approximately equally to this eigenvector but that one seems to be out of synchronisation with the rest. This latter economy is in fact Germany.

[1] The Luxemburg economy is trivially small and is not in this analysis.

The trace of the correlation matrix is conserved, and is equal to the number of independent variables for which time series are analysed. That is, for the core EU correlation matrix the trace is equal to 5 (since there are 5 time series). The closer the 'market' eigenmode (*i.e.* eigenmode 1) is to this value the more information is contained within this mode *i.e.* the more correlated the movements of GDP. The market eigenmode corresponds to the largest eigenvalue. The degree of information contained within this eigenmode, expressed as a percentage, is therefore $100\lambda_{\max}/N$.

To follow the evolution of the degree of business cycle convergence over time we may analyse how this quantity evolves temporally. The analysis is undertaken with a fixed window of data. Within this window the spectral properties of the correlation matrix formed from this data set are calculated. In particular the maximum eigenvalue is calculated. This window is then advanced by one period and the maximum eigenvalue noted for each period.

The choice of an appropriate window to span the periodicity of what constitutes the business cycle is not completely straightforward. Business activity is influenced by a very large number of events, and these events may be very diverse in character and scope. Individual cycles therefore vary both in terms of amplitude and period. This lack of regularity may be analysed formally using random matrix techniques (Ormerod and Mounfield 2000). The evidence for the existence of a business cycle at all relies more upon factors such as the fact that output changes in different sectors of an economy tend to move together (Lucas 1977) than upon regularities in either amplitude or period of the economy as a whole.

A major study of the US economy (Burns and Mitchell 1946) many years ago concluded that the period ranged from some two to twelve years, a range which still commands broad assent amongst economists, though the upper bound might now be felt to be slightly high. We initially carried out results for a window of 10 years, although the results for a window of 8 years are virtually identical, and it is these which we present here. The results are in fact robust to the choice of window, until a window as short as 5 years is chosen, when greater instability begins to be introduced, due to measurement noise induced by the reduced number of observations.

The results for the core EU economies are set out in figure 1. Each window contains 32 quarterly observations, and so we have 65 windows in total. The period 1980Q1–1987Q4 corresponds to the first data point in figure 1, 1980Q2–1988Q1 to the second, and so on through to 1997Q1–2004Q4.

Even in the early part of the period, the 'market' eigenvalue took up some 70 per cent of the total of the eigenvalues, indicating a strong degree of convergence of the business cycles of the EU core economies. There was a temporary reduction of convergence around the time of German re-

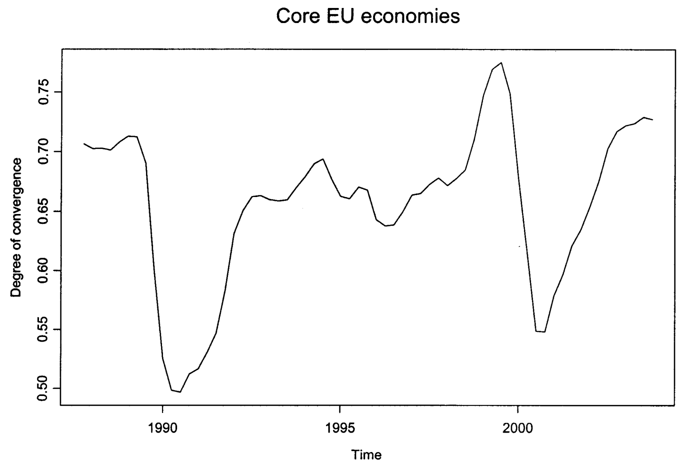

Fig. 1. The temporal evolution of the degree of information content in the maximum eigenvalue of the empirical correlation matrix formed from the time series of quarterly GDP growth for the core EU economies of France, Germany, Italy, Belgium and the Netherlands. Each window of data spans 32 quarterly observations. The period 1980Q1–1987Q4 corresponds to the first data point in figure 1, 1980Q2–1988Q1 to the second, and so on through to 1997Q1–2004Q4.

unification in the early 1990s, but the economies rapidly re-converged and by 2000 the principal eigenvalue accounted for nearly 80 per cent of the total information content within the correlation matrix, indicating a movement towards even greater convergence of the business cycles of the EU core economies over time. This result was reported by (Ormerod and Mounfield 2002). However, since then, the deep seated problems of the German economy have led to considerable variation in the degree of convergence over time.

There were more general divergencies during the 1990s, as figure 2 shows. Here, we plot the core EU economies without Germany.

The patterns in the two charts are broadly similar, but the degree of convergence excluding Germany is considerably greater, as the values of $100\lambda_{\max}/N$ indicates. Its mean value, for example, for all the core economies is 0.65, but excluding Germany this rises to 0.75. On a formal Kolmogorov–Smirnov test, the null hypothesis that the distribution of $100\lambda_{\max}/N$ is the same both including and excluding Germany is rejected even at a p-value of 0.00.

We now move on to examine the case of Spain. After many years isolated under dictatorship, the Spanish authorities have attached great importance to modernising their economy and society in a European context. Policy

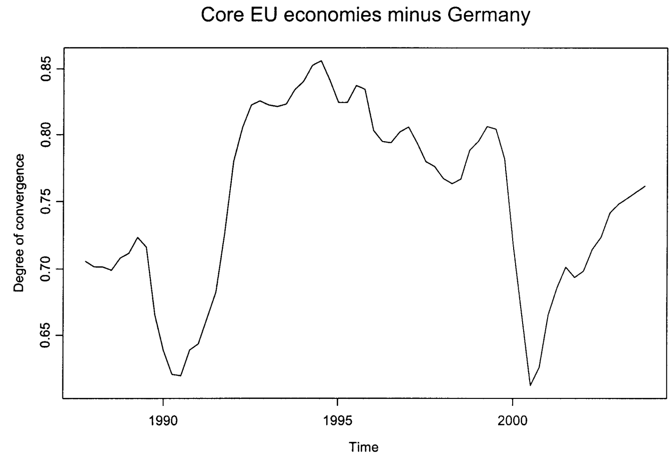

Fig. 2. The temporal evolution of the degree of information content in the maximum eigenvalue of the empirical correlation matrix formed from the time series of quarterly GDP growth for the EU economies of France, Italy, Belgium and the Netherlands. Each window of data spans 32 quarterly observations.

has been strongly supportive of European integration. The extent to which business cycle convergence has been achieved with the EU core, excluding Germany, is plotted in figure 3.

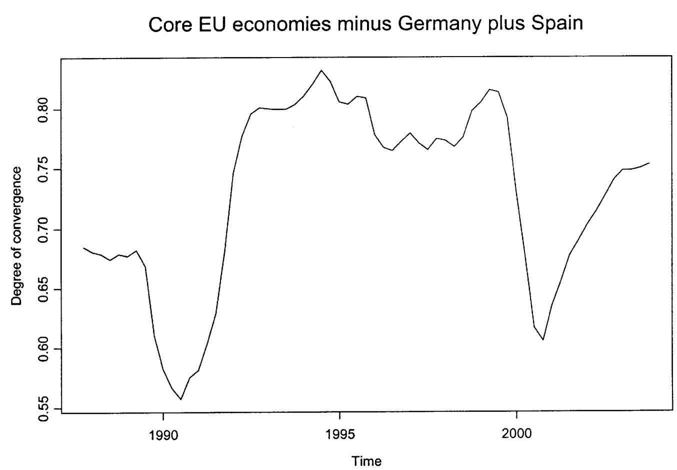

Fig. 3. The temporal evolution of the degree of information content in the maximum eigenvalue of the empirical correlation matrix formed from the time series of quarterly GDP growth for the core EU economies of France, Italy, Belgium and the Netherlands plus the time series of GDP growth for the Spanish economy.

Qualitatively, the pattern over time is very similar indeed to that of figure 2. In other words, this suggests strong evidence to support the view that the Spanish economy has become closely converged with the core EU economies (excluding Germany) in terms of its movements over the business cycle. On a formal Kolmogorov–Smirnov test, the null hypothesis that the distribution of $100\lambda_{\max}/N$ is the same both including and excluding Spain is only rejected at a p-value of 0.22, well above the conventional level of statistical significance.

Figure 4 shows the results for France, Italy, Belgium, Netherlands and Spain plus the UK.

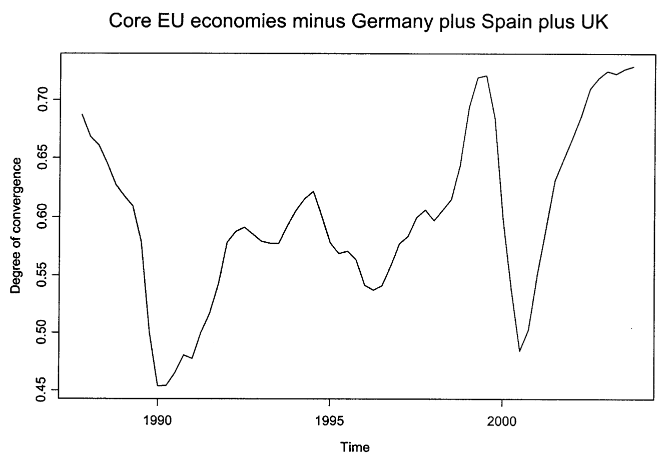

Fig. 4. The temporal evolution of the degree of information content in the maximum eigenvalue of the empirical correlation matrix formed from the time series of quarterly GDP growth for the core EU economies of France, Italy Belgium, the Netherlands and Spain plus the time series of GDP growth for the UK economy.

Including the UK, the mean value of falls to 0.65, compared to the value of 0.73 when the data set includes just France, Italy, Belgium, Netherlands and Spain. Given that the trace of the matrix is conserved, the addition of a purely random variable to the latter data set would on average reduce $100\lambda_{\max}/N$ to 0.60. So the results suggest very clearly that the UK economy is not really converged with most of the other main EU economies in terms of the timing of its business cycle.

4. Conclusion

In this paper, we analyse the convergence or otherwise of the business cycle in the main economies of the European Union, using the annual growth rates of quarterly real GDP over the 1980Q1–2004Q4 period. The correlations between the growth rates are analysed using random matrix theory, which enables us to identify the extent to which the correlations contain true information rather than noise.

For most of the core EU countries, France, Italy, Belgium and the Netherlands, we find that the business cycles have shown strong synchronisation over the whole of the 1980–2004 period. The inclusion of Spain in the data set does not alter this conclusion.

In contrast, Germany appears to have become considerably less synchronised with these other 'core' EU economies. The problems which have existed for the German economy since the early 1990s have reduced very considerably the extent to which its business cycle moves in line with those of other core EU economies.

Further, there is little evidence that the UK business cycle has moved into synchronisation with the main core of the EU.

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