PROTON–NEUTRON PAIRING IN LIPKIN–NOGAMI APPROACH*

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State-dependent δ-force is used to analyze isovector ($T = 1$) and isoscalar ($T = 0$) superfluidity in the framework of the generalized BCS model with an approximate Lipkin–Nogami particle-number projection. Calculations are performed with the single-particle levels generated in axially symmetric Skyrme–Hartree–Fock code with SIII force for several medium-mass $N \sim Z$ nuclei.

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1. Introduction

The BCS formalism capable to deal simultaneously like-particle and proton–neutron correlations was already introduced in the sixties [1–4]. In recent years, due to the progress in experimental techniques, a revival of interest on the subject is taking place. There are many valuable approaches proposed by different authors to include *np* correlations, however, most of the mean-field calculations is done imposing different symmetries and simplifications, *e.g.* disregarding Coulomb interaction or using schematic pairing forces. Here we study the *np* interaction with the δ-force commonly used to describe pairing in *pp* and *nn* channels. The pairing correlations are permitted between nucleons in time-reversed orbits ($\alpha\bar{\alpha}$ pairing) as the $\alpha\alpha$ pairing plays a role only in well deformed nuclei [4]. In this paper the effects of including *np* correlations are studied in BCS and Lipkin–Nogami (LN) approaches in the ground states of several even–even Ge isotopes.

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2. Pairing Hamiltonian

In the following we consider the pairing Hamiltonian with the generalized pairing interaction of the form

$$\hat{v} = -\sum_{T,S} V^{TT_z}\delta(\vec{r}_1 - \vec{r}_2)\hat{\Pi}^T\hat{\Pi}^S\,, \tag{1}$$

where $\hat{\Pi}^T$ and $\hat{\Pi}^S$ operators project onto proper isospin–spin subspaces. V^{TT_z} are the coupling strengths to be determined. Since the nucleon–nucleon interaction is isospin invariant and we consider $N \sim Z$ nuclei it is justified to choose the coupling constant equal for all T_z components in $T=1$ channel. The interaction in $T=0$ channel is supposed to be stronger. The generalized pairing Hamiltonian can be written as

$$\hat{H} = \sum_{kl,\tau\tau',T} g^T_{k\tau,l\tau'}\, a^\dagger_{k\tau} a^\dagger_{\bar{k}\tau'} a_{\bar{l}\tau'} a_{l\tau}\,, \tag{2}$$

where $g^T_{k\tau,l\tau'}$ is the antisymmetrized matrix element of the interaction (1) and $\tau\tau' \in \{p,n\}$. The generalized BCS equations which allow to treat the protons and neutrons as nonseparable systems are of the form [6]

$$\begin{pmatrix} \tilde{\varepsilon}_{pk} & 0 & \Delta^{pp}_k & \Delta^{np}_k \\ 0 & \tilde{\varepsilon}_{nk} & \Delta^{np\star}_k & \Delta^{nn}_k \\ \Delta^{pp}_k & \Delta^{np}_k & -\tilde{\varepsilon}_{pk} & 0 \\ \Delta^{np\star}_k & \Delta^{nn}_k & 0 & -\tilde{\varepsilon}_{nk} \end{pmatrix} \begin{pmatrix} u_{kpq} \\ u_{knq} \\ v_{kpq} \\ v_{knq} \end{pmatrix} = E_{kq} \begin{pmatrix} u_{kpq} \\ u_{knq} \\ v_{kpq} \\ v_{knq} \end{pmatrix}, \tag{3}$$

where $q = 1,2$, $\tilde{\varepsilon}_{\tau k} = e_{\tau k} - \lambda_\tau$. The diagonalization of the matrix (3) yields quasiparticle energies E_{kq} and occupation amplitudes u, v. The pairing gaps read

$$\begin{aligned} \Delta^{\tau\tau}_m &= \sum_{k,q} g^{T=1}_{m\tau,k\tau} v_{k\tau q} u^\star_{k\tau q}\,, \\ \Delta^{np}_m &= \Delta^{1np}_m + i\Delta^{0np}_m\,, \\ \Delta^{1np}_m &= \sum_{k,q} g^{T=1}_{mp,kn} \mathfrak{Re}\left(v_{kpq} u^\star_{knq}\right)\,, \\ \Delta^{0np}_m &= \sum_{k,q} g^{T=0}_{mp,kn} \mathfrak{Im}\left(v_{kpq} u^\star_{knq}\right)\,, \end{aligned} \tag{4}$$

and the pairing energy is calculated as

$$E_{\mathrm{pair}} = -\sum_{kl,\tau\tau',T} g^T_{kl}\, \kappa^{\tau\tau'}_{l\bar{l}}\, \kappa^{\tau\tau'\star}_{k\bar{k}}\,, \tag{5}$$

where $\kappa_{k\bar{k}}^{\tau\tau'} = \sum_q v_{k\tau q}^{\star} u_{k\tau' q}$. The Fermi levels for protons and neutrons are determined from the particle number equation

$$N_\tau = 2\sum_{kq} v_{k\tau q} v_{k\tau q}^{\star}\,. \tag{6}$$

These BCS equations are solved iteratively till the required accuracy for the pairing energy and average particle number is achieved.

3. Lipkin–Nogami approach

It is especially interesting to investigate the LN approach in the case of isoscalar and isovector pairing interactions where transitions superfluid to normal phases are present in BCS calculations. Let us remind shortly the outline of the Lipkin–Nogami [7] model for the case of a generalized pairing interaction. We follow here the considerations of Ref. [8] where the case of *pp* and *nn* pairing was studied. A more detailed discussion on the subject can be also found in [9, 10].

The Lipkin–Nogami method aims to minimize the expectation value of the operator

$$\hat{H}' = \hat{H} - \sum_\tau \lambda^\tau \Delta\hat{N}_\tau - \sum_{\tau,\tau'} \lambda_2^{\tau\tau'} \Delta\hat{N}_\tau \Delta\hat{N}_{\tau'}\,, \tag{7}$$

where $\Delta\hat{N}_\tau = \hat{N}_\tau - \langle\hat{N}_\tau\rangle$. The coefficients $\lambda_2^{\tau\tau'}$, which are kept constant during the variational procedure, are determined using subsidiary conditions

$$\langle \hat{H}'(\Delta\hat{N}_\tau\Delta\hat{N}_{\tau'} - \langle\Delta\hat{N}_\tau\Delta\hat{N}_{\tau'}\rangle)\rangle = 0\,, \tag{8}$$

which lead to the set of equations

$$\mathcal{G}^{\tau\tau'} + \sum_{\sigma\sigma'} \lambda_2^{\sigma\sigma'} \mathcal{N}_{\sigma\sigma'}^{\tau\tau'} = 0\,, \tag{9}$$

where

$$\mathcal{G}^{\tau\tau'} = \sum_{\{4\}} \langle 0|\hat{H}_{40}|4\rangle\langle 4|(\hat{N}_\tau\hat{N}_{\tau'})_{04}|0\rangle\,, \tag{10}$$

$$\mathcal{N}_{\sigma\sigma'}^{\tau\tau'} = \sum_{\{4\}} \langle 0|(\hat{N}_\sigma\hat{N}_{\sigma'})_{40}|4\rangle\langle 4|(\hat{N}_\tau\hat{N}_{\tau'})_{04}|0\rangle\,. \tag{11}$$

Here $|4\rangle\langle 4|$ denotes the projection onto a four-quasiparticle space, $\sigma\sigma' \in \{p, n\}$. The resulting LN equations can be written as

$$\begin{pmatrix} \tilde{\varepsilon}_{pk}^{(\mathrm{LN})} & 0 & \Delta_k^{pp(\mathrm{LN})} & \Delta_k^{np(\mathrm{LN})} \\ 0 & \tilde{\varepsilon}_{nk}^{(\mathrm{LN})} & \Delta_k^{np\star(\mathrm{LN})} & \Delta_k^{nn(\mathrm{LN})} \\ \Delta_k^{pp(\mathrm{LN})} & \Delta_k^{np(\mathrm{LN})} & -\tilde{\varepsilon}_{pk}^{(\mathrm{LN})} & 0 \\ \Delta_k^{np\star(\mathrm{LN})} & \Delta_k^{nn(\mathrm{LN})} & 0 & -\tilde{\varepsilon}_{nk(\mathrm{LN})} \end{pmatrix} \begin{pmatrix} u_{kpq} \\ u_{knq} \\ v_{kpq} \\ v_{knq} \end{pmatrix} = E_{kq}^{(\mathrm{LN})} \begin{pmatrix} u_{kpq} \\ u_{knq} \\ v_{kpq} \\ v_{knq} \end{pmatrix},$$

where the pairing gaps and single-particle energies are renormalized as follows:

$$\begin{aligned} \tilde{\varepsilon}_{\tau k}^{(\mathrm{LN})} &= \tilde{\varepsilon}_{\tau k} + 2\lambda_2^{\tau\tau} \rho_{kk}^{\tau\tau} \,, \\ \Delta_k^{\tau\tau'(\mathrm{LN})} &= \Delta_k^{\tau\tau'} - 2\lambda_2^{\tau\tau'} \kappa_{k\bar{k}}^{\tau\tau'} \,, \end{aligned} \tag{12}$$

with particle densities $\rho_{kk}^{\tau\tau} = \sum_q v_{k\tau q} v_{k\tau q}^\star$. Pairing correlation energy in the Lipkin–Nogami model is given by

$$E_{\mathrm{LN}} = E_{\mathrm{pair}} - 2 \sum_{k\tau, \bar{k}\tau'} \lambda_2^{\tau\tau'} \kappa_{k\bar{k}}^{\tau\tau'} \kappa_{k\bar{k}}^{\tau\tau'\star} \,. \tag{13}$$

4. Results

The usual way of choosing the coupling strengths for *pp* and *nn* correlations is adjusting them to odd–even mass differences. For *np* correlations very little is known about the actual strength of the interaction. It can be fitted to experimental pairing gaps [6] or to the Wigner energy [11] which is expected to be the result of *np* correlations in $T = 0$ channel. Both methods of adjusting pairing strengths give similar results [12]. Here, to discuss the behaviors of different modes we present the results as functions of the ratio of coupling strengths in $T = 1$ and $T = 0$ channels: $x = V^{T=0}/V^{T=1}$. All the results shown were obtained for $V^{T=1} = 400\,\mathrm{MeVfm}^3$ in the pairing window containing all the levels with the energies less than $e_{\mathrm{F}} + 5\,\mathrm{MeV}$, e_{F} indicating the energy of the Fermi level.

Fig. 1 shows the spectral pairing gaps in different channels as functions of x parameters in BCS and LN models for ^{64}Ge. As can be seen, the *np* solution in LN approach arises for larger values of x parameter as compared to BCS results, but the *nn* and *pp* gaps do not vanish with the appearance of the *np* mode. Contrary to the BCS model, in particle conserving approach both $T = 1$ and $T = 0$ channels contribute to the *np* solution. An excerpt of the state-dependent LN pairing gaps as functions of the level number in ^{64}Ge is shown in Fig. 2.

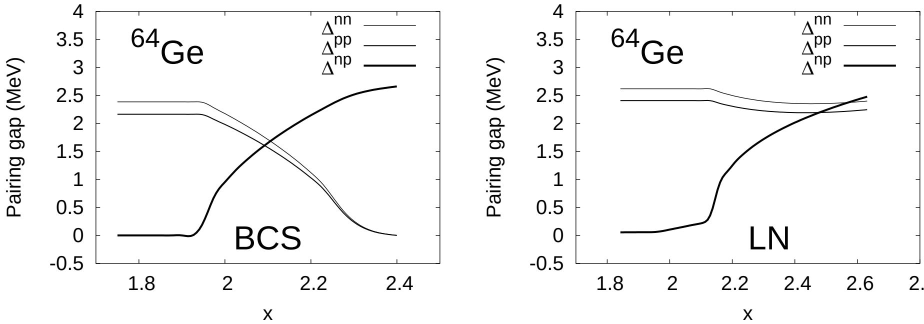

Fig. 1. Pairing gaps as functions of coupling strengths ratio.

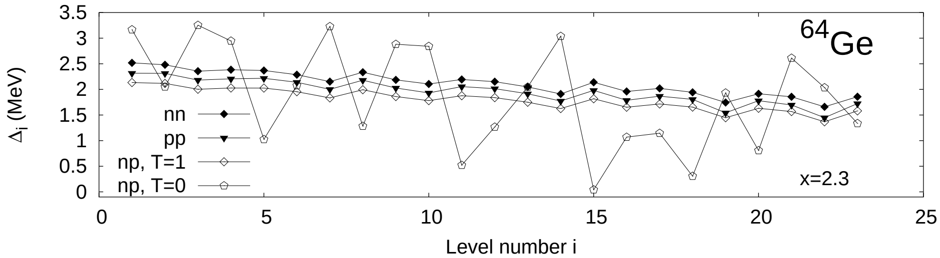

Fig. 2. State-dependent pairing gaps as functions of the level number.

Fig. 3 shows the normalized pairing energy as a function of x in BCS and LN models. Due to the transition between different modes in BCS approach the correlation energy is lower when only one type of pairs is present. This is mostly the case of ^{64}Ge where with the increase of $V^{T=0}$ strength the system prefers to form only np pairs. For nuclei with non zero $T_z = (N-Z)/2$ values there is less sharp transition and the np mode occurs only in coexistence with particle-like modes. In LN approach the three pairing gaps can coexist in a wide range of x values, therefore, activating the np mode leads to a gain in pairing energy. It is worth to notice that λ_2^{np} is negative and the correction to the pairing energy associated with np mode is positive. Nevertheless, the pp and nn channels are enhanced in such a way that the pairing energy increases with the increase of x value.

The main results can be summarized as follows:
(i) BCS method does not lead to a coexistence of $T = 1$ and $T = 0$ np superfluid phases but the mixing of $T = 0$ np and like-particle modes is found. In the particle conserving approach both $T = 0$ and $T = 1$ channels contribute to the np pairing gap.
(ii) The BCS solutions with coexisting $T = 0$ and $T = 1$ superfluid phases are sometimes higher in energy than the solutions with $T = 1$ or $T = 0$ pairs only.

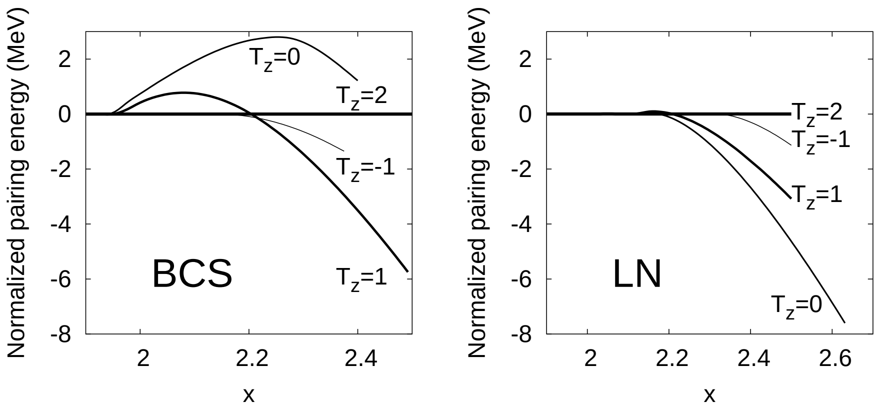

Fig. 3. Normalized pairing energy as a function of x in BCS and LN models. Different curves correspond to different Ge isotopes indicated by $T_z = (N - Z)/2$ values.

(iii) Particle number projection act destructively on np correlations and enhances pp and nn pairing modes ($\lambda_2^{\tau\tau'} < 0, \lambda_2^{\tau\tau} > 0$). The $T = 0$ phase occurs for a larger value of x parameter than in the BCS case but contrary to the BCS method, in the LN model activating the $T = 0$ pairing always leads to a gain in the pairing energy.
(iv) The above considerations are common for studied nuclei with $T_z = -1, 0, 1$ values. In the considered range of V^T parameters for $T_z = 2$ only trivial np solutions are found.
(v) The BCS and LN results obtained in this work are similar to those of Refs. [6, 10]. However, contrary to those results the coexistence of $T = 0$ and $T = 1$ phases in BCS approach in the ground state of $N = Z$ nucleus is found. No np collective solution is observed in Ge isotopes with $T_z \geq 2$ which is in contradiction to the results of Ref. [6].

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REFERENCES

[1] A. Goswami, *Nucl. Phys.* **60**, 228 (1964).
[2] A. Goswami, L.S. Kisslinger, *Phys. Rev.* **140**, B26 (1965).
[3] H.T. Chen, A. Goswami, *Phys. Lett.* **B24**, 257 (1967).
[4] A.L. Goodman, G.L. Struble, A. Goswami, *Phys. Lett.* **B26**, 260 (1968).
[5] A.L. Goodman, *Nucl. Phys.* **A186**, 475 (1972).

[6] F. Šimkovic, Ch.C. Moustakidis, L. Pacearescu, A. Faessler, *Phys. Rev.* **68**, 054319 (2003).

[7] H.J. Lipkin, *Ann. Phys.* **31**, 525 (1960); Y. Nogami, *Phys. Rev.* **134**, B313 (1964).

[8] P. Magierski, S. Ćwiok, J. Dobaczewski, W. Nazarewicz, *Phys. Rev.* **C48**, 1686 (1993).

[9] W. Satuła, R. Wyss, P. Magierski, *Nucl. Phys.* **A578**, 45 (1994).

[10] W. Satuła, R. Wyss, *Nucl. Phys.* **A676**, 120 (2000).

[11] W. Satuła, R. Wyss, *Phys. Lett.* **B393**, 1 (1997).

[12] K. Sieja, A. Baran, P. Quentin, to be published in *Phys. Scripta*.