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CVC TESTS AND CKM UNITARITY*

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The most precise and convincing confirmation of the conservation of the vector current (CVC) comes from measurements of superallowed nuclear β decay. It also provides the most demanding test available of the unitarity of the Cabibbo–Kobayashi–Maskawa (CKM) matrix, a basic pillar of the Electroweak Standard Model. Current experiments focus on tests of the small correction terms that must be applied to the data, with the goal of improving the precision of these tests. A recent result raises unexpected questions.

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1. Introduction

Since the conservation of the weak vector current (CVC) was first hypothesized in 1958 [1], tests of its validity have been of two types: comparisons of the weak magnetism form factor observed in β decay with the corresponding electromagnetic form factor; and comparisons among values of the vector coupling constant, G_{V}, measured from different superallowed β transitions in a wide range of nuclei. So far, the former have confirmed the expectations of CVC to a few percent; the latter have demonstrated CVC expectations, the constancy of G_{V}, to the much more demanding precision of 0.03%.

Tests of the weak magnetism form factor require difficult measurements of the detailed shape of the β spectrum or of asymmetry in the β-decay of polarized nuclei. To achieve reasonable precision, most experiments of both types have relied on the determination of differences between mirror β^+ and β^- decays, and have focused primarily on the decays of ^{12}B and ^{12}N (although the spectrum shape for ^{20}F has also been measured carefully [2]).

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When these results were summarized 10 years ago [3] the spectrum-shape results agreed with CVC to $\pm 10\%$ and the asymmetry results to $\pm 3\%$. There has been no substantive improvement since that time.

Two orders of magnitude higher precision has been achieved in testing the constancy of G_{V} [4] *via* superallowed nuclear β decay. The measured strength (ft value) of the superallowed β transition between two 0^+ analog states is a sensitive measure of the vector coupling constant, with the relationship between ft and G_{V} being given by [4]

$$\mathcal{F}t \equiv ft(1+\delta_{\mathrm{R}}')(1+\delta_{\mathrm{NS}}-\delta_{\mathrm{C}}) = \frac{K}{2G_{\mathrm{V}}^2(1+\Delta_{\mathrm{R}}^{\mathrm{V}})}, \tag{1}$$

with

$$\begin{aligned} \frac{K}{(\hbar c)^6} &= 2\pi^3\hbar\ln 2/(m_e c^2)^5 \\ &= (8120.271 \pm 0.012) \times 10^{-10}\mathrm{GeV}^{-4}\mathrm{s}, \end{aligned} \tag{2}$$

where f is the statistical rate function, which depends on the measured Q_{EC} value; t is the measured partial half-life for the transition; δ_{C} is the isospin-symmetry-breaking correction; δ_{R}' and δ_{NS} are components of the transition-dependent part of the radiative correction; and $\Delta_{\mathrm{R}}^{\mathrm{V}}$ is the transition-independent part. Here we have also defined $\mathcal{F}t$ as the "corrected" ft value. The four calculated correction terms, $\Delta_{\mathrm{R}}^{\mathrm{V}}$, δ_{R}', δ_{NS} and δ_{C}, are all of order 1% or less; only two of them, δ_{NS} and δ_{C}, depend on nuclear structure.

Once G_{V} is known to be constant, its average value can be used to test a fundamental principle of the electroweak standard model, the unitarity of the Cabibbo–Kobayashi–Maskawa (CKM) matrix. The up–down element of that matrix, V_{ud}, is given by $V_{ud} = G_{\mathrm{V}}/G_{\mathrm{F}}$, where G_{F} is the weak-interaction constant for the purely leptonic muon decay. The value of V_{ud} is a key component of the most demanding test available for the unitarity of the CKM matrix, the sum of the top-row elements: $V_{ud}^2 + V_{us}^2 + V_{ub}^2$. In what follows, I shall focus on the results from superallowed β decay, on the fundamental tests that they make possible, and on the prospects for improving these tests.

2. Current status of superallowed decays

A new and complete survey of world data completed late last year [4] demonstrates that, to date, superallowed ft values have been measured to <0.4% precision for twelve different parent nuclei ranging from ^{10}C to ^{74}Rb. Nine of these cases are actually known to 0.1% or better. These measured transitions yield twelve independent determinations of G_{V}, from which its

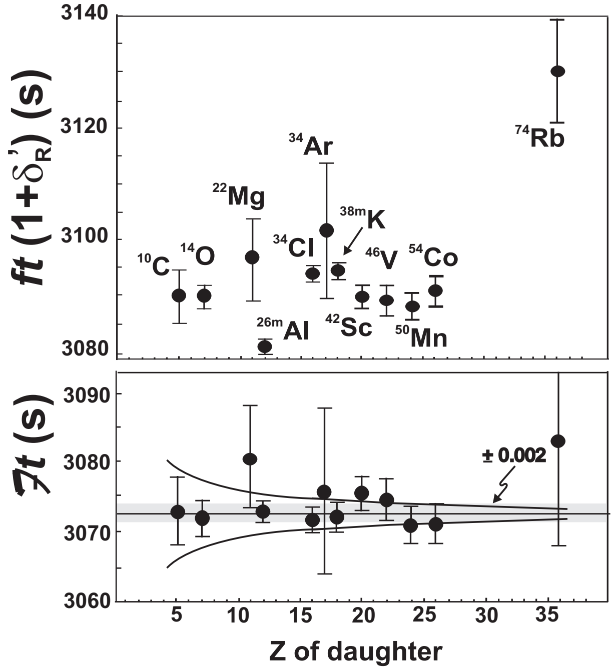

Fig. 1. In the top panel are plotted the experimental ft values corrected only for δ_R', those radiative corrections that are independent of nuclear structure. In the bottom panel, the corresponding $\mathcal{F}t$ values are given; they differ from the top panel simply by the inclusion of the nuclear-structure-dependent corrections, δ_{NS} and δ_C. The horizontal grey band indicates the average $\mathcal{F}t$ value with its uncertainty. The curved lines show the approximate loci the $\mathcal{F}t$ values would follow if the induced scalar coupling constant were $f_S = \pm 0.002$.

constancy can be confirmed. As illustrated in the bottom panel of Fig. 1, the $\mathcal{F}t$ values — and hence G_V^2 — are found to be constant to three parts in 10^4. Also illustrated in the figure is an important by-product of this result: the data confirm the absence of induced scalar currents, another predicted outcome of CVC. The curved lines in the bottom panel represent the approximate loci the $\mathcal{F}t$ values would follow if the induced scalar coupling constant, f_S, took the values ± 0.002 in electron mass units. A statistical analysis of the data yields the limit $f_S < 0.0013$.

With the constancy of G_V thus established, its average value can next be used to derive V_{ud}, the CKM matrix element. The result obtained, $V_{ud} = 0.9738(4)$, when combined with values for V_{us} and V_{ub} taken from the most recent Particle Data Group survey [5], yields a unitarity sum for the top-row elements of the CKM matrix:

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.9966 \pm 0.0014, \quad (3)$$

which fails unitarity by 2.4 standard deviations. This disagreement, which has persisted for at least a decade now, has stimulated considerable recent activity in remeasurements of V_{us}. Although there is not yet consensus on the best new value for V_{us} — largely because of inconsistencies in the different calculations for the SU(3) symmetry-breaking corrections — the most likely outcome will be a higher result than the old one, leading to a unitarity sum of 0.9995 ± 0.0012.

Whether the deviation from unitarity is ultimately confirmed or removed, however, the unitarity test must continue to play a key role, either in characterizing new physics beyond the standard model or in setting a limit on its existance. Improvements in the precision with which V_{ud} is known will continue to be an important goal of nuclear experiments.

3. Current experimental directions

In principle, nuclear β decay is not the only way to determine V_{ud}: both neutron and pion decays are currently being used to probe the vector current. (See Refs. [5,6] for a list of references up to 2003; recent pion results appear in Ref. [7].) So far, though, both of these approaches have been limited in their precision by experimental challenges and inconsistencies. Indeed the most recent measurement of the neutron half-life [8] disagrees by 6.5 standard deviations from the average of all previous measurements [5,6]. As a result, although the neutron- and pion-derived values for V_{ud} are consistent with the nuclear value, their uncertainties are larger by factors of five and seven, respectively, and are dominated by experimental factors. In contrast, the uncertainty attached to the value of V_{ud} obtained from nuclear β-decay is predominantly theoretical in origin, arising from documented and assigned uncertainties in the calculated correction terms. The results for V_{ud} and the error budgets for all three techniques are shown in Fig. 2.

It is evident in Fig. 2 that the transition-independent radiative correction, $\Delta_{\mathrm{R}}^{\mathrm{V}}$, is currently the dominant uncertainty overall for the nuclear result and is the dominant theoretical uncertainty for the neutron one. New calculations of that correction [9] will reduce its uncertainty by 50% (indicated by light and dark grey shading in the figure). With this reduction, the nuclear-structure-dependent corrections, δ_{NS} and δ_{C}, will become significant contributors to the overall V_{ud} uncertainty. The focus of contemporary nuclear experiments has already been to test these structure-dependent corrections but this new development makes such tests even more important. Only in this way can we hope ultimately to reduce their uncertainties as well.

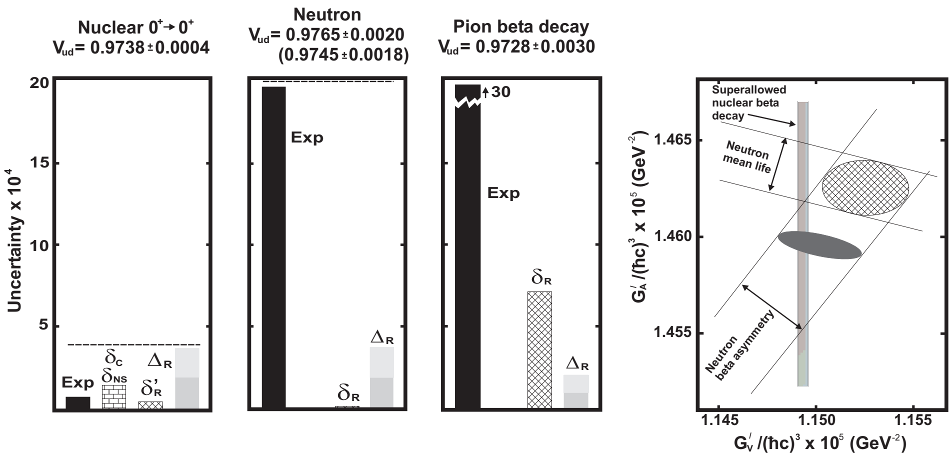

Fig. 2. Error budgets are shown for each of the three different methods to determine V_{ud}, illustrating the relative importance of experimental and theoretical uncertainties. The right panel compares the experimental results for neutron decay with those of superallowed decays. When all neutron data are included, the one-standard-deviation limits are given by the crosshatched oval; the solid oval is the result if the most recent neutron half-life measurement [8] is excluded. The former leads to the unbracketed value given for V_{ud} obtained from neutron decay; the latter leads to the bracketed value.

The principle of these tests can be grasped from a comparison of the two panels in Fig. 1. In the top panel of the figure are plotted the values of $ft(1+\delta'_{\mathrm{R}})$ for all 12 well-known superallowed transtions. The corresponding values of $\mathcal{F}t$ appear in the bottom panel. As can be seen from Eq. (1), these two quantities differ only by the inclusion of the nuclear-structure-dependent corrections, $\delta_{\mathrm{NS}} - \delta_{\mathrm{C}}$, in the latter. Obviously these corrections act very well to remove the considerable "scatter" that is apparent in the top panel and is effectively absent in the bottom one. It is important to note that the calculations of δ_{NS} and δ_{C} [10] employ the best available shell-model wave functions, which are based on a wide range of spectroscopic data. They were further tuned to agree with measured binding energies, charge radii and coefficients of the isobaric multiplet mass equation (except in the case of ^{74}Rb, where these properties are not yet measured). This means that the origins of the structure-dependent correction terms are completely independent of the superallowed decay data, so the consistency of the corrected $\mathcal{F}t$ values appearing in the bottom panel of the figure is a powerful validation of the calculated corrections used in their derivation.

The validation of the nuclear-structure-dependent correction terms, exemplified by the comparison of the two panels in Fig. 1, can be improved by the addition of new transitions selected from amongst those with large calculated corrections. If the ft values measured for cases with large calculated corrections also turn into corrected $\mathcal{F}t$ values that are consistent with the others, then this must verify the calculations' reliability for the existing cases, which have smaller corrections. In fact, the cases of ^{34}Ar and ^{74}Rb, which have only recently been measured, were chosen for this very reason and, although their precision does not yet equal that of the others, they do indicate that the corrections so far are living up to expectations.

In general, there are three types of experiments now aimed at improving the existing tests of the structure-dependent corrections. One focuses on making improvements in the precision of the nine currently best-known cases; the other two focus on exploring new series of 0^+ superallowed emitters: the even-Z, $T_z = -1$ nuclei with $18 \leq A \leq 42$, (^{18}Ne, ^{22}Mg, ^{26}Si, ^{30}S, ^{34}Ar, ^{38}Ca, and ^{42}Ti) and the odd-Z, $T_z = 0$ nuclei with $A \geq 62$ (^{62}Ga, ^{66}As, ^{70}Br and ^{74}Rb). The attraction of these new regions is that the calculated values of $(\delta_{\mathrm{C}} - \delta_{\mathrm{NS}})$ are larger, or show larger variations from nuclide to nuclide, than the nine best-known cases.

Both new series of emitters present experimental challenges not encountered with the ones previously studied. In the past, precise Q_{EC} values were obtained via direct reactions — for example (p,n) or (^{3}He,t) — on the daughter nuclei, which were stable. All new cases have unstable daughters, so precise Q_{EC} values require careful on-line Penning-trap measurements of the masses of both the parent and daughter nuclei. Because of the precision required ($< \pm 500$ eV) and the short half-lives of most emitters, such measurements are pushing close to the limits of current technology. Even so, the Q values of ^{18}Ne [11], ^{22}Mg [12, 13], ^{34}Ar [14] and ^{62}Ga [15] have all recently been determined by Penning traps with sufficient precision that their statistical rate functions, f, are all now known to better than $\pm 0.1\%$.

There are other challenges too. In all but one case, the superallowed emitters studied previously have branching ratios of $>99.3\%$ to the superallowed transition. High precision in this value can be readily achieved by a much less precise measurement of, or a limit on, the competing transitions, which can then be subtracted from 100%. The new emitters are not so straightforward. Those with $T_z = -1$ feed odd–odd daughters with at least a few 1^+ states that are populated by strong Gamow–Teller transitions. The branching ratio to the superallowed transition must therefore be measured directly to 0.1% precision. For this purpose calibration procedures have been developed to determine the efficiency of a HPGe detector [16], which now allow the intensities of β-delayed γ rays to be measured with nearly this precision. Results have been published [17] for the branching ratio in the decay of ^{22}Mg and soon will also be for ^{34}Ar decay [18].

The new emitters with $A \geq 62$ offer a different branching-ratio challenge. Their decays are of higher energy (>9 MeV) and, although their daughters are even–even, their level density is high enough that numerous weak Gamow–Teller branches compete with the superallowed branch. Though the total Gamow–Teller strength can be significant, many of the individual branches are unobservably weak [19]. This "Pandemonium" effect (see Ref. [20]) can be partially corrected for by careful measurement of weak β-delayed γ rays but ultimately one must rely on calculation to account for those γ rays that remain undetected. With this approach, branching ratios for both ^{74}Rb and ^{62}Ga have been determined with good precision [21, 22].

Finally, precise half-life measurements on these new emitters are also more difficult than were those previously measured. The most obvious reason is that these new emitters are all farther from stability and, in general, shorter lived. Simply achieving the production rate and source purity required for clean high-statistics counting can be a challenge in itself. So far, the necessary conditions have been achieved for ^{22}Mg (with a recoil separator and additional "range" purification — see Ref. [17]) and for ^{74}Rb (with an on-line isotope separator — see Ref. [23]). A further complication also arises for cases, like that of ^{34}Ar, in which the daughter has a half-life that is comparable to the parent's. Since 4π β counting is the method of choice for precision measurements, the fact that parent and daughter activities are virtually indistinguishable from one another makes it difficult to achieve statistical definition for the parent decay in the presence of the daughter's growth and decay. Fortunately, this problem has now been overcome [24].

4. An unexpected result

One might reasonably have expected that measurements aimed at improving the uncertainties on the best-known superallowed transitions would lead to the least dramatic results. After all, a large amount of high quality data is already incorporated in these nine ft values. Just one more measurement would not have been expected to have much impact on the overall result. However, there were a few instances where some improvements in an uncertainty seemed achievable. One such instance was the Q value for ^{46}V decay. Figure 3 shows the measurements that contribute to the Q values for each of the best-known emitters. Until recently, only two measurements existed for ^{46}V and one of these had rather poor precision by modern standards.

Accordingly, a new measurement was made with an on-line Penning trap [25] of the masses of ^{46}V and ^{46}Ti, from which the Q value of the superallowed transition could be determined, the first Q value among this group of transitions ever to have been measured with a trap. As expected,

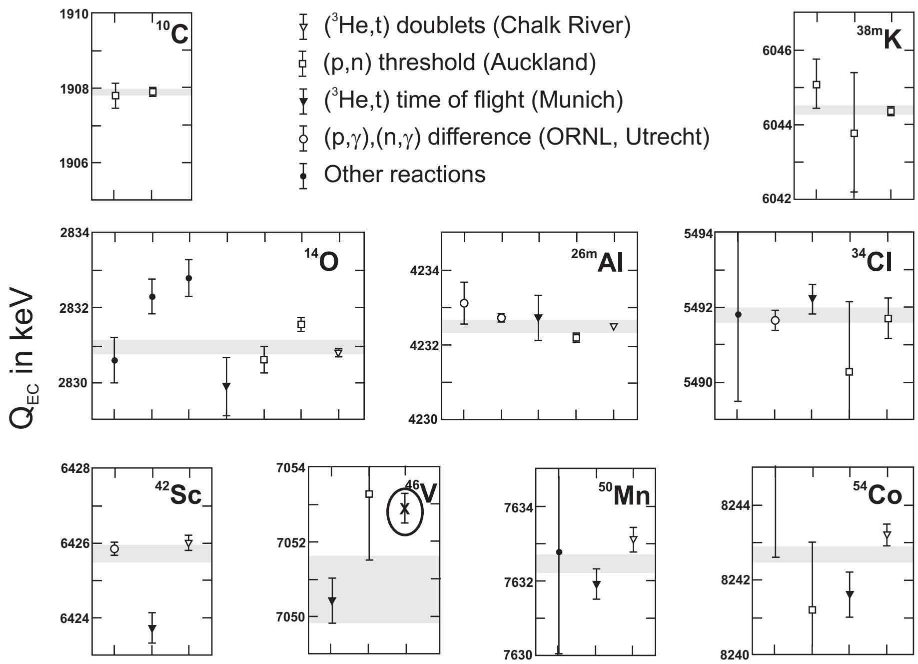

Fig. 3. All Q-value measurements that contribute to the survey of world data published this year [4] are plotted in chronological order, and identified by the type of reaction(s) employed. The only Penning-trap measurement is the recent ^{46}V result of Savard *et al.* [25]; it is indicated with an "X" and is circled.

the new measurement reduced the uncertainty considerably but unexpectedly the result itself was significantly different from the previously accepted value. If the new result is simply combined with the two previous measurements of the ^{46}V Q value it makes only a modest change in the average and no change at all in the resultant value of V_{ud}. However, Savard *et al.* [25] went a step further. They noted that the Penning-trap value for the ^{46}V Q value only really disagrees with a single previous result [27], one obtained from a (^{3}He,t) reaction study that included six other Q values in the prime series of superallowed emitters, all quoted with similarly small error bars. All seven results are plotted as inverted solid triangles in Fig. 3.

Savard *et al.* [25] argue that most of the Q values from Ref. [27] deviate significantly from the average of all other measurements for the same transitions, and they opt to remove all seven results in that reference from the high-precision data set. If this is done, there is a serious deterioration of $\mathcal{F}t$ value plot, with a bump appearing at Z=20 and 22. At the very least, the new measurement raises suspicion about previous Q-value results and emphasizes the critical importance of extending these Penning-trap measurements to other cases.

Suspicion has already been further aroused by a preliminary result [26] for the masses of ^{42}Sc and ^{14}O, which may also lead to $Q_{\rm EC}$-values that are higher than the previous averages from reaction measurements [4]. At issue now is the question of whether Q values measured in Penning traps are consistently higher than those determined by other methods, or whether ^{46}V is an isolated occurrence. If the latter is the case, then the restoration of CVC consistency would seem to depend on there being a deficiency in the current nuclear-structure dependent corrections [10] obtained for ^{46}V. If the former is the case, then all Q-value measurements, past and present, need to be scrutinized even more carefully for undetected systematic effects.

Whatever the outcome of this puzzle, the effect on V_{ud} will be relatively small (at most, slightly more than one standard deviation) and it will be in the direction of reducing it. Such a reduction will, of course, also lead to a reduction in the unitarity sum.

5. Conclusions

Based on a recent survey of world data [4], superallowed β decay confirms the expectations of CVC — that the vector coupling constant is unrenormalized in nuclear matter and that there is no induced scalar current. It also provides the most precise value for V_{ud}, the up–down element of the CKM matrix and a key component of the most precise test of CKM unitarity. Because this test remains inconclusive at the 0.1% level, considerable activity in nuclear physics is now focused on sharpening both V_{ud} and the unitarity test, with the goal of reducing uncertainty on the nuclear-structure-dependent corrections, $\delta_{\rm C}$ and $\delta_{\rm NS}$. New results for Q values (masses), branching ratios and half-lives are becoming available, but still more are needed.

So far, most new measurements confirm the validity of the calculated correction terms but recent Penning-trap mass measurements among the previously best-known superallowed transitions are raising an important question about a possible systematic effect in Q-value measurements that has not yet been identified. Although the resolution of this question is not likely to change the value of V_{ud} very significantly, it is important that it be answered as soon as possible. Evidently, the nuclear result for V_{ud} can still be improved.

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