# THE NUCLEAR INTERACTION OF $\varSigma$ HYPERONS\* \*\*

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The analyses of the strangeness exchange  $(K^-, \pi)$  and the associated production  $(\pi^-, K^+)$  reactions are presented. They indicate — together with the observed properties of  $\Sigma$  atoms — that the  $\Sigma$  single particle potential  $V_{\Sigma}$  is repulsive inside nuclei and has a shallow attractive pocket at the nuclear surface. This conclusion is consistent with the Nijmegen model F of the hyperon–nucleon interaction. It is demonstrated how the strong-interaction shifts and widths measured in  $\Sigma$  atoms may be used to obtain information on the nucleon density distributions.

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### 1. Introduction

Our present knowledge of the  $\Sigma$  hyperon interaction with nuclear matter, represented by the single particle (s.p.) potential  $V_{\Sigma}$ , comes from the following sources:

— Final state interaction of  $\Sigma$  hyperons in the strangeness exchange  $(K^-, \pi)$  reactions (see, e.g., [1], and [2]).

— Final state interaction of  $\Sigma$  hyperons in the associated production  $(\pi, K^+)$  reactions (see [3], and [4]).

— Free space hyperon-nucleon scattering data, to which the hyperonnucleon interaction potential  $V_{\Sigma N}$  may be fitted (see, *e.g.*, [5–8]), and with this potential one may calculate  $V_{\Sigma}$  (see, *e.g.*, [9–11]).

— Strong interaction shifts  $\varepsilon_a$  and widths  $\Gamma_a$  of the observed  $\Sigma^-$  atomic levels [12, 13].

All these sources imply that the  $\Sigma N$  interaction is well represented by the Nijmegen model F of the baryon-baryon interaction [6], which leads

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to  $V_{\Sigma}$  which is repulsive at densities  $\rho$  of nuclear matter of the order of the equilibrium density  $\rho_0 = 0.166 \text{ fm}^{-3}$  and is slightly attractive at low densities encountered at nuclear surface [11, 14, 15].

Since the strong interaction shifts and widths in  $\Sigma^-$  atoms are sensitive to the proton and neutron density distributions  $\rho_n$  and  $\rho_p$ , we may use the  $\Sigma^-$  atomic data to gain information on these density distributions.

In Sec. 2, we describe how the properties of  $V_{\Sigma}$  and  $V_{\Sigma N}$  follow from the analysis of the strangeness exchange and associated production reactions. In Sec. 3, we discuss the possibility of gaining information on  $\rho_n$  and  $\rho_p$  from the  $\Sigma^-$  atomic data. Conclusions are presented in Sec. 4.

# 2. Information on $V_{\Sigma}$ and $V_{\Sigma N}$

Gaining information on  $V_{\Sigma}$  from the  $\Sigma$  production processes was burdened for a long time by the inaccurate early CERN results for the strangeness exchange  $(K^-, \pi)$  reactions [16]. The situation was clarified, when new experiments were performed at Brookhaven [17] with an order of magnitude better statistics. A simple analysis of the pion spectra measured in Brookhaven in the  $(K^-, \pi^+)$  reaction on the <sup>9</sup>Be target was performed in [2] in impulse approximation. The results, shown in Fig. 1, suggest that  $V_{\Sigma}$  is repulsive inside nuclei with the strength of about 20 MeV.



Fig. 1. Pion spectrum from  $(K^-, \pi^+)$  reaction on <sup>9</sup>Be at  $\theta_{\pi} = 4^{\circ}$  at  $p_K = 600 \text{ MeV}/c$ . The curves A, B, C, D were calculated with a square well potential  $V_{\Sigma}$  with the strength 20, 10, -10, and -20 MeV, respectively.

A similar analysis of the kaon spectrum measured in KEK [3] in the associated production  $(\pi^-, K^+)$  reaction on the <sup>28</sup>Si target was performed in [4]. The results shown in Fig. 2 lead to a similar conclusion with a slightly stronger repulsion of  $V_{\Sigma}$ .



Fig. 2. Kaon spectrum from  $(\pi^-, K^+)$  reaction on <sup>28</sup>Si at  $\theta_K = 6^\circ$  at  $p_{\pi} = 1.2 \text{ GeV}/c$ . See text for explanation. The curves A, B, C were calculated with a square well potential  $V_{\Sigma}$  with the strength -20, 20, and 40 MeV, respectively.

An important feature of  $V_{\Sigma}$  is its isospin dependence [14]. For a  $\Sigma^{\pm}$  moving with momentum  $k_{\Sigma}$  in nuclear matter of density  $\rho$  with neutron excess  $\alpha = (N - Z)/A$ , we have:

$$V_{\Sigma^{\pm}}(\rho, k_{\Sigma}) = V_0(\rho, k_{\Sigma}) \mp \frac{1}{2} V_{\tau}(\rho, k_{\Sigma}) \alpha .$$
<sup>(1)</sup>

As discussed in [14], the  $\pi^-$  spectrum observed in the Brookhaven  $(K^-, \pi^-)$ experiments [17] indicates that the final state interaction of the  $\Sigma$  hyperon is here less repulsive than in the  $(K^-, \pi^+)$  reaction. This suggests a sizable strength of the  $\Sigma$  Lane potential  $V_{\tau}$ , consistent with the value of  $V_{\tau}|_{\rho=\rho_0}$ ~ 80 MeV estimated in the phenomenological analysis [12] of  $\Sigma^-$  atoms.

We may exploit our information on  $V_{\Sigma}$  to find the most reliable version of the  $\Sigma$ -nucleon interaction potential  $V_{\Sigma N}$ . In the present discussion, we consider the Nijmegen models of of the baryon-baryon interactions: models D [5], F [6], soft-core (SC) model [7], and the new soft-core (NSC) model [8]. Within the Brueckner theory, one may obtain with these models the corresponding effective interactions in nuclear matter. This was done by Yamamoto *et al.* [11], and we use here their effective YNG interaction. It is a configuration space representation of the *G*-matrix calculated in the low order Brueckner (LOB) theory. With the help of the YNG interaction, we calculate  $V_0$  and  $V_{\tau}$ , by applying the expressions derived in [14]. Our results obtained for  $V_0(\rho, k_{\Sigma} = 0)$  and  $V_{\tau}(\rho, k_{\Sigma} = 0)$  are shown in Fig. 3 and 4<sup>1</sup>. We notice that at densities  $\rho$  of the order of the nuclear matter densities  $\rho_0$ , only model F leads to a repulsive  $V_0 \sim 20$  MeV, in agreement with the results of the analyses of the  $(K^-, \pi)$  and  $(\pi, K^+)$ reactions described above, whereas all the remaining models lead to an attractive  $V_0$ . At the same time, the attractive character of  $V_0$  at low densities (relevant in  $\Sigma^-$  atoms) revealed by model F guaranties that it leads to



Fig. 4. Potential  $V_{\tau}(\rho, k_{\Sigma} = 0)$ .

<sup>&</sup>lt;sup>1</sup> The dependence on  $k_{\Sigma}$ , which is rather weak (especially in case of  $V_{\tau}$ ), is discussed in [18].

a strong-interaction energy shifts in  $\Sigma^-$  atoms towards increased binding of the atomic levels, in agreement with experiment (see [13]). As far as the Lane potential is concerned, we see that at  $\rho \sim \rho_0$ , model F leads to  $V_{\tau} \sim 80 \text{ MeV}$ , in agreement with the previously mentioned estimates, whereas models D and SC lead to a weaker  $V_{\tau}$ , and model NSC leads to negative value of  $V_{\tau}$ .

We conclude that among the Nijmegen baryon–baryon interaction models, only model F appears to be a realistic representation of the  $\Sigma N$  interaction.

## 3. Information on $\rho_n$ and $\rho_p$

If we know the complex  $\Sigma$  s.p. potential in nuclear matter (its imaginary part may be directly related to the cross section for the  $\Sigma\Lambda$  conversion process  $\Sigma + N \to \Lambda + N$ ) and apply the local density approximation, we may follow [13] and calculate the strong-interaction shifts  $\varepsilon$  and widths  $\Gamma$  of  $\Sigma^-$  atomic levels [12,13], provided we know the neutron and proton density distributions  $\rho_n$  and  $\rho_p$ . Here, we want to hint at a possibility of exploiting the sensitivity of the calculated values of  $\varepsilon$  and  $\Gamma$  to the form of  $\rho_n$  and  $\rho_p$ (especially in the peripheral region of the nucleus) to gain information on  $\rho_n$  and  $\rho_p$ .

We consider the case of the  $\Sigma^-$ Pb atom because of the relatively high accuracy of the three data points measured in [19]: the energy shift  $\varepsilon$  and the width  $\Gamma$  of the lower level with the principal and orbital quantum numbers  $n = 9, \ l = 8$ , and the width  $\Gamma^u$  of the upper level with  $n = 10, \ l = 9$ . We want to find nucleon densities in <sup>208</sup>Pb,  $\rho_n(r)$  and  $\rho_p(r)$  which lead to the best agreement between the measured values of  $\varepsilon, \Gamma$ , and  $\Gamma^u$  and these values calculated with model F of the Nijmegen interaction.

Let us consider model F and, e.g. model D. We start with HF densities  $\rho_n$ and  $\rho_p$  calculated by Skalski [20] with the Skyrme interaction SkM<sup>\*</sup>. Values of  $\varepsilon$ ,  $\Gamma$ , and  $\Gamma^u$  calculated with these densities lead to  $\chi^2|_D = 7.9$  and  $\chi^2|_F = 65.2$ , respectively, for model D and F. The fact that the wrong model D leads to a better agreement with experiment than the correct model F indicates that the HF densities require corrections. The steps in which these corrections are introduced, are described in detail in [21]. They lead to the modified proton density distribution  $\tilde{\rho}_p$  with the unchanged root-meansquare radius (with the neutron distribution unchanged). The energy shift and widths  $\varepsilon$ ,  $\Gamma$ , and  $\Gamma^u$  calculated with these densities give:  $\tilde{\chi}^2|_D = 22.7$ and  $\tilde{\chi}^2|_F = 7.6$ . Thus the corrected densities lead to the desired situation in which the realistic model of the  $\Sigma N$  interaction leads to the best agreement with the  $\Sigma^-$ Pb atomic data. The essential features of the corrected densities are visualized in Fig. 5 which shows  $\alpha(r) = [\rho_n - \rho_p]/[\rho_n + \rho_p]$ . Whereas with the HF densities the nuclear periphery consists mainly of neutrons, with the corrected densities the neutron to proton ratio at the nuclear periphery is approximately equal N/Z.



Fig. 5. The local neutron excess  $\alpha(r)$  calculated with the HF density  $\rho_p$  and with the modified density  $\tilde{\rho}_p$ .

#### 4. Summary

— The analysis of the new strangeness exchange and associated production data indicates that the  $\Sigma$  s.p. potential  $V_{\Sigma}$  inside the nuclear core is repulsive.

— Among the Nijmegen models of the  $\Sigma N$  interaction only model F leads to a  $V_{\Sigma}$  repulsive at nuclear densities appearing inside nuclei.

— Assuming that model F is a realistic picture of the  $\Sigma N$  interaction, the analysis of the  $\Sigma$  atomic data may lead to information on  $\rho_n$ ,  $\rho_p$  at the nuclear periphery.

### REFERENCES

- [1] C.B. Dover, D.J. Millener, A. Gal, *Phys. Rep.* **184**, 1 (1989).
- [2] J. Dąbrowski, J. Rożynek, Acta Phys. Pol. B 29, 2147 (1998).
- [3] H. Noumi et al., Phys. Rev. Lett. 89, 072301 (2002).
- [4] J. Dąbrowski, J. Rożynek, Acta Phys. Pol. B 35, 2303 (2004).
- [5] N.M. Nagels, T.A. Rijken, J.J. de Swart, *Phys. Rev.* D12, 744 (1975);
   15, 2547 (1977).
- [6] N.M. Nagels, T.A. Rijken, J.J. de Swart, *Phys. Rev.* **D20**, 1663 (1979).
- [7] P.M.M. Maessen, T.A. Rijken, J.J. de Swart, Phys. Rev. C40, 2226 (1989); Nucl. Phys. A547, 245c (1992).
- [8] T.A. Rijken, V.G.J. Stoks, Y. Yamamoto, Phys. Rev. C59, 21 (1999).
- [9] J.Dąbrowski, J.Rożynek, Phys. Rev. C23, 1706 (1981).

- [10] Y. Yamamoto, H. Bandō, Progr. Theor. Phys., Suppl. 81, 9 (1985).
- [11] Y. Yamamoto, T. Motoba, H. Himeno, K. Ikeda, S. Nagata, Progr. Theor. Phys., Suppl. 117, 361 (1994).
- [12] C.J. Batty, E. Friedman, A. Gal, Phys. Rep. 287, 385 (1997).
- [13] J. Dąbrowski, J. Rożynek, G.S. Anagnostatos, Eur. Phys. J. A14, 125 (2002).
- [14] J. Dąbrowski, Phys. Rev. C60, 025205 (1999).
- [15] J. Dąbrowski, Nucl. Phys. A691, 58c (2001).
- [16] R. Bertini *et al.*, *Phys. Lett.* B90, 375 (1980); B136, 29 (1984); B158, 19 (1985).
- [17] R. Sawafta, Nucl. Phys. A585, 103c (1995); S. Bart et al., Phys. Rev. Lett. 83, 5238 (1999).
- [18] J. Dąbrowski, Acta Phys. Pol. B 36, 3063 (2005).
- [19] R.J. Power et al., Phys. Rev. C47, 1263 (1993).
- [20] J. Skalski, private communication; Acta Phys. Pol. B 34, 1977 (2003).
- [21] J. Dąbrowski, J. Rożynek, Eur. Phys. J. A25, 137 (2005).