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ENERGY DISTRIBUTIONS IN SZEKERES TYPE I AND II SPACE-TIMES

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In this study, in context of general relativity we consider Einstein, Bergmann–Thomson, Møller and Landau–Lifshitz energy-momentum definitions and we compute the total energy distribution (due to matter and fields including gravitation) of the universe based on Szekeres class I and class II space-times. We show that Einstein and Bergmann–Thomson definitions of the energy-momentum complexes give the same results, while Møller's and Landau–Lifshitz's energy-momentum definition does not provide same results for Szekeres class II space. The definitions of Einstein, Bergmann–Thomson and Møller definitions of the energy-momentum complexes give similar results in Szekeres class I space-time.

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1. Introduction

One of the most interesting problems which remains unsolved since Einstein proposal of general theory of relativity, is the energy-momentum localization. After Einstein [1] obtained an expression for the energy-momentum complexes many physicists, such as Landau and Lifshitz [2], Tolman [3], Papapetrou [4], Weinberg [5], Qadir–Sharif [6] and Bergmann–Thomson [7] had given different definitions for the energy-momentum complex. These definitions were restricted to evaluate energy distribution in quasi-Cartesian coordinates. This motivated by Møller [8] and many other, like Komar [9] and Penrose [10], to construct coordinate independent definitions. Møller proposed an expression which could be utilized to any coordinate systems.

Because of this, the notion of energy-momentum prescriptions was severely criticized for a number of reasons. Firstly, the nature of symmetric and locally conserved object is non-tensorial one; thus its physical interpretation appeared obscure [11]. Secondly, different energy-momentum complexes could yield different energy-momentum distributions for the same gravitational background [12]. Finally, energy-momentum complexes were local objects while it was generally believed that the suitable energy-momentum of the gravitational field was only total, *i.e.* it cannot be localized [13]. There have been several attempts to calculate energy-momentum prescriptions associated with different space-times [14, 15]. Virbhadra [16] showed that the definitions of Einstein, Tolman and Landau and Lifshitz (LL) give the same energy distribution for the Kerr–Newman metric. Later, Aguirregabiria *et al.* [17] proved that definitions of Einstein, LL, Weinberg and Papapetrou give the same result for any metric of Kerr–Schild class. Later, Virbhadra [18] emphasized that these complexes in fact coincide for space-times more general than the Kerr–Schild class. He also computed energy distribution for a general non-static spherically symmetric space-time of Kerr–Schild class and found that all these definitions give the same result as given by the Penrose quasi-local definition of energy. Vargas [19], by using teleparallel gravity analogs of Einstein and Landau–Lifshitz energy-momentum definitions found that energy is zero in Friedmann–Robertson–Walker space-times. This result agrees with the previous works of Cooperstock–Israelit [20], Rosen [21], Banerjee–Sen [22] who investigated the problem of the energy in Friedmann–Robertson–Walker universe in Einstein's theory of general relativity. After this works, Saltı and Havare [23] considered Bergmann–Thomson's definition in both general relativity and teleparallel gravity for the viscous Kasner-type metric.

The basic purpose of this paper is to obtain the total energy for Szekeres class I and class II metrics by using the energy-momentum expression of Einstein, Bergmann–Thomson, Møller and Landau–Lifshitz in general relativity. We will proceed according to the following scheme. In Section 2, we give the Szekeres class I type and class II space-times and some kinematical quantities associated with these metrics. In Section 3, we give the energy-momentum definitions of Bergmann–Thomson, Einstein, Møller and Landau–Lifshitz's in general relativity, respectively. In Section 4, we calculate the total energy-momentum densities for the Szekeres space-times. Finally, we summarize and discuss our results. Throughout this paper, the Latin indices *(i,j,...)* represent the vector number and the Greek $(\mu, \nu \ldots)$ represent the vector components; all indices run 0 to 3. We use geometrized units where $G = 1$ and $c = 1$.

2. The Szekeres class I and Szekeres class II space-times

Szekeres [24] derived a remarkable set of inhomogeneous exact solutions of Einstein's field equations without cosmological constant. The source of curvature of the models is an expanding, irrotational, and geodesic dust. These solutions are divided into two classes usually denoted by I and II. The class I solutions are usually presented in a way that is formally analogous to the Tolman–Bondi spherically-symmetric solutions, which they generalize. This class of solutions has primarily been used to model non-spherical collapse of an inhomogeneous dust cloud [25]. The class II solutions are usually considered as generalizations of the Kantowski–Sachs [26] and Friedmann–Robertson–Walker (FRW) solutions and have primarily been studied as cosmological models [27]. Those of class II are more important as cosmological models, because they can closely approximate, over a finite time interval, the FRW dust models.

In this section, we introduce the Szekeres class II and Szekeres class I metrics and then using these space-times we make some required calculations and find some kinematical quantities.

2.1. The Szekeres class II model

The Szekeres class II space-time is defined by the line element [28]

$$ds^2 = -dt^2 + Q^2 dx^2 + R^2(dy^2 + h^2 dz^2)\,, \tag{1}$$

where $Q=Q(x,y,z,t)$, $R=R(t)$ and $h=h(y)$ are functions to be determined. The kinematical quantities in Szekeres class II space-time [29] are given as follows:

The expansion (θ) is

$$\theta = \frac{Q_t R + 2R_t Q}{RQ}\,. \tag{2}$$

The shear scalar (σ^2), and the rotation (Ω^2) of the four velocity vector u_i are determined as

$$\sigma^2 = \frac{1}{3}\frac{(Q_t R - R_t Q)^2}{Q^2 R^2}\,, \tag{3}$$

$$\Omega^2 = 0\,. \tag{4}$$

The acceleration vector ($\dot{u}_i$) and the proper volume ($U^3 = \sqrt{-g}$) are given by,

$$\dot{u}_i = (0,0,0,0)\,, \tag{5}$$

$$U^3 = R^2 Q h\,, \tag{6}$$

where g is the determinant of the metric and x, y, z and t indices describe the derivative with respect to x, y, z and t. Thus, we see that the model given in (1) has expansion, non vanishing shear and vanishing rotation and acceleration.

2.2. The Szekeres class I model

The Szekeres class I space-time is defined by the line element

$$ds^2 = -dt^2 + e^{2B}(dx^2 + dy^2) + e^{2A}dz^2\,, \tag{7}$$

where $A = A(x, y, z, t)$, $B = B(x, y, z, t)$ are functions to be determined.

The expansion (θ) is

$$\theta = 2B_t + A_t\,. \tag{8}$$

The shear scalar (σ^2), and the rotation (Ω^2) of the four velocity vector u_i are determined as

$$\sigma^2 = \frac{1}{3}(A_t - B_t)^2\,, \tag{9}$$

$$\Omega^2 = 0\,. \tag{10}$$

The acceleration vector $(\dot{u}_i)$ and the proper volume $(U^3 = \sqrt{-g})$ are given by,

$$\dot{u}_i = (0, 0, 0, 0)\,, \tag{11}$$

$$U^3 = e^{2B+A}\,, \tag{12}$$

where g is the determinant of the metric. Thus, we see that the model given in Eq. (7) has expansion, non vanishing shear and vanishing rotation and acceleration, like in the case of the class II space-time. When $Q = R = e^B$ and $h = e^{A-B}$, these results agree with Tomimura and Motta [29].

3. Energy-momentum in general relativity

In this section, we introduce Bergmann–Thomson, Einstein, Møller and Landau–Lifshitz energy-momentum definitions, respectively.

3.1. Bergmann–Thomson's energy-momentum formulation

The energy-momentum prescription of Bergmann–Thomson is given by

$$\Xi^{\mu\nu} = \frac{1}{16\pi}\Pi^{\mu\nu\alpha}_{,\alpha}\,, \tag{13}$$

where

$$\Pi^{\mu\nu\alpha} = g^{\mu\beta} V_{\beta}^{\nu\alpha} \tag{14}$$

with

$$V_{\beta}^{\nu\alpha} = -V_{\beta}^{\alpha\nu} = \frac{g_{\beta\xi}}{\sqrt{-g}} \left[-g(g^{\nu\xi} g^{\alpha\rho} - g^{\alpha\xi} g^{\nu\rho}) \right]_{,\rho} . \tag{15}$$

Ξ_0^0 is the energy density, Ξ_μ^0 are the momentum density components, and Ξ_0^μ are the components of the energy current density. The Bergmann–Thomson energy-momentum definition satisfies the following local conservation laws

$$\frac{\partial \Xi^{\mu\nu}}{\partial x^\nu} = 0 \tag{16}$$

in any coordinate system. The energy and momentum components are given by

$$P^\mu = \int \int \int \Xi^{\mu 0} dx dy dz \, . \tag{17}$$

Further Gauss's theorem furnishes

$$P^\mu = \frac{1}{16\pi} \int \int \Pi^{\mu 0 \alpha} \kappa_\alpha dS \, . \tag{18}$$

κ_α stands for the 3-components of unit vector over an infinitesimal surface element dS. The quantities P^i for i=1,2,3 are the momentum components, while P^0 is the energy.

3.2. Einstein's energy-momentum formulation

The energy-momentum complex as defined by Einstein is given by

$$\Theta_\mu^\nu = \frac{1}{16\pi} H_{\mu,\alpha}^{\nu\alpha} , \tag{19}$$

where

$$H_\mu^{\nu\alpha} = \frac{g_{\mu\beta}}{\sqrt{-g}} \left[-g(g^{\nu\beta} g^{\alpha\xi} - g^{\alpha\beta} g^{\nu\xi}) \right]_{,\xi} . \tag{20}$$

Θ_0^0 is the energy density, Θ_α^0 are the momentum density components, and Θ_0^α are the components of energy current density. The Einstein energy and momentum density satisfies the local conservation laws

$$\frac{\partial \Theta_\mu^\nu}{\partial x^\nu} = 0 \tag{21}$$

and energy and momentum components are given by

$$P^{\mu} = \int\int\int \Theta^{0}_{\mu} dxdydz\,. \tag{22}$$

Further Gauss's theorem furnishes

$$P^{\mu} = \frac{1}{16\pi}\int\int H^{0\alpha}_{\mu}\eta_{\alpha} dS\,. \tag{23}$$

η_{α} stands for the 3-components of unit vector over an infinitesimal surface element dS. The quantities P^{i} for i=1,2,3 are the momentum components, while P^{0} is the energy.

3.3. Møller's energy-momentum formulation

The energy-momentum complex of Møller [30] is given by

$$M^{\nu}_{\mu} = \frac{1}{8\pi}\chi^{\nu\alpha}_{\mu,\alpha} \tag{24}$$

satisfying the local conservation laws:

$$\frac{\partial M^{\nu}_{\mu}}{\partial x^{\nu}} = 0\,, \tag{25}$$

where the antisymmetric super-potential $\chi^{\nu\alpha}_{\mu}$ is

$$\chi^{\nu\alpha}_{\mu} = \sqrt{-g}[g_{\mu\beta,\gamma} - g_{\mu\gamma,\beta}]g^{\nu\gamma}g^{\alpha\beta}\,. \tag{26}$$

The locally conserved energy-momentum complex M^{ν}_{μ} contains contributions from the matter, non-gravitational fields. M^{0}_{0} is the energy density and M^{0}_{α} are the momentum density components. The momentum four-vector of Møller is given by

$$P_{\mu} = \int\int\int M^{0}_{\mu} dxdydz\,. \tag{27}$$

After using the Gauss's theorem, this definition transforms into

$$P_{\mu} = \frac{1}{8\pi}\int\int \chi^{\nu\alpha}_{\mu}\mu_{\alpha} dS\,, \tag{28}$$

where μ_{α} is the outward unit normal vector over the infinitesimal surface element dS. P_{i} give momentum components P_1, P_2, P_3 and P_0 gives the energy.

3.4. Landau–Lifshitz energy-momentum formulation

Energy-momentum prescription of Landau–Lifshitz is given by

$$\Omega^{\mu\nu} = \frac{1}{16\pi} S^{\mu\nu\alpha\beta}_{\ ,\alpha\beta} \,, \tag{29}$$

where

$$S^{\mu\nu\alpha\beta} = -g(g^{\mu\nu} g^{\alpha\beta} - g^{\mu\alpha} g^{\nu\beta}) \,. \tag{30}$$

Ω_0^0 is the energy density, Ω_μ^0 are the momentum density components, and Ω_0^μ are the components of energy current density. The Landau–Lifshitz energy-momentum complex satisfies the local conservation laws

$$\frac{\partial \Omega^{\mu\nu}}{\partial x^\nu} = 0 \tag{31}$$

in any coordinate system. The energy and momentum components are given by

$$P^\mu = \int\int\int \Omega^{\mu 0} dx dy dz \,. \tag{32}$$

Further Gauss's theorem furnishes

$$P^\mu = \frac{1}{16\pi} \int\int S^{\mu\alpha 0\nu}_{\ ,\nu} \eta_\alpha dS \,, \tag{33}$$

where η_α stands for the 3-components of unit vector over an infinitesimal surface element dS. The quantities P^i for i=1,2,3 are the momentum components, while P^0 is the energy.

4. The total energy and momentum of Szekeres universes

This section gives us the total energy of the universe based on class II and class I metrics in theory of relativity, respectively.

4.1. Solutions in Szekeres class II model

4.1.1. Bergmann–Thomson energy-momentum

Considering the line element (1) for Eqs. (14) and (15), the required components of $\Pi^{\mu\nu\alpha}$ are

$$\begin{aligned} \Pi^{000} &= 0 \,, & \Pi^{002} &= -2(hQ_y + Qh_y) \,, \\ \Pi^{003} &= -\frac{2Q_z}{h} \,, & \Pi^{101} &= -\frac{4RhR_t}{Q} \,, \\ \Pi^{202} &= \frac{2h(Q_tR + R_tQ)}{R} \,, & \Pi^{303} &= \frac{2(Q_tR + R_tQ)}{hR} \,. \end{aligned} \tag{34}$$

Substituting this result into Eq. (13), we find that

$$\Xi_1^0 = -\frac{RhR_tQ_x}{4\pi}\,, \tag{35}$$

$$\Xi_2^0 = \frac{R(RQ_th_y + hR_tQ_y + QR_th_y + RhQ_{t,y})}{8\pi}\,, \tag{36}$$

$$\Xi_3^0 = \frac{hR(Q_zR_t + Q_{t,z}R)}{8\pi}\,, \tag{37}$$

$$\Xi_0^0 = \frac{1}{8\pi}\frac{2hQ_yh_y + h^2Q_{yy} + hQh_{yy} + Q_{zz}}{h}\,, \tag{38}$$

where x, y, z and t indices describe the derivative with respect to x, y, z, t.

4.1.2. Einstein energy-momentum

The required non-vanishing components of $H_\mu^{\nu\alpha}$ are

$$\begin{aligned} H_0^{00} &= 0\,, & H_1^{01} &= -4QRhR_t\,,\\ H_2^{02} &= -2Rh(Q_tR + R_tQ)\,, & H_3^{03} &= -2Rh(Q_tR + R_tQ)\,,\\ H_0^{02} &= -2(hQ_y + Qh_y)\,, & H_0^{03} &= -\frac{2Q_z}{h}\,. \end{aligned} \tag{39}$$

Substituting these results into Eq. (19), we get following energy and momentum densities in the form

$$\Theta_1^0 = -\frac{RhR_tQ_x}{4\pi}\,, \tag{40}$$

$$\Theta_2^0 = \frac{R(RQ_th_y + hR_tQ_y + QR_th_y + RhQ_{t,y})}{8\pi}\,, \tag{41}$$

$$\Theta_3^0 = \frac{hR(Q_zR_t + Q_{t,z}R)}{8\pi}\,, \tag{42}$$

$$\Theta_0^0 = \frac{1}{8\pi}\frac{2hQ_yh_y + h^2Q_{yy} + hQh_{yy} + Q_{zz}}{h}\,. \tag{43}$$

4.1.3. Møller energy-momentum

The required non-vanishing components of $\chi_\mu^{\nu\alpha}$ are

$$\chi_1^{01} = -2R^2hQ_t\,, \quad \chi_2^{02} = -2RQhR_t\,, \quad \chi_3^{03} = -2RQhR_t \tag{44}$$

and if we take these results into Eq. (24), we obtain following energy and momentum densities

$$\begin{aligned} M_1^0 &= -\frac{1}{4}\frac{R^2 h Q_{t,x}}{\pi}\,, && (45)\\ M_2^0 &= -\frac{1}{4}\frac{RR_t(hQ_y+Qh_y)}{\pi}\,, && (46)\\ M_3^0 &= -\frac{1}{4}\frac{RhR_tQ_z}{\pi} && (47)\\ M_0^0 &= 0\,. && (48) \end{aligned}$$

4.1.4. Landau–Lifshitz energy-momentum

The energy of non-vanishing components of $S^{\mu\nu\alpha\beta}$ are

$$\begin{aligned} S^{1010} &= -R^4h^2\,, & S^{2020} &= -R^2Q^2h^2\,,\\ S^{3030} &= -Q^2R^2\,, & S^{0010} &= -Q^2R^4h^2\,,\\ S^{0022} &= -Q^2R^4h^2\,, & S^{0030} &= -Q^2R^4h^2 \end{aligned} \qquad (49)$$

and if we take these results into Eq. (29), we obtain following energy and momentum densities

$$\begin{aligned} \Omega_1^0 &= \frac{1}{8\pi}\frac{R^4h^2(Q_{xx}Q-3Q_x^2)}{Q^2}\,, && (50)\\ \Omega_2^0 &= -\frac{R^2}{8}\frac{(Q_y^2h^2+4hQ_yh_y+Q_{yy}h^2Q+Q^2h_y^2+hQ^2h_{yy})}{\pi}\,, && (51)\\ \Omega_3^0 &= -\frac{R^2(Q_z^2+QQ_{zz})}{8\pi}\,, && (52)\\ \Omega_0^0 &= \frac{R^2}{8}\frac{(Q_y^2h^2+4hQ_yh_y+Q_{yy}h^2Q+Q^2h_y^2+hQ^2h_{yy}+Q_z^2+QQ_{zz})}{\pi}\,. && (53) \end{aligned}$$

4.2. Solutions in Szekeres class I model

4.2.1. Bergmann–Thomson energy-momentum

Considering the line element (7) for Eqs. (14) and (15), the required components of $\Pi^{\mu\nu\alpha}$ are

$$\begin{aligned} \Pi^{001} &= -2e^A(B_x+A_x)\,, & \Pi^{002} &= -2e^A(B_y+A_y)\,,\\ \Pi^{003} &= -4e^{(2B-A)}B_z\,, & \Pi^{101} &= 2e^A(B_t+A_t)\,,\\ \Pi^{202} &= 2e^A(B_t+A_t)\,, & \Pi^{303} &= 4e^{(2B-A)}B_t\,. \end{aligned} \qquad (54)$$

Substituting this result into Eq. (13), we find that

$$\begin{aligned}
\Xi_1^0 &= \frac{1}{8\pi}\left[e^{(2B+A)}(A_xB_t + A_xA_t + B_{t,x} + A_{t,x})\right], && (55)\\
\Xi_2^0 &= \frac{1}{8\pi}\left[e^{(2B+A)}(B_tA_y + A_yA_t + B_{t,y} + A_{t,y})\right], && (56)\\
\Xi_3^0 &= -\frac{1}{4\pi}\left[e^{(2B+A)}(B_tA_z - 2B_zB_t - B_{t,z})\right], && (57)\\
\Xi_0^0 &= \frac{1}{8\pi}\left[e^{A}(B_xA_x + A_x^2 + B_{xx} + A_{xx} + A_yB_y + A_y^2 + B_{yy} + A_{yy})\right.\\
&\quad \left. +2e^{2B-A}(2B_z^2 - B_zA_z + B_{zz})\right]. && (58)
\end{aligned}$$

4.2.2. Einstein energy-momentum

The required non-vanishing components of $H_\mu^{\nu\alpha}$ are

$$\begin{aligned}
H_0^{03} &= -4e^{(2B-A)}B_z\,, & H_1^{01} &= -2e^{(2B+A)}(B_t + A_t)\,,\\
H_2^{02} &= -2e^{(2B+A)}(B_t + A_t)\,, & H_3^{03} &= -4e^{(2B+A)}B_t\,,\\
H_0^{01} &= -2e^{A}(B_x + A_x)\,, & H_0^{02} &= -2e^{A}(B_y + A_y)\,. && (59)
\end{aligned}$$

Substituting these results into Eq. (19), we get following energy and momentum densities in the form

$$\begin{aligned}
\Theta_1^0 &= \frac{1}{8\pi}\left[e^{2B+A}(2B_tB_x + 2B_xA_t + A_xB_t + A_tA_x + B_{t,x} + A_{t,x})\right], && (60)\\
\Theta_2^0 &= \frac{1}{8\pi}\left[e^{2B+A}(2B_tB_y + 2B_yA_t + B_tA_y + A_yA_t + B_{t,y} + A_{t,y})\right], && (61)\\
\Theta_3^0 &= \frac{1}{4\pi}\left[e^{2B+A}(A_zB_t + 2B_zB_t + B_{t,z})\right], && (62)\\
\Theta_0^0 &= \frac{1}{8\pi}\left[e^{A}(B_xA_x + A_x^2 + B_{xx} + A_{xx} + B_yA_y + A_y^2 + B_{yy} + A_{yy})\right.\\
&\quad \left. +2e^{(2B-A)}(2B_z^2 - B_zA_z + B_{zz})\right]. && (63)
\end{aligned}$$

4.2.3. Møller energy-momentum

The required non-vanishing components of $\chi_\mu^{\nu\alpha}$ are

$$\chi_1^{01} = -2e^{(2B+A)}B_t\,, \quad \chi_2^{02} = -2e^{(2B+A)}B_t\,, \quad \chi_3^{03} = -2e^{(2B+A)}A_t \qquad (64)$$

and if we take these results into Eq. (24), we obtain following energy and momentum densities

$$M_1^0 = -\frac{1}{4\pi}\left[e^{(2B+A)}(2B_tB_x + A_xB_t + B_{t,x})\right], \tag{65}$$
$$M_2^0 = -\frac{1}{4\pi}\left[e^{(2B+A)}(2B_tB_y + B_tA_y + B_{t,y})\right], \tag{66}$$
$$M_3^0 = -\frac{1}{4\pi}\left[e^{(2B+A)}(2B_zA_t + A_tA_z + A_{t,z})\right], \tag{67}$$
$$M_0^0 = 0. \tag{68}$$

4.2.4. Landau–Lifshitz energy-momentum

The energy of non-vanishing components of $S^{\mu\nu\alpha\beta}$ are

$$\begin{aligned} S^{1010} &= -e^{2(B+A)}, \qquad S^{2020} = -e^{2(B+A)}, \\ S^{3030} &= -e^{4B}, \qquad S^{0010} = -e^{2(2B+A)}, \\ S^{0020} &= -e^{2(2B+A)}, \qquad S^{0030} = -e^{2(2B+A)} \end{aligned} \tag{69}$$

and if we take these results into Eq. (29), we obtain following energy and momentum densities

$$\Omega_1^0 = -\frac{1}{8\pi}\left[e^{2(B+A)}(A_{xx} + 2A_x^2)\right], \tag{70}$$
$$\Omega_2^0 = -\frac{1}{8\pi}\left[e^{2(B+A)}(A_{yy} + 2A_y^2)\right], \tag{71}$$
$$\Omega_3^0 = -\frac{1}{8\pi}\left[e^{4B}(8B_z^2 - 8B_zA_z + 2B_{zz} + 2A_z^2 - A_{zz})\right], \tag{72}$$
$$\begin{aligned} \Omega_0^0 = \frac{1}{8\pi}\Big[&e^{2(B+A)}(B_{xx} + 2B_x^2 + 4B_xA_x + A_{xx} + 2A_x^2 + B_y + 2B_y^2 \\ &+4B_yA_y + A_{yy} + 2A_y^2) + e^{4B}(8B_z^2 + 2B_{zz})\Big]. \end{aligned} \tag{73}$$

5. Summary and discussion

The subject of energy-momentum localization in the general theory of relativity has been very exciting and interesting; however it has been associated with some debate. Recently, some researchers have been interested in studying the energy content of the universe in various models.

The object of present paper is to show that it is possible to solve the problem of localization of energy in general relativity by using the energy and momentum complexes. In this paper, we get the energy distributions of the inhomogeneous and anisotropic Szekeres cosmological models, we have considered four different energy and momentum complexes in general relativity: *e.g.* Bergmann–Thomson, Einstein, Møller and Landau–Lifshitz.

We have found that:

(I) The total energy and momentum (due to matter plus field) distribution in Einstein and Bergmann–Thomson formulations are the same in Szekeres class II type space-time. Nevertheless, the Møller and Landau–Lifshitz energy-momentum complexes disagree with Einstein and Bergmann–Thomson energy-momentum complexes, but different definitions of this formulation agree with each other.

(II) The total energy distribution in Einstein and Bergmann–Thomson formulations ($\Theta_0^0 = \Xi_0^0$) are exactly same in Szekeres class I space-time and there is a proportion with momentum complexes with Einstein, Bergmann–Thomson and Møller distributions in Szekeres class I type space-time while disagree with Landau–Lifshitz energy-momentum definition.

(III) Møller energy distributions are identically zero in inhomogeneous and anisotropic Szekeres space-time.

(IV) The results advocate the importance of energy-momentum complexes.

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