COMPLEXITY CHARACTERISTICS OF CURRENCY NETWORKS*

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A large set of daily FOREX time series is analyzed. The corresponding correlation matrices (CM) are constructed for USD, EUR and PLN used as the base currencies. The triangle rule is interpreted as constraints reducing the number of independent returns. The CM spectrum is computed and compared with the cases of shuffled currencies and a fictitious random currency taken as a base currency. The Minimal Spanning Tree (MST) graphs are calculated and the clustering effects for strong currencies are found. It is shown that for MSTs the node rank has power like, scale free behavior. Finally, the scaling exponents are evaluated and found in the range analogous to those identified recently for various complex networks.

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1. Introduction

Analysis of correlations among financial assets is of great interest for practical, as well as for fundamental reasons. Practical aspects are mainly related to the theory of optimal portfolios [1]. The theoretical interest results from the fact that such study may shed more light on the universal aspects of complex systems organization. The world currency network can definitely be considered as complex.

In this paper we analyze daily FOREX (FX) time series of 60 currencies (including gold, silver and platinum) from the period December 1998–May 2005, provided by University of British Columbia [2]. The 5σ filter was applied to avoid spikes due to errors.

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For a value $x_i(t)$ of the ith asset ($i = 1, \ldots, N$) at time t, one defines its return $G_i(t)$ as

$$G_i(t) = \ln x_i(t+\tau) - \ln x_i(t) \simeq \frac{x_i(t+\tau) - x_i(t)}{x_i(t)}\,, \tag{1}$$

where the return time τ is also called the time lag. The normalized returns, $g_i(t)$ are defined as

$$g_i(t) = \frac{[G_i(t) - \langle G_i(t)\rangle_T]}{\sigma(G_i)}\,, \tag{2}$$

where $\langle \ldots \rangle_T$ denotes averaging over variable t with the averaging window T and $\sigma(G_i)$ is the standard deviation (volatility) of G_i.

The stock market time series $x_i(t)$ are always expressed in terms of the local currency. However, for the FX data we have exchange rates, instead. Denoting currencies by n consecutive capital Latin letters $A, B, C, \ldots$ the corresponding FX data $x_i(t)$ can be expressed as their quotients: $x_{AB}(t) = A(t)/B(t)$. Neglecting friction caused by fees (this is usually negligible in open market transactions) one obtains two types of constraints among n currencies

$$\frac{A(t)}{B(t)}\,\frac{B(t)}{A(t)} = 1\,, \qquad \frac{A(t)}{B(t)}\,\frac{B(t)}{C(t)}\,\frac{C(t)}{A(t)} = 1\,, \tag{3}$$

where the second constraint is called the triangle rule [3]. Eqs. (3) can be rewritten in terms of returns that gives the following identities

$$\begin{aligned} G_{AB}(t) &= -G_{BA}(t)\,, \\ G_{AB}(t) + G_{BC}(t) + G_{CA}(t) &= 0\,. \end{aligned} \tag{4}$$

For n currencies there are in principle $n(n-1)$ possible exchange rates $x(t)$ and corresponding returns $G(t)$. Due to the first of Eqs. (3) half of them are simply related to the remaining values. The triangle effect can be shown to give additional $(n-1)(n-2)/2$ independent constraints. This leaves us with $(n-1)$ independent exchange rates and returns for n currencies, $i = 1, \ldots, n-1$. One currency can be chosen as a reference currency (denominators) and we shell call it the *base currency*. Taking different currencies as the base currency one can obtain a different "picture" of the market though in principle all these pictures should contain the same information.

In this paper we construct correlation matrices (CMs) for the FX time series and the corresponding Minimal Spanning Trees (MSTs). Finally, the scale free distribution of node multiplicity is found and the corresponding scaling exponents are estimated. The complex network approach seems to be one of the most promising do deal with such extremely complicated systems, as was suggested recently [4, 5].

2. Correlation matrices

The correlation matrix (CM) C_{ij} is defined in terms of returns (1) as

$$C_{ij} = \frac{\langle G_i(t)G_j(t)\rangle_T - \langle G_i(t)\rangle_T \langle G_j(t)\rangle_T}{\sigma(G_i)\sigma(G_j)}\,. \tag{5}$$

The (symmetric) correlation matrix can also be computed in terms of the normalized returns. To this end one has to form N time series $\{g_i(t_0), g_i(t_0+\tau), \ldots, g_i(t_0+(T-1)\tau)\}$ of length T. Hence, we can built an $N \times T$ rectangular matrix $\boldsymbol{M}$. The correlation matrix (5) can be written in matrix notation as

$$\boldsymbol{C} \equiv [C]_{ij} = \frac{1}{T}\,\boldsymbol{M}\widetilde{\boldsymbol{M}}\,, \tag{6}$$

where tilde, $\widetilde{\boldsymbol{M}}$ stands for the matrix transposition. To avoid artificial reduction of the rank of this matrix, one should have sufficiently large time window for averaging: $T \geq N$.

By construction the trace of a correlation matrix equals to the number of time series

$$\mathrm{Tr}\;\boldsymbol{C} = N\,. \tag{7}$$

When some of the time series become strongly dependent, zero eigenvalues emerge (zero modes).

The eigenspectrum of CM for USD, EUR and PLN as the base currency is plotted in Fig. 1. For comparison, two additional sets of time series were generated. As the first one, the USD based time series were taken and all of them were randomly and independently shuffled. This set is denoted as (rnd). As all time correlations are destroyed the case (rnd) is clearly different

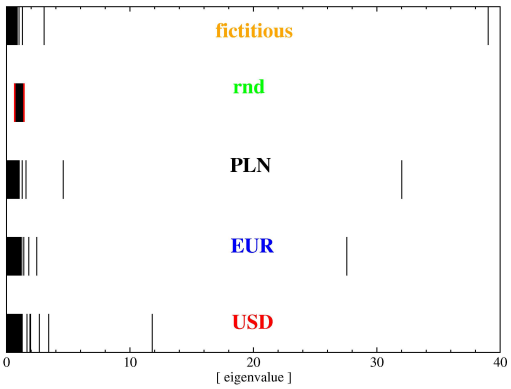

Fig. 1. Eigenspectra of correlation matrices for USD, EUR, PLN, shuffled USD and a random fictitious currency taken as the base currency, respectively.

than all other cases. In particular, it is very close to the random matrix spectrum, where the theoretical upper and lower limit for the spectrum is given by [6]

$$\lambda_{\min} = 1 + \frac{1}{q} - \frac{2}{\sqrt{q}}\,, \qquad \lambda_{\max} = 1 + \frac{1}{q} + \frac{2}{\sqrt{q}}\,, \tag{8}$$

where $q = T/N$. In our case Eq. (8) gives $\lambda_{\min} = 0.67$ and $\lambda_{\max} = 1.41$, in perfect agreement with the plot.

In the second case, a fictitious currency was generated with returns identical to Gaussian uncorrelated noise and it was used as the base currency for our time series. In this case time correlations of all other real currencies were preserved and it is denoted as "fictitious" (fict). The CM spectrum here is qualitatively similar to real currencies.

For the real currencies the maximal eigenvalue is smallest for USD, larger for EUR, much larger for PLN and the largest for a fictitious random currency taken as the base currency, respectively. The magnitude of separation of the largest eigenvalue from $\lambda_{\max}$ can be considered a measure of collectivity of the underlying dynamics [7]. Similar effects are observed for the stock market correlations [8].

3. Minimal Spanning Tree graphs

Looking at large numerical matrices is not very enlightening. Instead, there are useful visualizations that can be used for their analysis. In particular, the Minimal Spanning Trees that were introduced in graph theory long ago [9, 10] and later rediscovered several times [11, 12]. Recently they were applied to analyze the stock correlations [13]. Here, to draw the MST graph the following metric has been proposed

$$d(i,j) = \sqrt{2(1 - C_{ij})}\,. \tag{9}$$

Nodes corresponding to assets with the closest correlation coefficients are successively linked with a line. As a result one obtains a tree-like connected graph. The corresponding MST graphs for USD, EUR and PLN are shown in Figs. 2–4, respectively. In Fig. 2 USD is absent and one can see nodes with relatively small degree (small number of links). On the other hand, for EUR taken as the base currency (Fig. 3) we have two large clusters — USD and SAR cluster, both with high degree. The SAR cluster is present because of the strong coupling of both currencies, USD and SAR. The latter currency is artificially fixed to USD. In Fig. 4 PLN is taken as the base currency. Here, we have a larger USD cluster and smaller clusters, including the EUR cluster. The picture here is in a sense intermediate.

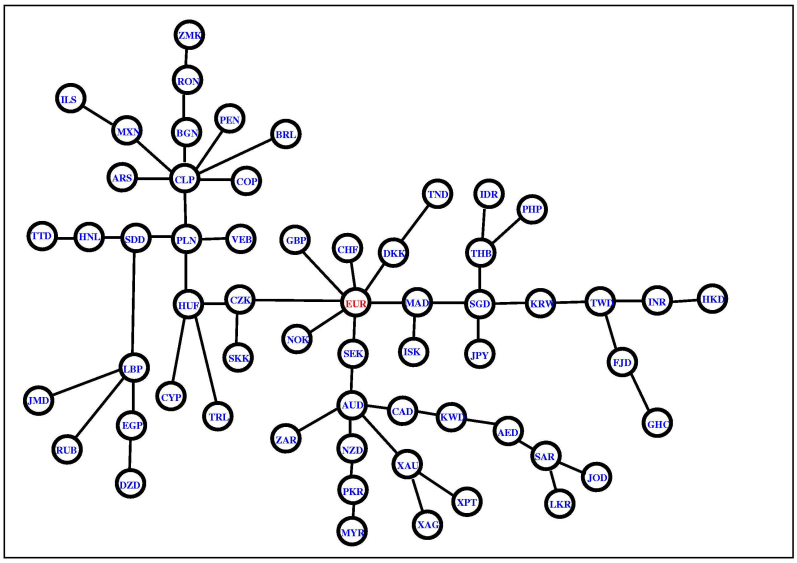

Fig. 2. Minimal Spanning Tree for USD as the base currency. In absence of USD only moderate size clusters are visible.

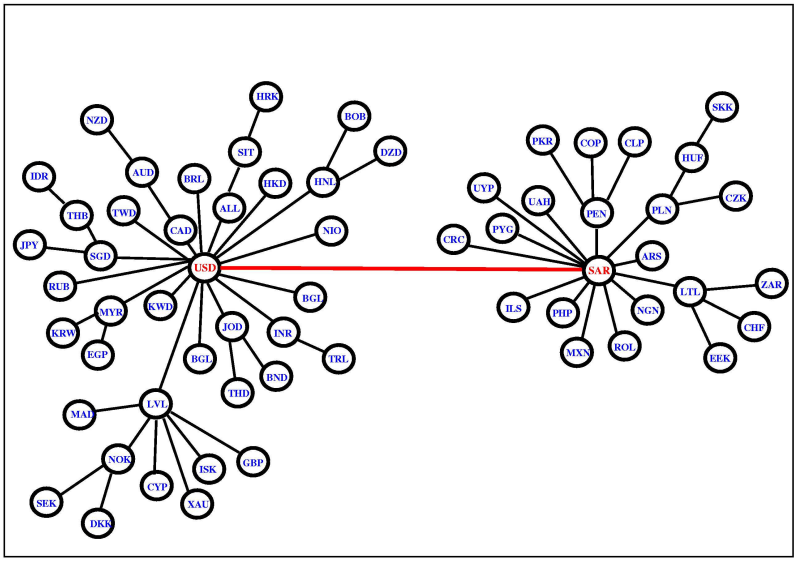

Fig. 3. Minimal Spanning Tree for EUR as the base currency. USD and SAR are in central positions of two large clusters.

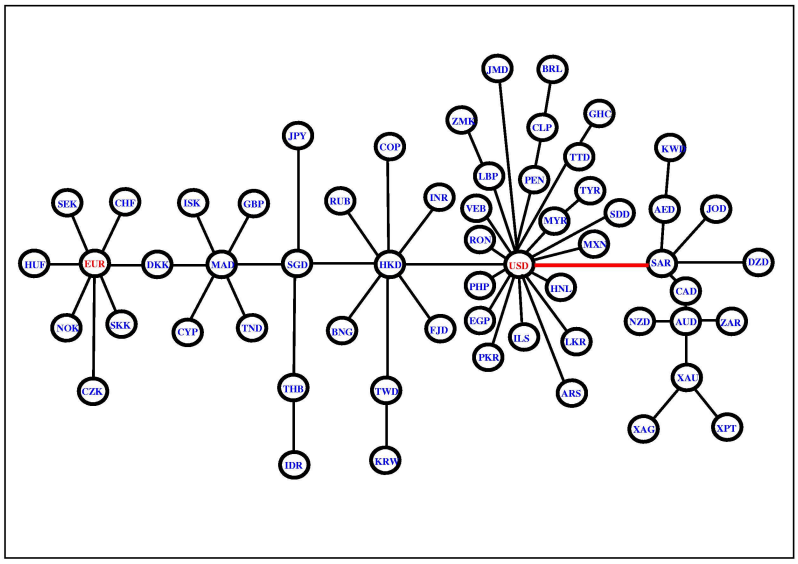

Fig. 4. Minimal Spanning Tree for PLN as the base currency with large USD cluster. Modest EUR cluster in the left part of the graph.

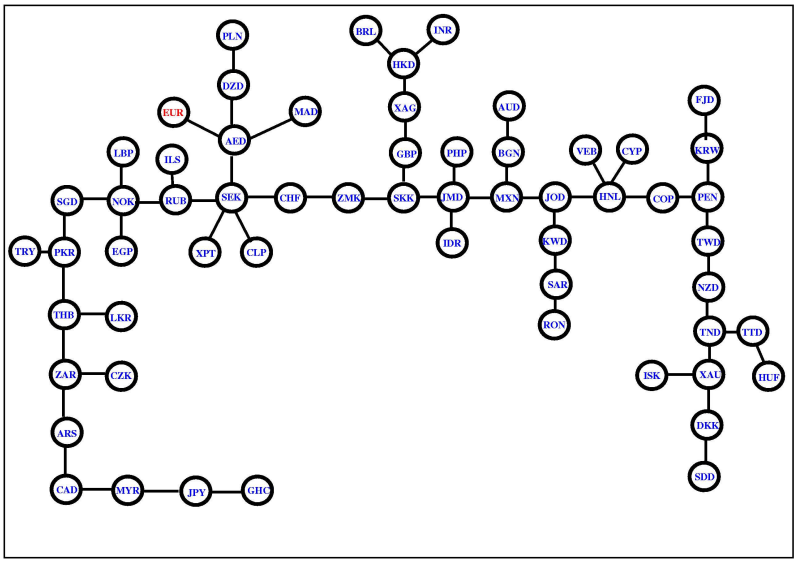

Fig. 5. Minimal Spanning Tree for shuffled time series.

We have also plotted MST for the correlation matrix with the USD as the base currency, but all the corresponding currency return time series are shuffled independently (Fig. 5). In this case all time correlations are killed. This corresponds to the (rnd) spectrum in Fig. 1. In this case larger clusters are absent, as one can expect. Finally, for a fictitious (fict) randomly generated currency (a prototype of a currency whose dynamics is completely disconnected from the rest) as the base currency one obtains MST graph as in Fig. 6. Here, its structure is qualitatively similar as for PLN taken as the base currency. This similarity even on a more quantitative level can be seen from Fig. 1.

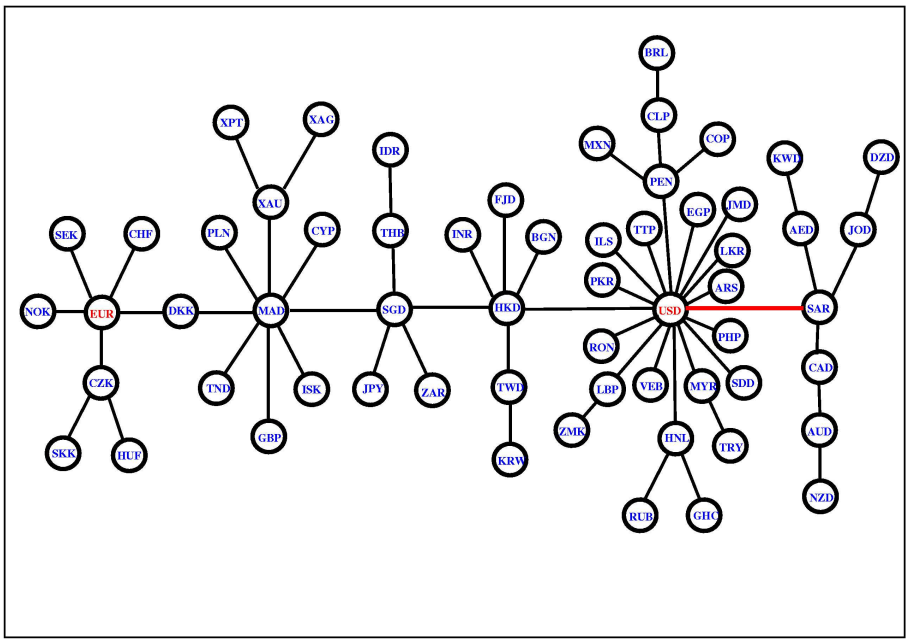

Fig. 6. Minimal Spanning Tree for a fictitious Gaussian currency as the base currency.

4. Power like scaling and conclusions

Because we have used considerably large number of currencies it is possible to estimate the integrated distribution of the nodes' degree for all plots. The most interesting question is the type of this distribution. For complex networks it has been found that these distributions usually have scale free power like scaling. Indeed, we have found good power like scaling in all cases except for the shuffled case, where all time correlations are wiped out. The

log–log plot of the integrated probability distribution for the nodes' degree (multiplicity) is plotted in Fig. 7. The corresponding dashed lines represent the power like fits. The numerical data are listed in Table I. In addition to the scaling exponent, α, its standard error, relative error and Pearsons r-coefficient are given. Except the shuffled case, standard error is of order of a few percent and the r-coefficient is > 0.97. This suggests a good power like scaling. The largest error is for USD. In this case, power like fit seems to be not so good. For the shuffled case, where time correlations are wiped out, one cannot see a power like scaling at all. The case of currencies expressed in terms of the USD seems to interpolate between the scale free and the shuffled cases. This may reflect the strong independence of the US currency.

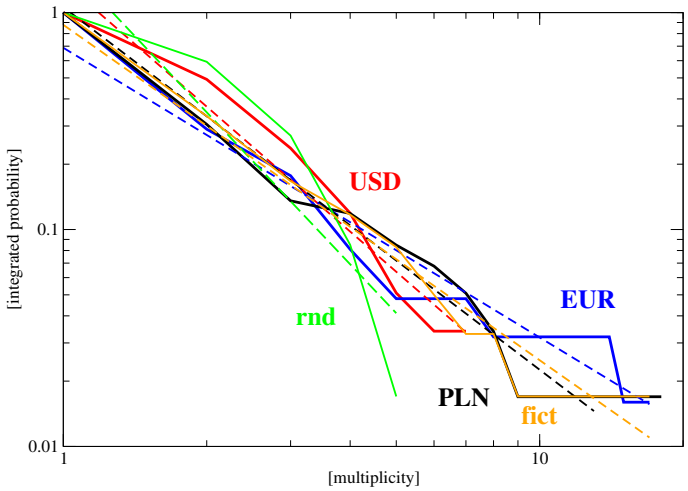

Fig. 7. Integrated probability distribution of nodes' multiplicity for the Minimal Spanning Tree graphs (Figs. 2–6). The linear fits are represented by corresponding dashed lines.

Numerical results for all fits can be found in Table I. It is worth to notice, that, except the shuffled case, for all cases we have obtained the scaling exponent in the range $1 < \alpha < 2$ (with average close to 1.6), the same range as for the finite average Lévy stable distributions [14]. What is more important, with rare exceptions, these exponents are similar to those found in different complex networks, such as WWW pages ($\alpha = 1.4$), physical internet networks with nodes representing hosts (1.38), routers (1.18) and peer-to-peer networks (1.19), protein–protein interaction network in the yeast (1.4), metabolic reactions network (1.15), movie actor collaboration network (1.3), phone calls (1.1), words co-occurence (1.7) — for references see [4, 5].

TABLE I

Numerical results for Minimal Spanning Trees represented by Figs. 2–6. α, its standard and relative error and Pearson's r are given.

base currency	α	std. error	%	r-coeff.
USD	1.913	$\pm$0.183	$\pm$9.6%	0.998
EUR	1.335	$\pm$0.086	$\pm$6.4%	0.970
PLN	1.488	$\pm$0.084	$\pm$5.7%	0.975
rnd	2.327	$\pm$0.627	$\pm$27 %	0.906
fictitious	1.546	$\pm$0.083	$\pm$5.4%	0.979

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