

CORRELATION MATRIX DECOMPOSITION
OF WIG20 INTRADAY FLUCTUATIONS*R. RAK^a, S. DROŹDZ^{a,b}, J. KWAPIEŃ^b, P. OŚWIĘCIMKA^b^aInstitute of Physics, University of Rzeszów
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Using the correlation matrix formalism we study the temporal aspects of the Warsaw Stock Market evolution as represented by the WIG20 index. The high frequency (1 min) WIG20 recordings over the time period between January 2001 and October 2005 are used. The entries of the correlation matrix considered here connect different distinct periods of the stock market dynamics, like days or weeks. Such a methodology allows to decompose the price fluctuations into the orthogonal eigensignals that quantify different modes of the underlying dynamics. The magnitudes of the corresponding eigenvalues reflect the strengths of such modes. One observation made in this paper is that strength of the daily trend in the WIG20 dynamics systematically decreases when going from 2001 to 2005. Another is that large events in the return fluctuations are primarily associated with a few most collective eigensignals.

PACS numbers: 89.20.-a, 89.65.Gh, 89.75.-k

1. Introduction

Nature of the temporal correlations in financial fluctuations constitutes one of the most fascinating issues of the contemporary physics. The pure Brownian-type motion [1] is definitely not an optimal reference [2]. Already the correlations in the financial volatility remain positive over a very long time horizon [3]. Even more involved are higher order correlations that give rise to the financial multifractality [4]. In this context a mention should also be given to the concept of financial log-periodicity — a phenomenon analogous to criticality in the discrete scale invariant version [5,6].

* Presented at the 2nd Polish Symposium on Econo- and Sociophysics, Kraków, Poland, April 21–22, 2006.

One more approach, initiated in Ref. [7], to quantify the character of financial time-correlations [8] is to use a variant of the correlation matrix. In this approach the entries of the corresponding matrix connect the high-frequency time series of returns representing different disconnected time-intervals like the consecutive days or weeks. The structure of eigenvectors of such a matrix allows then to quantify several characteristics of time correlations that remain invisible by more conventional methods.

Using this methodology here we present a systematic study for the Polish stock market index WIG20 over the period 02.01.2001–31.10.2005. The corresponding WIG20 chart, expressed both in terms of the Polish Zloty (PLN) and in terms of the US\$, *versus* two world leading stock market indices: the Nasdaq and the S&P500, is shown in Fig. 1. The Warsaw Stock Exchange trading time during this period was 10:00–16:10 and the WIG20 recorded with the frequency of 1 min.

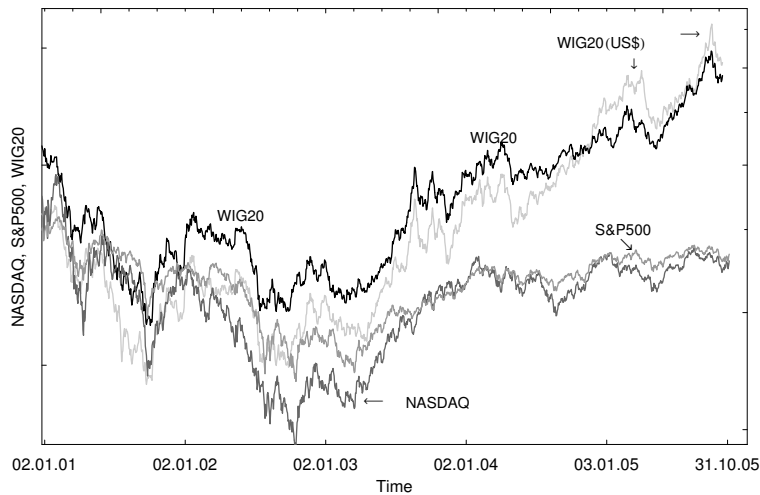


Fig. 1. The WIG20 (Warsaw Stock Exchange), the S&P500 and the Nasdaq indices from 2001.01.02 until 2005.10.31.

2. Formalism

In the present study the correlation matrix is thus defined as follows. To each element in a certain sequence N of relatively long consecutive time-intervals of equal length K labelled with α one uniquely assigns a time series $x_\alpha(t_i)$, where t_i ($i = 1, 2, \dots, K$) is to be understood as discrete time counted from the beginning for each α . In the financial application $x_\alpha(t_i)$ is going to represent the price time-series, α the consecutive trading days (or weeks) and t_i the trading time during the day (week). As usual it is then natural

to define the returns $R_\alpha(t_i)$ time-series as $R_\alpha(t_i) = \ln x_\alpha(t_i + \tau) - \ln x_\alpha(t_i)$, where τ is the time lag. The normalized returns are defined by

$$r_\alpha(t_i) = \frac{R_\alpha(t_i) - \langle R_\alpha(t_i) \rangle_t}{v}, \quad (1)$$

where v is the standard deviation of returns over the period T and $v^2 = \sigma^2(R_\alpha) = \langle R_\alpha^2(t) \rangle_t - \langle R_\alpha(t) \rangle_t^2$, and $\langle \dots \rangle_t$ denotes averaging over time.

One thus obtains N time series $r_\alpha(t_i)$ ($\alpha = 1, \dots, N$) of length $T = K - 1$, *i.e.* an $N \times T$ matrix \mathbf{M} . Then, the correlation matrix is defined as $\mathbf{C} = (1/T)\mathbf{M}\mathbf{M}^T$. By diagonalizing \mathbf{C}

$$\mathbf{C}\mathbf{v}^k = \lambda_k\mathbf{v}^k, \quad (2)$$

one obtains the eigenvalues λ_k ($k = 1, \dots, N$) and the corresponding eigenvectors $\mathbf{v}^k = \{v_\alpha^k\}$. In the limiting case of entirely random correlations the density of eigenvalues $\rho_C(\lambda)$ is known analytically [9–11], and reads

$$\rho_C(\lambda) = \frac{Q}{2\pi\sigma^2} \frac{\sqrt{(\lambda_{\max} - \lambda)(\lambda - \lambda_{\min})}}{\lambda},$$

$$\lambda_{\min}^{\max} = \sigma^2(1 + 1/Q \pm 2\sqrt{1/Q}), \quad (3)$$

with $\lambda_{\min} \leq \lambda \leq \lambda_{\max}$, $Q = T/N \geq 1$, and where σ^2 is equal to the variance of the time series (unity in our case).

For a better visualisation, each eigenvector can be associated with the corresponding time series of returns by the following expression:

$$z_k(t_i) = \sum_{\alpha=1}^N v_\alpha^k r_\alpha(t_i), \quad k = 1, \dots, N, \quad i = 1, \dots, T. \quad (4)$$

These new time series thus decompose the return fluctuations into the orthogonal components that reflect distinct patterns of oscillations common to all the time intervals labelled with α . They are, therefore, called the eigensignals [7, 8].

3. Results

3.1. Correlations among trading days

The above methodology is here applied to the WIG20 1 min recordings during the period between January 02, 2001 and October 31, 2005. This whole time period is split and analysed separately for the consecutive calendar years \mathcal{Y} that it covers ($\mathcal{Y} = \{2001, 2002, 2003, 2004, 2005\}$). The number

N_y of trading days correspondingly equals 249, 243, 249, 255, and 210. The WIG20 intraday variation is systematically taken between the trading time 10:01:30 (at this time the index is always already determined) and 16:10:00. This corresponds to $T = 368$ during one trading day. Using these data sets we construct the $N_y \times N_y$ correlation matrices \mathbf{C}_y .

The structure of eigenspectrum λ_k^y of such matrices for all the five calendar years is shown in Fig. 2. The pure noise range — as prescribed (Eq. (3)) by the corresponding Wishart ensemble of random matrices [10, 11] — is indicated by the shaded area. As one can see, the majority of eigenvalues of our empirical correlation matrices is located within this area which signals that noise is dominating. Typically there exist however, several eigenvalues that stay significantly above it. They are associated with some collectivity effects that in the present case are to be interpreted as an appearance of certain repeatable structures in the intraday dynamics of financial fluctuations. Definitely one such structure is the daily trend. As far as the WIG20

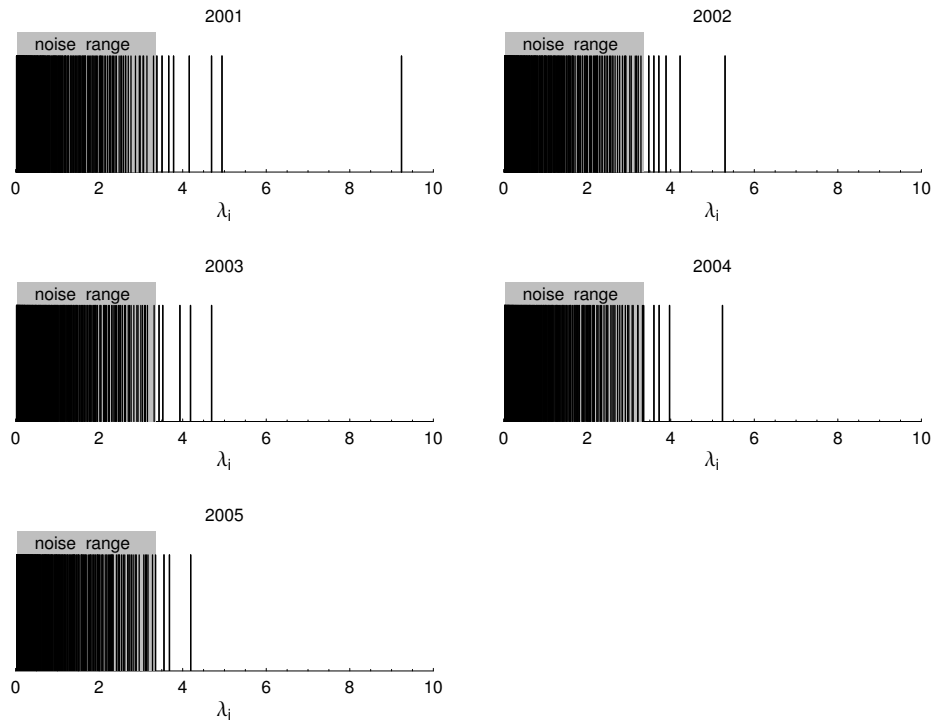


Fig. 2. Empirical eigenvalue spectrum of the correlations matrices \mathbf{C}_y (vertical black lines) calculated for WIG20(Warsaw Stock Exchange) index over the five consecutive calendar years. The noise range, as determined by a random Wishart matrix with $Q = 368/N_y$, is indicated by the shaded field.

dynamics is concerned it is however, even more interesting to see that when going from 2001 to 2005 those large eigenvalues gradually decrease and get closer to the noise area. This effect is more systematically documented in Fig. 3 which shows the evolution of the four largest eigenvalues of the $N \times N = 250 \times 250$ correlation matrix, which corresponds to 250 consecutive trading days, and this time window is moved with a step of one month (20 trading days).

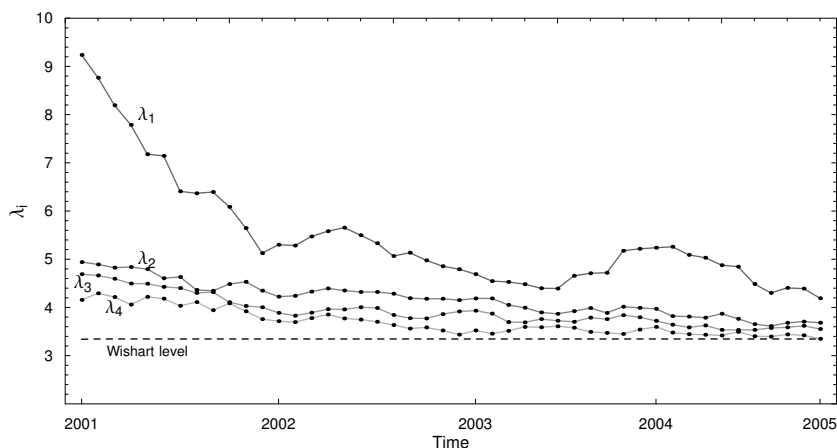


Fig. 3. Eigenvalues evolution of the sequence of $N \times N = 250 \times 250$ WIG20 correlation matrices translated with the step of one month. The dashed line corresponds to noise level.

The structure of eigenspectrum is expected to be closely related to the distribution of matrix elements of the correlation matrices. For the same five calendar years as in Fig. 2 the corresponding distributions are shown in Fig. 4 *versus* their Gaussian best fits. Indeed, in 2001 this distribution deviates most from a Gaussian and even develops a power-law tail with the slope of $\gamma \approx 6$ ($P(x) \sim x^{-\gamma}$). It is in this case that the largest eigenvalue of the correlation matrix is repelled most (upper most left panel of Fig. 2) from the rest of the spectrum due to an effective reduction of the rank of the matrix [12]. Later on the distribution of the matrix elements is much better fit by a Gaussian and some deviations remain on the level of essentially single entries.

An optimal way to visualise the character of repeatable intraday structures is to look at the eigensignals as defined by Eq. (4). They can be calculated for all the eigenvectors. An explicit numerical verification confirms that they are orthogonal indeed, *i.e.*, the correlation matrix constructed out of them is diagonal.

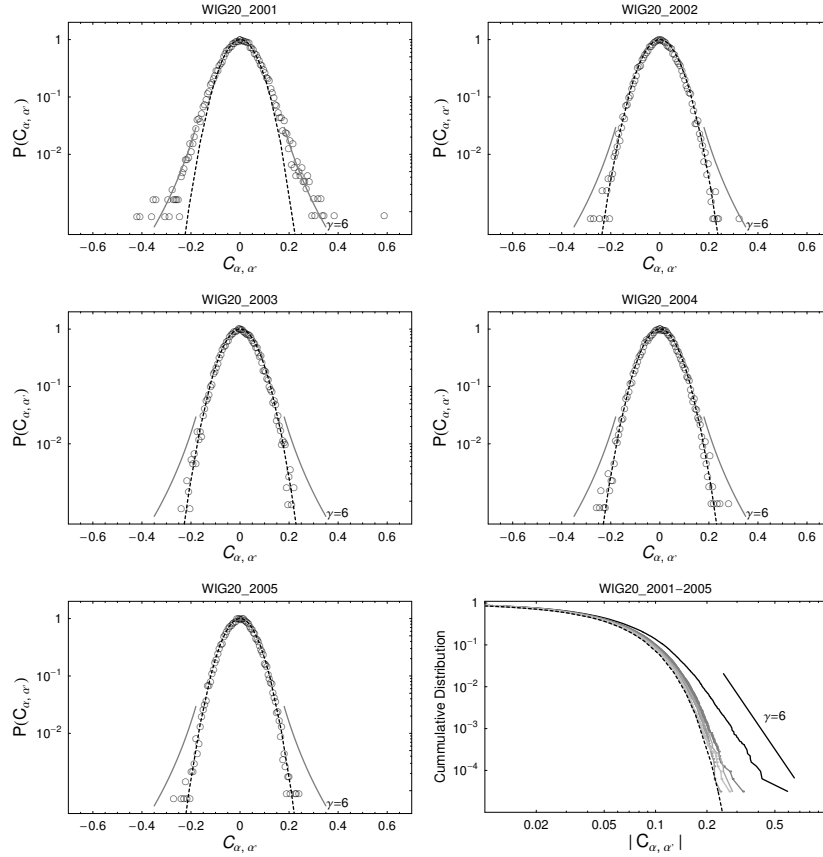


Fig. 4. Distribution of matrix elements $C_{\alpha, \alpha'}$ of the $N_{\mathcal{Y}} \times N_{\mathcal{Y}}$ correlation matrices $\mathbf{C}_{\mathcal{Y}}$ for the WIG20 variation during the intraday trading time 10:01:30–16:10:00. The solid lines indicate the power law fits to the tails of \mathbf{C}_{2001} with the power index $\gamma = 6$ ($P(x) \sim x^{-\gamma}$). The dashed lines correspond to a Gaussian best fit.

The most relevant examples of such eigensignals, corresponding to the two largest eigenvalues, for all the five years considered here, are shown in Fig. 5. They both display a strong enhancement of market activity just after the opening and lasting up to 60 min.

Interestingly, an analogous enhancement before closing as observed [8] for the other markets, in case of the WIG20 can be seen only rudimentary. For comparison one typical eigensignal ($z_{200}(t_j)$) corresponding to the bulk of eigenspectrum is shown in the bottom of Fig. 5. Its amplitude of oscillations can be seen to be about an order of magnitude smaller than for the previous leading eigensignals.

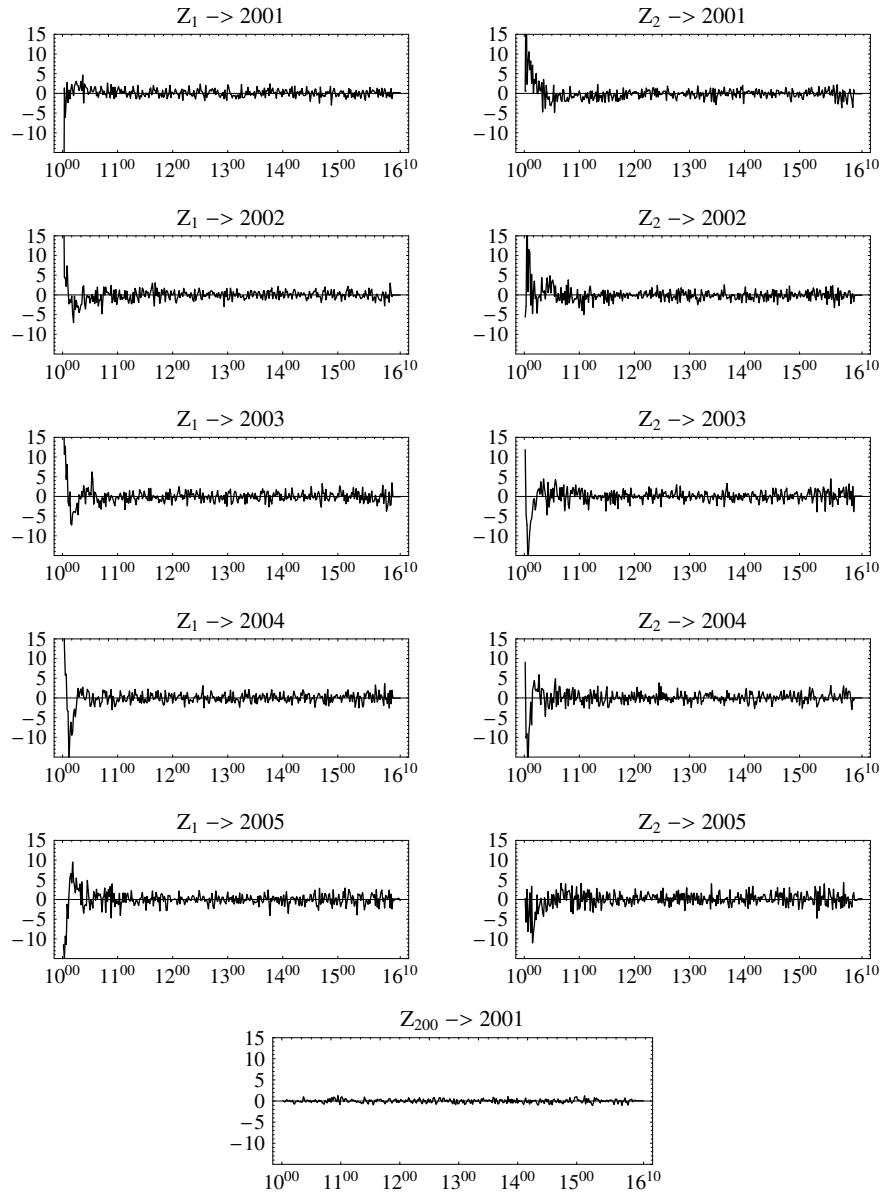


Fig. 5. Intraday eigensignals corresponding to the two largest eigenvalues (λ_1 , λ_2) calculated for five calendar years of WIG20 (Warsaw Stock Exchange) index variation during the intraday trading time 10:01:30–16:10:00. The last graph is the same intraday eigensignal but corresponding to λ_{200} for the year 2001.

3.2. Correlations among trading weeks

Ability of the above formalism to detect and decompose some potential repeatable structures in the financial patterns prompts a question of correlations among different trading weeks. Our WIG20 data set (Fig. 1) comprises $N = 207$ full trading weeks and thus allows to construct a 207×207 correlation matrix. The lengths T of the corresponding time series of 1 min returns between Monday opening and Friday closing equals 1840. The upper panel of Fig. 6 shows the resulting spectrum of eigenvalues. Most of them fall into the Wishart random matrix spectrum range (shaded area) but at least three eigenvalues of our empirical correlation matrix stay apart. The eigensignal associated with the largest eigenvalue (λ_1) is shown in the lower panel of this figure. It shows an enhanced market activity at the connections between the days. Interestingly however this activity is much stronger in the middle of the week than in its beginning or in the end.

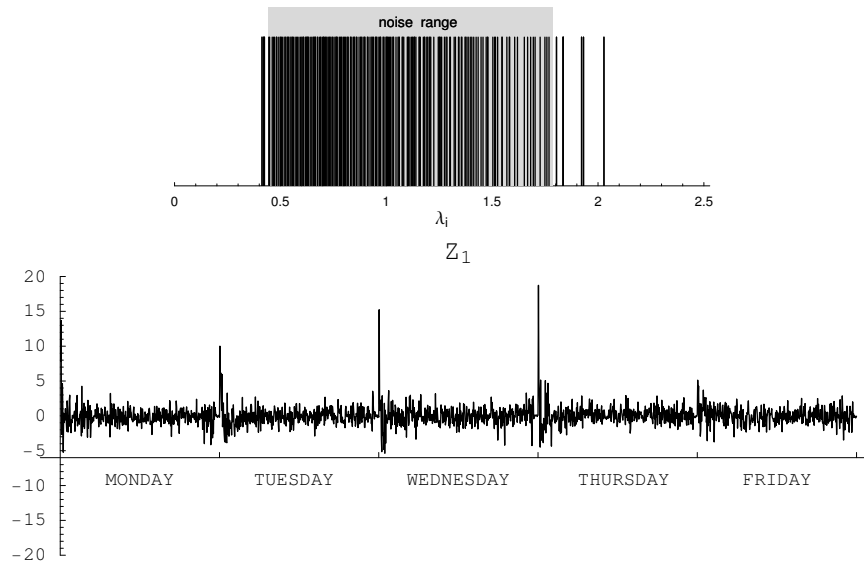


Fig. 6. (Top) Empirical eigenvalue spectrum of the 207×207 correlation matrix calculated among the weekly time intervals for the whole period of the WIG20 recordings as shown in Fig. 1. The corresponding noise range of a random Wishart matrix with $Q = 1840/207$ ($\lambda_{\max} \approx 1.78$ and $\lambda_{\min} \approx 0.44$) is marked by the shaded field. (Bottom) Intra-week (Monday 10:01:30 — Friday 16:10:00) eigensignal associated with the largest eigenvalue λ_1 .

Finally, such a decomposition of financial fluctuations allows an instructive insight into the statistics of returns distributions. This in itself constitutes one of the central issues of econophysics. The related well identified

stylized fact is the so-called inverse cubic power-law [13–15]. There exist also some consistency arguments that favour this law [16]. Fig. 7 shows the cumulative return distributions associated with several weekly eigensignals. Those distributions that originate from the bulk of eigenspectrum can be seen not to deviate much from a Gaussian even for such a short time lag of 1 min. The fatter tails result from the fluctuations filtered out by the eigensignals connected with the largest eigenvalues. In particular, the most extreme events can be seen in the first eigensignal, the one whose main component constitute fluctuations commonly considered a daily or a weekly trend.

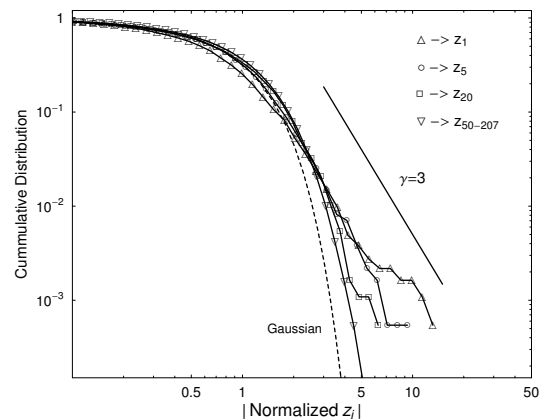


Fig. 7. Cumulative distributions of the moduli of normalised intraweek eigensignals. Dashed line corresponds to a Gaussian distribution and the solid line indicates a slope corresponding to the inverse cubic power-law.

4. Summary

The way of using the correlation matrix formalism, as presented here, opens a promising novel window to view the character of financial fluctuations. In particular, the related concept of eigensignals allows to filter out all repeatable synchronous patterns in the market daily or weekly activity. They are connected with a few largest eigenvalues of the corresponding correlation matrix. It is those eigensignals that appear to be responsible for the fat tails in the return distributions. The overwhelming rest of the spectrum stays within the borders prescribed by the random ensemble of Wishart matrices and fluctuations of the corresponding eigensignals are essentially of the Gaussian type. As far as the WIG20 dynamics is concerned it is interesting to notice a gradual weakening of the daily trend effects when going from 2001 to 2005. A question remains whether this effect is characteristic to this specific market or it takes place in the other markets as well.

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