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EFFICIENCY OF PAIR FORMATION IN A MODEL SOCIETY*

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In a recent paper a set of differential equations was proposed to describe a social process, where pairs of partners emerge in a community. The choice was performed on a basis of attractive resources and of random initial preferences. An efficiency of the process, defined as the probability of finding a partner, was found to depend on the community size. Here we demonstrate, that if the resources are not relevant, the efficiency is equal to unity; everybody finds a partner. With this new formulation, about 80 percent of community members enter into dyads; the remaining 20 percent form triads.

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1. Introduction

Sociophysics is a branch of the statistical and computational physics. It can be defined as an attempt to apply tools of theoretical physics to the social sciences. The problem, if and to which extent it is possible/allowed to describe social processes mathematically, cannot by resolved *a priori*. The arguments for the negative answer are well known in sociology, as they are used in the discussion between the empirical and the theoretical traditions [1]. As these arguments are continuously developed during about 200 recent years, it seems possible that still the subject is not closed. As for the physicists, however, mathematics is at least not worse than any other language; then the discussion is empty. Last but not least, the empirical

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tradition embraces a good part of sociology. In this field, however, it is natural to treat the physicists as outsiders, who "reinvent existing tools and rediscover established empirical results" [2, 3], and who eventually will be integrated into the larger social network community. We imagine that as for the sociological perspective, these notions could be applied to the sociophysics as a whole. Feeling offended or not, we are not willing to defy the integration.

The problem to be discussed here can be strictly formulated as a set of abstract differential equations, but its social interpretation is maybe more appealing. The term we use: "pair formation" means that a set of N objects, initially random, transforms in time towards a stable state with some pairs of objects. If no object remains unpaired, the efficiency of the process is 100 percent or 1.0. A fraction of unity means that some objects are not paired, *i.e.* the process is less efficient. The most straightforward equivalent of the process is when pairs of sexual partners appear in a group of young people. However, as we are going to demonstrate, the mathematics involved does not contain an assumption on individual differences, as for example sex.

Up to our knowledge, the problem of social pairing was discussed in Refs. [4, 5]. The subject of these papers is some kind of social optimization: the task is to find a partner with best parameters. There, the evaluation of these parameters can take into account individual needs. This algorithmic approach is well established in the literature of the subject [6,7]. Our starting point is maybe different, as we base on an assumption that final equilibrium state is developed as a result of initial preferences which are random. In this picture, the preferences are first followed, then rationalized *ex-post*. We note that this point of view was explored recently in a model of random sexual contacts [8]. As it was formulated by Vilfredo Pareto, people do what they believe is desirable for them [9].

In our previous paper [10], an attempt was made to evaluate the efficiency of the pairing process as dependent on the size $2N$ of the whole group (this variable was termed N in [10]). The difference between Ref. [10] and the present work is in the details of the model equations; in words, it can be summarized as follows. The core of the pairing process is that each group member selects another one based on the resources of the selected member and on her/his willingness to share them. Further, the willingness of the selecting member to share her/his own resources is reoriented in time as to give most resources to the most generous member, just selected. This can be seen as an application of the concept of reciprocity, well established in social sciences [11–13]. In this way, a positive feedback appears between particular members and weak initial inclinations are transformed into a monopoly on mutual feeding. Those who do not reorient quickly enough, because their initial inclinations were ill-directed or too distributed

— fail and remain unpaired. This version of the model has been presented in Ref. [10]. The velocity α of reorientation was another parameter, and the efficiency was investigated as dependent on α and N.

Within the above given terms, the goal of this work can be presented as follows. We already know that an individual selection is based on two agents: resources of the selected partner and her/his willingness to share them with the selecting member. Now, let us eliminate the criterion of resources, with only the one of willingness left. In other words, it does not matter any more if a member has anything to offer; the only relevant question is to whom she/he wants to offer what she/he has. Then, a striking effect is observed: the efficiency of the pairing process becomes equal to unity. No unpaired members is left.

The model equations are given in the next section, together with some example of the obtained plots. Last section contains our conclusion, aimed as sociological — in the physical sense.

2. Model and its results

The resources of i-th unit is defined as $p(i) \geq 0$. It evolves according to the following rule.

$$\frac{dp(i)}{dt} = N^2 - \left[\sum_{k=1}^{N} p(k)\right]^2 - \sum_{j\neq i}^{N}[r(j,i)p(j) - r(i,j)p(i)]\,, \tag{1}$$

where t is time and $r(i,j)$ is what i-th unit indents to offer to j-th one. The matrix $r(i,j)$ also evolves; in the model version presented in Ref. [10] it was evolving according to the following rule:

$$\frac{dr(i,j)}{dt} = \alpha\left(r(j,i)p(j) - \frac{\sum_k r(k,i)p(k)}{N-1}\right), \tag{2}$$

whereas in the present work the latter equation is limited to

$$\frac{dr(i,j)}{dt} = \alpha\left(r(j,i) - \frac{\sum_k r(k,i)}{N-1}\right). \tag{3}$$

As we see, in the previous model the matrix $r(i,j)$ (willingness matrix) evolves in time as dependent on the resources $p(i)$. In the second formulation this dependence is removed.

In Fig. 1 we present an example of the results for small value of α. In the former version of the model [10], the efficiency of the process for this value of α was about 0.2. With the application of Eq. (3), the efficiency is equal

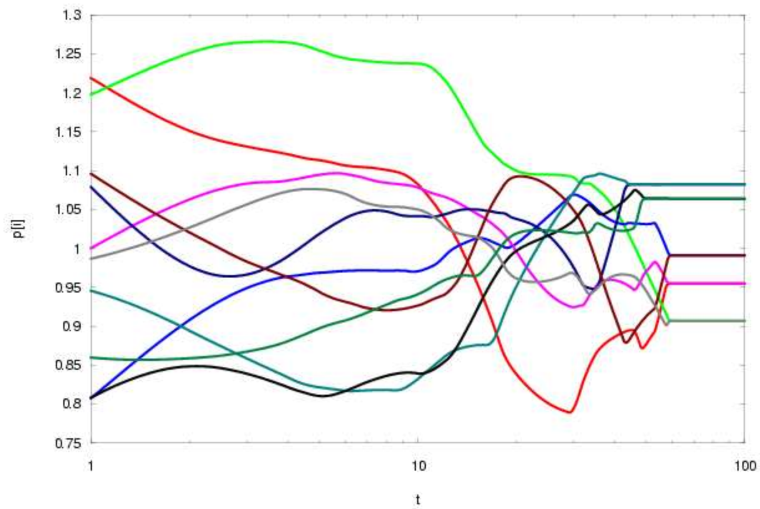

Fig. 1. The amount of resources as dependent on time for N=10 and α=0.2.

to 1.0 (100 percent) for all of our numerical evidence. As in the previous model, the paired units get the same amounts of resources.

Surprisingly, a pair is not the only pattern which appears as the result of the simulation. We observe also triads of two different types; let us call them type 1 and 2. In the stable state of triad 1, one of contributing units

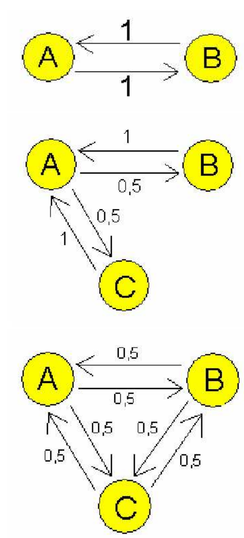

Fig. 2. The scheme of a pair and two kinds of triads.

exchanges resources symmetrically with the remaining two, while these two do not interact with each other. As a consequence, the amount of resources of one unit is twice larger than the amounts of the others. In triad 2, each of three units exchanges her/his resources with the others; then, their amounts of resources are equal. These triads are presented schematically in Fig. 2, together with the pair. We checked that the probability that an unit enters to a triad is about 0.2, and this result seemingly does not depend on the parameters N and α. Small numerical evidence suggests that groups larger than triads can be stable as well, but they are rare.

3. Conclusion

Summarizing, we report a new strategy of searching a partner to exchange resources. Our new element is to disregard the actual amount of resources of a potential partner, and to take into account only her/his willingness to offer them. Although this strategy is obviously far from being optimal with respect of acquired amount of resources, its advantage is that everybody is fed with some resources in the final state. This is the main goal of this paper.

The results of the model seem to present a nontrivial alternative for what is considered as normal behaviour in biology. Sexual rivalization and competition of genes — these ideas entered into common knowledge. The price we pay — we, living creatures — is that some less perfect units cannot pass their genetic material to an offspring. In the competitive world and in a closed group, they just are not able to find partners, tolerant to their weaknesses. Although in human world all that can be changed, at least in principle, the literature and other media provide a large pile of descriptions of this situation.

Would it then be of interest to modify our sexual behaviour? Truly, we do it already. As it is described by evolutionary psychologists, once the aim is not an offspring but just sex, our demands decrease [14]. This behaviour means that our sexual needs are alienated from their biological basis. Then, they can be used for other purposes: political, economic, social or just entertainment. This is one of differences between animals and people.

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