Vol. 37 (2006) ACTA PHYSICA POLONICA B No 12

MATTER EVOLUTION AND SOFT PHYSICS IN $A+A$ COLLISIONS*

Yu.M. Sinyukov

Bogolyubov Institute for Theoretical Physics
Metrologichna 14b, Kiev 03143, Ukraine

(Received October 27, 2006)

The principal problems of the matter evolution in hydrodynamic picture of $A+A$ collisions are considered. They concern the process of freeze-out, possible duality in the hydrodynamic and kinetic descriptions, and formation of the initial conditions at pre-thermal partonic stage, in particular, developing of initial transverse velocities and angular momentum. Also time evolution of the momentum spectra, Bose–Einstein correlation functions and averaged phase-space densities (APSD) are analyzed within hydrodynamic and kinetic models. The results shed light on the behavior of the observables, in particular, the interferometry volumes and APSD in $A+A$ collisions at different energies and for different nuclei. The new class of freeze-out parameterizations, accounting for continuous in time particle emission, is considered and their hydrodynamical realization is discussed.

PACS numbers: 24.10.Nz, 24.10.Pa, 25.75.–q, 25.75.Gz

1. Introduction

This is a review of recent results [3–6] as well as new one [7] that connect peculiarities of hadronic observables, related to "soft physics" of relatively small momenta, with the time evolution of the corresponding physical quantities and initial conditions in hydrodynamic and transport models of heavy ion collisions. It is known that the bulk of hadronic observables are formed at the very last period of the matter evolution, so called kinetic freeze-out when the hadronic system decays. We argue that the slope of particle transverse momentum spectra, interferometry volumes and averaged phase-space densities (APSD), if measured at any time of the expansion of hadron-resonance gas, are approximately conserved upon isoentropic and chemically frozen evolution. This result allows us to study the hadronic matter at the early

* Presented at the XLVI Cracow School of Theoretical Physics, Zakopane, Poland May 27–June 5, 2006.

stages of the evolution, in particular, near the hadronization point. We discuss the treatment of experimental data for pions that accounts for such a behavior of the above quantities. Our analysis sheds also some light on the RHIC HBT puzzle. We illustrate the general results by the concrete realistic hydrodynamic calculations that well describe the experimental data and allow us to make some conclusions for the initial conditions of the hydrodynamic expansion, which are formed at pre-thermal partonic stage. We discuss a problem of validity of the hydrodynamic approach to multiparticle production. We start the lectures from the final stage of matter evolution — analysis of the freeze-out process — and finish by the problem of initial conditions.

2. Sudden freeze-out vs continuous emission: the duality in hydro-kinetic approach

In the middle of XX century the three main features of high energy nucleon and nuclear collisions, namely, strong interactions, multi-particle production and exponential cut of transverse momentum spectra, put an idea into Heisenberg, Watagin and Fermi minds [1] that the systems created in those collisions could be considered as thermal. Then the hydrodynamic approach for multi-hadron production proposed by Landau [2] in 1953 appears as a natural development of that idea allowing one to provide a detailed space-time description of hadronic and nuclear collisions basing on the local energy-momentum conservation laws for thermal systems. According to the Landau criterion of freeze-out, the hydro-evolution stops and particles become free when fluid elements reach the temperature that is near the mass of the lightest hadron (pion): $T_{\rm f.o.} = m_\pi$. The descriptions of very wide class of high energy collisions: from $p+p(\overline{p})$ to heavy ion $A+A$ (see, *e.g.*, [8]) within this naive picture of ideal hydro-expansion with an extremely simple freeze-out criterion was unexpectedly very successful. One can call such a success even mysterious, and corresponding puzzles are known. As for $A+A$ collisions they are: first, too short, less than 1 fm/c, for a loss of coherence and thermalization of the colliding partonic system, and second, obviously oversimplified treatment of the freeze-out. For $A+A$ collisions the problem of short thermalization time arises from an analysis of elliptic flows within ideal hydrodynamics [9]; the problem of freeze-out becomes clear from the results of many studies of $A+A$ collisions based on cascade (transport) models. Such results contradict the idea of a sudden freeze-out at some fixed temperature. In fact, the particles escape from the system during the whole period of its evolution (see, *e.g.*, [10]), at first mostly from the surface layer, and do not demonstrate the local equilibration at the late stages.

2.1. Observables in terms of distribution and emission functions

In the papers [11, 12] we propose the way of treating the problem. To clarify it let us analyze spectrum formation by using two different objects: the distribution function (in quantum case Wigner functions) f and emission functions S. The inclusive spectra can be describe by the distribution function at asymptotically large time $t_{\rm out}$ as follows:

$$p^0 \frac{dN}{d\boldsymbol{p}} = \langle a_p^+ a_p \rangle \,, \; p_1^0 p_2^0 \frac{dN}{d\boldsymbol{p_1} d\boldsymbol{p_2}} = \langle a_{p_1}^+ a_{p_2}^+ a_{p_1} a_{p_2} \rangle , \tag{1}$$

where the irreducible averages of the creation and annihilation operators in (1) are,

$$\langle a_{p_1}^+ a_{p_2} \rangle = \int\limits_{\sigma_{\rm out}} d\sigma_\mu p^\mu \exp{(iqx)} f(x,p) \,. \tag{2}$$

Here $p = (p_1 + p_2)/2$, $q = p_1 - p_2$ and the hypersurface $\sigma_{\rm out}$ just generalizes $t_{\rm out}$.

In simple cases evolution of distribution function can be described by Boltzmann equation that in absence of external forces has the form:

$$\frac{p^\mu}{p^0} \frac{\partial f(x,p)}{\partial x^\mu} = F^{\rm gain}(x,p) - F^{\rm loss}(x,p). \tag{3}$$

The term $F^{\rm gain}$ and $F^{\rm loss}$ are associated with the number of particles which respectively came to the point (x,p) and leave this point because of collisions. The term $F^{\rm loss}(x,p) = R(x,p) f(x,p)$ can easily be expressed in terms of the rate of collisions of the particle with momentum p. The term $F^{\rm gain}$ has more complicated integral structure and depends on the differential cross-section.

Let us introduce the escape function $f_{\rm esc}(x,p)$, describing the portion of the particles that will never interact after the time t. According to the probability definition

$$f_{\rm esc}(x,p) = P(x,p) f(x,p) \,, \tag{4}$$

where the escape probability $P(x,p)$, or probability for any *given* particle at x with momentum p not to interact any more, propagating freely, can be expressed explicitly, in terms of the rate of collisions along the world line $x' \equiv (t', \boldsymbol{x} + (\boldsymbol{p}/p_0)(t'-t))$ of the free particle with momentum p, through the opacity integral

$$P(x,p) = \exp\left(- \int\limits_t^\infty dt' R(x',p) \right) \tag{5}$$

Since $f_{\rm esc}(x,p)$ is associated with the particles suffering last collisions at the space-time point x, it is formed by the term $PF^{\rm gain}$, or emission function S, and described by the following equation [11]

$$p^{\mu}\frac{\partial}{\partial x^{\mu}}f_{\rm esc}(x,p)=p^{0}P(x,p)F^{\rm gain}(x,p)\,\equiv S(x,p)\,. \tag{6}$$

For initially finite system with a short-range interaction among particles, the system becomes free, in fact, at large enough times $t_{\rm out}$, so $P(x,p)\rightarrow 1$ and $f_{\rm esc}(x,p)\rightarrow f(x,p)$ in this limit. Therefore, to describe the inclusive spectra of particles the asymptotic equality $f_{\rm esc}(x,p)=f(x,p)$ can be used, replacing the total distribution function f in all the irreducible averages in (2) by $f_{\rm esc}$. Applying the Gauss theorem and recalling that $\partial_{\mu}[p^{\mu}\exp{(iqx)}]=0$ for particles on mass shell, one obtains, using respectively general equations (2), (6) and (3) and supposing their analytical continuation off mass shell,

$$\langle a^{+}_{p_1}a_{p_2}\rangle \;=\; p^{\mu}\int\limits_{\sigma_0}d\sigma_{\mu}f_{\rm esc}(x_0,p)e^{iqx}+\int\limits_{\sigma_0}^{\sigma_{\rm out}}d^4x\,S(x,p)e^{iqx}\,, \tag{7}$$

$$\begin{aligned}\langle a^{+}_{p_1}a_{p_2}\rangle \;=\;& p^{\mu}\int\limits_{\sigma_0}d\sigma_{\mu}f(x_0,p)e^{iqx}+p^0\int\limits_{\sigma_0}^{\sigma_{\rm out}}d^4x(F^{\rm gain}(x,p)\\ &-F^{\rm loss}(x,p))e^{iqx}\,,\end{aligned} \tag{8}$$

where $S(x,p)$ is defined through the product $PF^{\rm gain}$ by Eq. (6), $f_{\rm esc}(x_0,p)$ corresponds to the portion of the particles, which is already free at initial time t_0, or, more generally, at the initial hypersurface σ_0, and $f(x_0,p)$ is the distribution function at σ_0.

Eqs. (2) and (7) describe the observables either in terms of asymptotic distribution function or 4-volume emission function $S=p^0PF^{\rm gain}$ together with direct emission $f_{\rm esc}(x_0,p)$ from an initial 3D hypersurface σ_0. As one can see the expression of the emission function through distribution (and cross section) is quite complicated.

The Landau criterion of freeze-out and corresponding Cooper-Frye prescription, defined by Eq. (2) with substitutions $\sigma_{\rm out}\rightarrow\sigma_{\rm f.o.}$ and $f\rightarrow f_{\rm l.eq.}$, treat particle spectra as a result of a rapid conversion of a locally equilibrated (l.eq.) hadron system into a gas of free particles at some hypersurface $\sigma_{\rm f.o.}$. Formally, it corresponds to taking the cross-section tending to infinity at $t<t_{\sigma_{\rm f.o.}}$ (to keep system in l.eq. state) and zero beyond $t_{\sigma_{\rm f.o.}}$. Then $P(t,\boldsymbol{x},p)=\theta(t-t_{\sigma_{\rm f.o.}}(\boldsymbol{x}))$ (and so $f_{\rm esc}=0$ at $t<t_{\sigma_{\rm f.o.}}$), and $S=p^0PF^{\rm gain}=p^0\delta(t-t_{\sigma_{\rm f.o.}}(\boldsymbol{x}))f_{\rm l.eq.}$ in Eq. (7).

It is worth to note that the proportionality between the S and $f_{\text{l.eq.}}$, like $S(x,p) = \rho(\tau) f_{\text{l.eq.}}(x,p)$ is used in many papers devoted to a description of the data in a hydrodynamically motivated way (see review [13]). One should understand, however, that in the realistic case of no sudden freeze-out *the emission function $S(x,p)$ looses completely its proportionality to the distribution function $f(x,p)$*: they are just two different mathematical (and physical, of course) objects! This may be easily demonstrated by the direct calculations, as in Ref. [11] and is also clear from Eqs. (4), (6): the escape probabilities $P(x,p)$ in finite systems are very sensitive to the asymmetry of positions $\boldsymbol{x}$ as for the effective system boundary and, thereby, the $S(x,p)$ becomes anisotropic in momentum $\boldsymbol{p}$ in the rest frame of a fluid element unlike to the hydrodynamic distribution function $f_{\text{l.eq.}}(x,p)$.

2.2. The duality of the hydro and kinetic approaches

As it was advocated in Ref. [14] the perfect hydrodynamics is a good approximation for the earlier stage of the matter evolution because of the big cross-section of the interaction among color and white quasi-hadronic states in QGP. As for the matter evolution at the post hadronization stage in $A+A$ collisions the approximate equivalence between chemically frozen hydrodynamics of hadron gas was found and its evolution within the cascade approach in the temperature region above 0.12 GeV was discussed in Ref. [15]. Below this region the formation of spectra is continuous in time with fairly long "tails" of the emission. The detailed analysis of the process of particle emission was made in Ref. [11]. It was shown that the process of particle liberation, described by the emission function, is, indeed, continuous in time, and cannot be approximated by a delta function, as it was discussed in the previous subsection. However, despite a very complicated structure of the emission function $S(x,p)$ the observed momentum spectra and Bose–Einstein correlation functions can be expressed by means of the Landau/Cooper-Frye prescription (1), (2) for the thermal distribution function $f_{\text{l.eq.}}(x,p)$ in the fireball *before* it starts to decay. (This initial moment, or more exactly — the initial hypersurface where the decaying process in the expanding fireball starts, one can consider as the last stage of hydro-evolution, more precisely, as the final hypersurface that borders with a region of applicability of hydrodynamics and where the mean free paths become already congruous with hydrodynamic lengths). It seriously simplifies the problem.

Such an example was considered also numerically within Universal Kinetic Model (UKM-R), a Monte Carlo event generator, realized as a numerical code written in object oriented C++ language [5]. As it is shown in Fig. 1 neither momentum spectra nor interferometry radii of expanding fireball depend on the evolution time, preserving their initial values until complete decay, despite a huge number of collisions, if the initial state of a gas corresponds to Maxwell–Boltzmann thermal function with a spacially spherical symmetric Gaussian distribution. The evolution of such a fireball was considered analytically in Refs. [16].

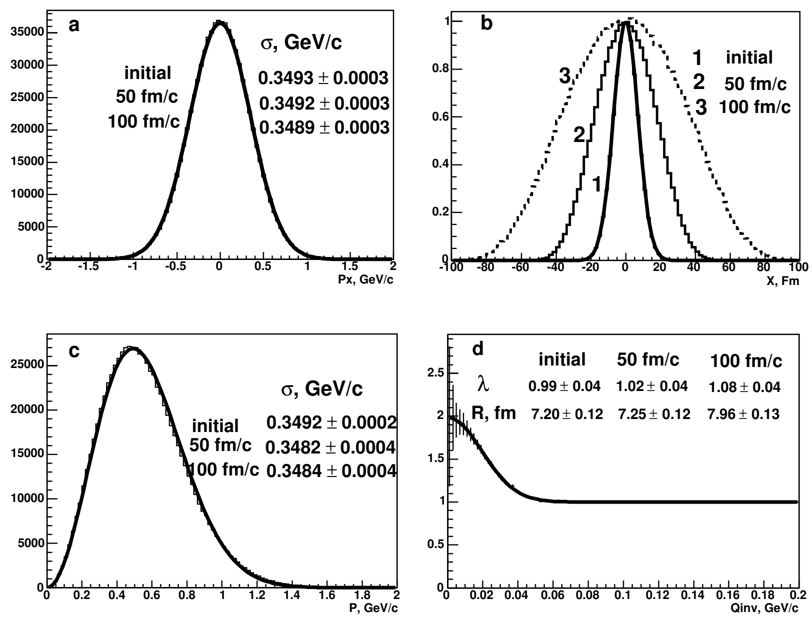

Fig. 1. Distributions of the x-components of particle three-momentum (a) and three-coordinate (b), particle momentum (c) and the corresponding CF's as functions of $q_{\rm inv}$ (d) obtained in the UKM-R simulation of the kinetic evolution of $N = 400$ heavy spin-0 bosons of mass $m = 0.938$ GeV/c^2 at the evolution time $t = 0$, 50 and 100 fm/c. The elastic cross section $\sigma_{\rm el} = 400$ mb, the initial Gaussian radius $R_0 = 7$ fm and the initial temperature $T_0 = 0.130$ GeV. Shown results of Gaussian fits of the momentum distributions and the CF's agree with the input initial values $\sigma = (mT_0)^{1/2} = 0.3492$ GeV/c and $\lambda = 1$, $R_0 = 7$ fm, respectively.

The explanation is based on the duality of equations (7) and (8). In the former one the spectra are expressed through the emission function (and corrected by initially free particles). The latter one describes spectra through the distribution functions at the initial time (and corrected by the term that is responsible for dissipation [11]). While the emission function, that is proportional to $F^{\rm gain}$, can have a significant non-zero value in wide space-time region, the integral of the difference $F^{\rm gain} - F^{\rm loss}$ over this region could be either zero, as in the example of expanding fireball discussed in Ref. [11], or small. Then the momentum spectra and BE correlations coincide, at least approximately, with what the system had at the initial moment. The latter is typical when the system expands in a spherically symmetric way. The analysis of different hydrodynamic solutions demonstrates that the velocity field of expanding systems tends typically to a spherically symmetric one at the late stage of evolution, at least, in the central region where the low p_{T} spectrum forms. Also, it was found in UKM-R simulation [5] for non-thermal and non-symmetric initial conditions that with the increasing elastic cross section the system more and more recovers spherical symmetry and thermal momentum distribution.

As for the high p_{T} spectrum formation, one can expect that corresponding particles are radiated mainly from the periphery of the system at the earlier times, because of large hydrodynamic velocities and fast transition to free streaming there as it was argued in Refs. [11] and [17]. Thereby, in a rough approximation, one can apply the generalized Cooper-Frye prescriptions (8) putting there $\sigma_0 = \sigma_{\mathrm{f.o.}}$ and taking into account the possible p-dependence of hypersurface $\sigma_{\mathrm{f.o.}}(p)$ at high $p_{\mathrm{T}} \geq 0.8$–1 GeV.

There is some kind of duality in a description of the spectra and the interferometry data based either on (thermal) distribution functions $f_{\mathrm{l.eq.}}(x, p)$ (2) which characterize the system just before decay begins or on the emission function $S(t, \boldsymbol{x}, \boldsymbol{p})$ (7) that describes the process of continuous particle liberation during the decay of system.

In fact, any kind of hydrodynamic, ideal or viscous, looses its applicability when the particle mean free paths become compatible with lengths of homogeneity in the system. Accounting for the pion component that is dominating in hadronic systems, the relative universality of the Landau freeze-out temperature $T \simeq m_\pi$ could be, at least partially, explained by exponential growth of the pion mean free path, $1/\sigma n(T(x)) \sim \exp(\beta m_\pi)$ with $\beta = 1/T(x)$ when the temperature falls down below m_π as it follows from the analytic representation of the thermal density $n(T)$. Thus the temperature $T \simeq m_\pi$ is just the lower boundary of the region of applicability of hydrodynamics in wide class of nucleon and nuclear collisions. It does not mean that the hadrons stop to interact in the post hydrodynamic stage, but one can expect that the momentum spectra do not change significantly, especially, if the above discussed conditions (small value of F^{gain}, symmetry of expansion at the late stage, *etc.*) are satisfied and so the integral of $F^{\mathrm{gain}} - F^{\mathrm{loss}}$ in Eq. (8) is small at that "post-freeze-out" stage.

Replacing the complicated emission process by the simple Landau [2] criterion of a sudden freeze-out at $T \approx 0.12$–0.15 GeV is, of course, a rather rough approximation. Nevertheless, as it was discussed in Refs. [3, 11], the momentum-energy conservation laws, peculiarities of almost isoentropic and chemically frozen evolution as well as some symmetry features of the late stage of hydro evolution could minimize the corresponding uncertainties.

Note, that the duality does not mean and even excludes the parametrization of emission function in the form $S \sim f_{\rm l.eq.}$ except for the case of real sudden freeze-out which is, probably, very non-realistic.

The discussed approximation is, unfortunately, not well controlled in practical utilization and we hope to develop an approximate method for spectra calculations within the hydro-kinetic approach [11] based on the escape probabilities and generalized relaxation time approximation. The method will combine the advantages of hydrodynamic approximation and microscopic (kinetic) approach. The former allows one to incorporate the complicated evolution of the system at the possible phase transitions encoded in the corresponding equation of state; the latter makes it possible to evaluate the observable spectra taking into account the non-equilibrated character of their formation. The developing of the method and corresponding numerical code is presently in progress.

3. Evolution of observables in hydrodynamic and kinetic models

For locally equilibrated weekly interacting gas of (quasi)particles the Wigner functions are just the local Bose–Einstein or Fermi–Dirac distribution functions if the Compton length of quanta in the gas is much less than the typical homogeneity length in expanding system, otherwise they are modified [18]. Supposing such distributions or, if it is possible, the Boltzmann approximation to them, we analyze temporal behavior of the "observables" in the hydrodynamic and transport models.

Many of features of the evolution originate from physical principles which are quite general for different classes of models. Qualitatively these features and physics behind them can be explained using non-relativistic analogy with simple rules providing a correspondence with the relativistic situation. To this end we use the exact Bjorken-like non-relativistic solution [19] of the ideal hydrodynamics. It has longitudinal (L) velocity $v_{\rm L} = x_{\rm L}/t$, initially zero transverse (T) velocity $v_{\rm T} = 0$ and the Gaussian transverse density profile (with radii R). It results in the following evolution of the locally equilibrated phase-space density [3]:

$$\begin{aligned} f^{\rm l.eq.}(t,\boldsymbol{r},\boldsymbol{p}) &= \left(\frac{1}{2\pi mT}\right)^{3/2} n \exp\left(-\frac{m}{2T}(\boldsymbol{v}-\boldsymbol{u})^2\right) , \\ n(t,\boldsymbol{r}) &= n_0 \frac{R_0^2 t_0}{R^2(t)t} \exp\left(-\frac{r_x^2}{2R^2} - \frac{r_y^2}{2R^2}\right) , \\ \boldsymbol{v} &= \frac{\boldsymbol{p}}{m} , \quad \boldsymbol{u}(t,\boldsymbol{r}) = \left(\frac{\dot{R}}{R}r_x, \frac{\dot{R}}{R}r_y, \frac{1}{t}r_z\right) , \end{aligned} \tag{9}$$

where the initial particle density in the central part of system, $r_x = r_y = 0$, is $n_0 = N/(2\pi)^{3/2}R_0^2 t_0$ and

$$T = T_0 \left(\frac{R_0^2 t_0}{R^2(t)t}\right)^{2/3}, \quad R\ddot{R} = \frac{T}{m}\,. \tag{10}$$

At each moment t the longitudinal spectrum, similarly to the relativistic boost-invariant model (here and below such a correspondence is marked by an arrow), is $m\frac{dN}{dp_{\rm L}} = \frac{dN}{dv_{\rm L}} = \text{const.} \to \frac{dN}{dy} = \text{const.}$, and the momentum spectrum is[1]

$$\frac{d^3N}{d^3p} = \frac{n_0 R_0^2 t_0}{m^2 T_{\rm eff}(t)} \exp\left(-\frac{p_{\rm T}^2}{2mT_{\rm eff}(t)}\right) \to f(\tau, T, m_{\rm T}) \exp\left(-\frac{m_{\rm T}}{T_{\rm eff}}\right), \tag{11}$$

where $m_{\rm T} \equiv \sqrt{m^2 + p_{\rm T}^2}$, n_0 is the initial particle density at $\boldsymbol{x}_{\rm T} = 0$, R_0 is the initial Gaussian radius of the fireball and the effective temperature, $T_{\rm eff}$, is

$$T_{\rm eff} = m\dot{R}^2(t) + T(t) \to T + mI^2\,. \tag{12}$$

The equation connects the slope of the transverse mass spectra with the intensity of transverse flows

$$I = \frac{\overline{R}_{\rm T}}{(v'_{\rm T})^{-1}} = \frac{\text{transverse radius}}{\text{hydrodynamical length}}\,. \tag{13}$$

For our exact solutions

$$\begin{aligned}
&\text{effective transv. radius} && R = \overline{R}_{\rm T}, \\
&\text{transv. velocity gradient} && v'_{\rm T} = \frac{\dot{R}}{\overline{R}_{\rm T}}, \\
&\text{intensity of transv. flow} && I = v'_{\rm T}\overline{R}_{\rm T}, \\
&\text{average squared of transv. velocity} && m\frac{\langle v_{\rm T}^2\rangle}{2} = I^2\,.
\end{aligned} \tag{14}$$

Qualitatively Eqs. (14) are rather general (for not very large $p_{\rm T}$) and we apply them also in what follows to clarify the experimental situation. The value of $T_{\rm eff}(t)$ does not change much since a decrease of temperature $T(t)$ during the system expansion is accompanied by an increase in time of the transverse flows described by $\dot{R}(t)$. The typical behaviors of the temperature and effective temperature in this model are presented in the left panel of Fig. 2.

---

[1] Note, that the presented relativistic analogies are valid under conditions (I is defined below): $m \gg T, m_{\rm T} - m \ll \frac{T}{I^2}(\frac{m}{T}I^2 + 1)^2$ [8]; otherwise (at $m_{\rm T} \to \infty$) one obtains ultrarelativistic blue shift factor in spectra [20].

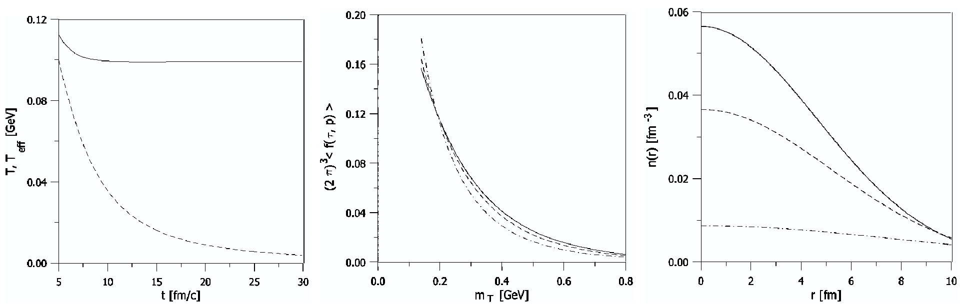

Fig. 2. Left: Evolution of the pion effective temperature $T_{\rm eff}$ (solid line) *versus* the temperature of the system T (dashed line) in the non-relativistic one-component Bjorken-like model with transverse expansion. The plots correspond to initial conditions: $T_0 = 0.1$ GeV, time $t_0 = 5$ fm$/c$, transverse radius $R_0 = 5$ fm and the initial transverse flow $\dot{R}(t_0) = 0.3$. Middle and right: The pion APSD $\langle f(\tau,p)\rangle$ (middle) and the pion densities $n(r)$ (right) taken at typical proper times: at hadronization, $\tau = 7.24$ fm$/c$, (solid line) and at kinetic freeze-out $\tau = 8.9$ fm$/c$ (dashed line). The dot-dashed line corresponds to the "asymptotic" time $\tau = 15$ fm$/c$ of hydrodynamic evolution. The initial conditions of hydrodynamic expansion of hadron-resonance gas are taken from Ref. [19].

The interferometry radii, out-, side- and long- take the forms (in this model $R_{\rm O} = R_{\rm S}$)

$$R_{\rm S}^2 = \frac{T(t)}{T_{\rm eff}(t)} R^2(t) \to \overline{R}_{\rm T}^2 \frac{1}{1 + m_{\rm T} I^2/T}; \quad R_{\rm L}^2 = t^2 \frac{T(t)}{m} \to \tau^2 \frac{T}{m_{\rm T}} . \qquad (15)$$

The last formulas are related to midrapidity and time of expansion is substituted by Bjorken proper time and $t \to \tau = \sqrt{t^2 - x_{\rm L}^2}$. The expression for the interferometry volume easily follows from dynamical equations (10)

$$V_{\rm int} \equiv R_{\rm O} R_{\rm S} R_{\rm L} = \frac{R_0^2 t_0 (T_0)^{3/2}}{T_{\rm eff}(t)\sqrt{m}} \qquad (16)$$

and it does not change significantly since it depends on almost constant function $T_{\rm eff}$ only. The average momentum phase-space density [22] in this model is

$$\langle f(t,\boldsymbol{p})\rangle = \frac{\int \left(f^{\rm l.eq.}(t,\boldsymbol{r},\boldsymbol{p})\right)^2 d^3r}{\int f^{\rm l.eq.}(t,\boldsymbol{r},p) d^3r} = \frac{d^3N}{d^3p} \Big/ (8\pi^{3/2} V_{\rm int}) . \qquad (17)$$

This quantity also depends on effective temperature $T_{\rm eff}(t)$ only and so does not change much. It is interesting and important to note that the phase-space density *totally averaged* over space and over momenta except the longitudinal one is completely conserved, it is integral of motion ($V_{\mathrm{int}}T_{\mathrm{eff}} =$ const.):

$$\langle f \rangle = \frac{\int \left(f^{\mathrm{l.eq.}}(t,\boldsymbol{r},\boldsymbol{p})\right)^2 d^3r d^2p_{\mathrm{T}}}{\int f^{\mathrm{l.eq.}}(t,\boldsymbol{r},p) d^3r d^2p_{\mathrm{T}}} \sim \frac{dN}{dv_{\mathrm{L}}} \Big/ \left(V_{\mathrm{int}}T_{\mathrm{eff}}m^2\right) = \mathrm{const}\,. \tag{18}$$

In the middle panel of Fig. 2 we demonstrate that the similar result holds for the evolution of the averaged pion phase-space density $\langle f(t,\boldsymbol{p})\rangle$ within much more complicated realistic (semi)relativistic model of Pb+Pb collisions at CERN energies 158 AGeV [19]. The model describes hydrodynamically the evolution of the chemically frozen hadron-resonance gas (with particle mass spectra limited by 2 GeV) from the hadronization point to thermal, or kinetic, freeze-out. As one can see, the values of $\langle f(t,\boldsymbol{p})\rangle$ are changed by less than 10% during the evolution and can be even higher at the final than at initial time at small transverse momenta. Contrary to this the particle densities of the thermal pions drop rather quickly in the central part of the system as is shown in the right panel of Fig. 2. Note, when densities become sufficiently small the hydrodynamic picture fails, nevertheless, formally calculated for this asymptotic regime the slope parameters, interferometry radii and other observables do not depend on time [11,21] in complete accordance with analogous (and obvious) results for a free streaming.

The study of the evolution of non-relativistic gas by UKM-R solver of Boltzmann equation [5] for wide class of initial conditions show that the momentum dispersion and the interferometry volume is practically conserved during the evolution. The approximate conservation effect takes place also in the evolution of the system of ultra-relativistic particles, except for some increase ($\sim$ 30% at 400 mb) of the final interferometry volume. However, since the mean mass of hadron-resonance gas is fairly large, $\bar{m} = 0.662$ [3,19], as compared with typical temperatures, 100–170 MeV, one can expect the interferometry volume should not change noticeably during the evolution. Thus UKM-R studies support similar results obtained in hydrodynamic approximation.

4. Analysis and treatment of experimental data: from AGS to RHIC

We apply now the ideas developed in previous sections to a general analysis of the relativistic evolution of a perfect and chemically frozen fluid. It has been shown (Ref. [3]) that the totally averaged (at fixed particle rapidity, *e.g.*, $y = 0$) phase-space density, or APSD, of thermal pions has the same properties as in a specific model — see Eq. (18) — it is, in general, an integral of motion. Note that this property is also preserved for free streaming. This conserved value can be expressed through the spectra and interferometry radii in the form [3]

$$(2\pi)^3 \langle f \rangle_{y=0} = \frac{\int d^3p \overline{f}_{\rm eq}^2}{\int d^3p \overline{f}_{\rm eq}} = \kappa \frac{2\pi^{5/2} \int \left(\frac{1}{R_{\rm O} R_{\rm S} R_{\rm L}} \left(\frac{d^2N}{2\pi m_{\rm T} dm_{\rm T} dy} \right)^2 \right) dm_{\rm T}}{dN/dy} \,. \tag{19}$$

Here $\overline{f}_{\rm eq} \equiv (\exp(\beta(p_0 - \mu) - 1)^{-1}$, and β and μ coincide with the inverse of the temperature and chemical potential which are supposed to be uniform on freeze-out hypersurface. The $\kappa = 1$ if one ignores resonance decays. Using the same approximation of the uniform freeze-out temperature and density we get the following expression for specific entropy s in mid-rapidity:

$$s = \frac{dS/dy}{dN/dy} = \frac{\int d^3p \left[-\overline{f}_{\rm eq} \ln \overline{f}_{\rm eq} + (1 + \overline{f}_{\rm eq}) \ln(1 + \overline{f}_{\rm eq}) \right]}{\int d^3p \overline{f}_{\rm eq}} \,. \tag{20}$$

The above ratio depends only on two parameters: temperature and chemical potential at freeze-out. The temperature can be obtained from the fit of the transverse spectra for different particle species and we will use the value $T = 120$ MeV as a typical "average value" for SPS and RHIC experiments. Another parameter, the chemical potential, can be extracted from an analysis of the APSD following to (19).

4.1. Possible signatures of new states of matter

The conservation of the APSD allows one to study the hadronization stage of the matter evolution based on Eq. (19). To some extend it concerns also specific entropy of thermal pions [4]. The results for the APSD at mid-rapidity for *all* negative pions ($\kappa = 1$) at the AGS, SPS, RHIC energies have been obtained in [4] and are presented in Fig. 3, left panel. One can see that the APSD grows significantly with energy at the AGS energies, then has the plateau starting from the lowest SPS energy, 20 AGeV, till 80 AGeV and then begins to grow again, apparently very slowly at RHIC as one can conclude from the not quite compatible experimental data of the STAR and PHENIX Collaborations, especially at $\sqrt{s}$=130 GeV. Since the conserved APSD values of *thermal* pions are almost proportional to the presented total values (with coefficients $\kappa(\sqrt{s}) \approx 0.6$–$0.7$ in (19) accounting for decays of long- and short-lived resonances [3]) they follow the same tendencies which are presented in Fig. 3. Then a plateau at low SPS energies indicates, apparently, a transformation of an excess of the initial energy to non-hadronic forms of matter: the pure hadronic stage with densities smaller than the initial ones appears later (after some expansion) at the hadronization temperature $T_{\rm c}$

defining the APSD of thermal pions[2]. A saturation of the APSD at RHIC energies can be treated as an existence of the limiting Hagedorn temperature of hadronic matter, or maximal temperature of deconfinement.

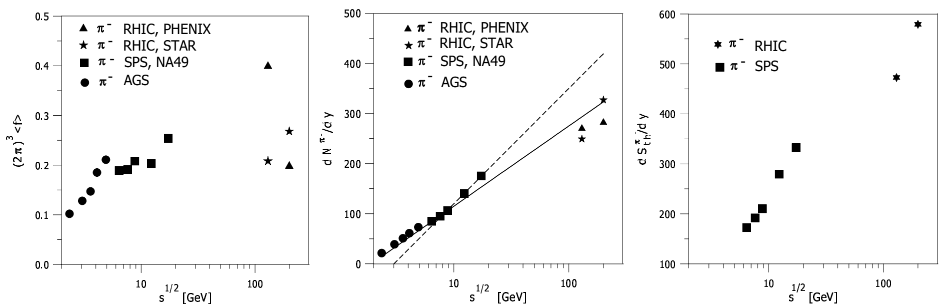

Fig. 3. Left: The average phase-space density of all negative pions at mid-rapidity, $(2\pi)^3 \langle f(y) \rangle$, as function of c.m. energy per nucleon in heavy ion central collisions. Middle and right: The rapidity densities of negative pions, dN^{π^-}/dy, and their entropy, $dS_{\rm th}^{\pi^-}/dy$, as function of c.m. energy per nucleon in heavy ion central collisions [4].

The APSD of negative *thermal* pions ($\kappa = 0.65$ for SPS and $\kappa = 0.7$ for RHIC energies respectively) are used then to extract the chemical potentials μ of them at thermal freeze-out at different SPS and RHIC energies, and after that to calculate the specific entropies, $s = \frac{dS/dy}{dN/dy}$ (20), and the entropies, $\frac{dS}{dy} = s * \frac{dN}{dy}$, of negative thermal pions. The entropies dS/dy of negative thermal pions are shown in Fig. 3, where the values at the RHIC energies are mean values of STAR and PHENIX data. Also we present there the rapidity densities of *all* negative pions at mid-rapidity in central nucleus–nucleus collisions for the AGS, SPS and RHIC energies. The lines in the middle panel of Fig. 3 represent the logarithmic law of energy dependence for negative pion multiplicities: $a \log_{10}(\sqrt{s_{\rm NN}}/b)$, where $a = 160$ (230), $b = 1.91$ (3) GeV for solid (dashed) lines respectively. A behavior of the entropy of thermal pions and measured pion multiplicities in central rapidity region *vs* energy demonstrates an anomalously high slope of an increase of the pion entropy/multiplicities at SPS energies compared to what takes place

[2] Note that a relatively *steep* rise of the pion APSDs at AGS energies is caused, probably, by transition from the nucleon to pion dominated matter within that energy range accompanied by decrease of $V_{\rm int}$ with beam energy [23]. However this effect cannot explain the plateau since the conservation of the pion APSD should lead to a rise of its value with energy density if the latter forms just hadronic matter. In addition, there is the another phenomenon, the so called "horn" in the ratios of strange particles to pions production at small SPS energies [24], that also cannot be only explained by the transition to pion dominant matter.

at the AGS and RHIC energies. This additional growth could be, probably, a manifestation of the QCD critical end point (CEP) between the highest SPS energy and high RHIC energies. The observed phenomenon can be caused by the dissipative effects that usually accompany phase transitions, such as an increase of the bulk viscosity [25], and also by peculiarities of pionic decays of σ mesons and other resonances with masses that are reduced, as compared to its vacuum values, in vicinity of the QCD CEP [26]. At the RHIC energies there is no anomalous rise of pion entropy/multiplicities, apparently, because the crossover transition takes place far from the CEP and no additional degrees of freedom appear at that scale of energies: quarks and gluons were developed at the previous energy scale.

4.2. Femtoscopy picture in hydrodynamic approach, RHIC HBT puzzle

The first results of the femtoscopy analysis of RHIC experiments [27] (as first announced by the STAR Collaboration) have revealed very unexpected results — the so-called HBT puzzle (see *e.g.* [28]). The puzzle implies first that the absolute values of the interferometry radii/volume do not change essentially at RHIC as compared to the SPS energies despite much higher multiplicities. It was in contrast with, expected at that time, possibility of the proportionality law between the interferometry volumes and multiplicities. At the same time there is an approximate proportionality between interferometry volume and different initial "volumes" which can be associated, *e.g.*, with number of participants (nucleons of nuclei) in the collision process [29] and, thus, with the multiplicity. Secondly, the ratio of outward to sideward transverse radii is opposite to that expected in standard hydrodynamic and hadronic cascade pictures. The ratio measured by STAR and PHENIX collaborations at RHIC BNL is close to unity in a wide momentum region. At the first sight these observations are in a contradiction with an existence of quark–gluon plasma and mixed phase as it implies a long time pion radiation [30] which usually results in the large ratio of outward to sideward transversal radii. As a result, now the phenomenological parameterizations, like the "blast wave model" just ignore the emission from the surface of expanding system despite the fact that it should last at least about the extracted life-time of the fireball: 10–12 fm/c. In addition, the correlation data on non-central $A+A$ collisions demonstrate that the transverse freeze-out geometry is extended out of the reaction plane [31] signaling a too rapid hydrodynamic system evolution to freeze-out which is in disagreement with the detailed transport calculations [32].

Let us use our results to clarify the situation at least qualitatively. We start from an approximate proportionality between "initial size" and interferometry one that is observed in non-central $A+A$ collisions and central ones for different nuclei [29]. One can estimate the integral in (19) using the phenomenological approximation that can be used for transverse spectra fit at least in some p_{T} region: $\frac{dN}{m_{\mathrm{T}} dm_{\mathrm{T}} dy} \propto \exp\left(-m_{\mathrm{T}}/T_{\mathrm{eff}}\right)$ (*c.f.* with the result (11)). Then, estimating the APSD, $\langle f \rangle$, and assuming that the integral C over dimensionless variable $m_{\mathrm{T}}/T_{\mathrm{eff}}$ depends on energies of collisions fairly smoothly, one can write, similarly to Eq. (18) that corresponds to nonrelativistic limit $m \gg T$, that

$$V_{\mathrm{int}} \simeq C \frac{dN/dy}{\langle f \rangle T_{\mathrm{eff}}^3} \,. \tag{21}$$

It is easy to see then that at any *fixed* energy $\sqrt{s_{\mathrm{NN}}}$ the V_{int} is nearly constant in time since the values dN/dy, APSD $\langle f \rangle$ and effective temperature T_{eff} in r.h.s. of Eq. (21), as it was discussed in Sec. 3, are approximately conserved for the thermal pions during the chemically frozen hydro-evolution. As a result, the HBT microscope at diverse energies "measures" the radii that are similar to the sizes of colliding nuclei. It explain the experimental observations.

As we mentioned above the experiments show clearly that in central collisions for the same colliding nuclei there is no proportionality law between V_{int} and dN^{π^-}/dy: the latter value grows with energy significantly faster than V_{int}. This fact is the main component of the HBT puzzle. To see the energy evolution we appeal again to Eq. (21). One can see a proportionality between V_{int} and the particle numbers dN/dy may be destroyed by a factor $\langle f \rangle T_{\mathrm{eff}}^3$. So, if the APSD and V_{int} only slightly grow with energy, mostly an increase of T_{eff}^3 could compensate a growth of dN/dy in Eq. (21). One can see that it is indeed the case: for example, the ratio of cube of effective temperatures of negative pions at $\sqrt{s_{\mathrm{NN}}} = 200$ GeV (RHIC) to one at 40 AGeV (CERN SPS) gives approximately 2, while the ratio of correspondent mid-rapidity densities is approximately equal to 3. It can be only in the case of an increase of the pion transverse flows in $A+A$ collisions with energy, if freeze-out temperature is approximately constant, $T = T_{\mathrm{f.o.}}(\sqrt{s}) \approx \mathrm{const.}$ If the intensity $I = v'_{\mathrm{T}} \overline{R}_{\mathrm{T}}$ (13) of flows grows, it leads to a reduction of the transverse interferometry radii, see (15). This effect can almost compensate a contribution to observed transverse radii, if the gradient of transverse velocity $I = v'_{\mathrm{T}}$ is not small, $v'_{\mathrm{T}} \overline{R}_{\mathrm{T}} > 1$ and does not decrease with energy: $v'_{\mathrm{T}}(\sqrt{s}) \geq \mathrm{const.}$ It was found in Ref. [8] that the gradient of transverse velocity at SPS energies can be estimated as: $v'_{\mathrm{T}} \approx 0.07(\mathrm{fm}/c)^{-1}$. It is enough to give an explanation of a constancy of the *transverse* sizes with energy, which is in agreement with observations also at RHIC.

The question which then arise is: why does the intensity of flow grow? It is clear that an increase of collision energy $\sqrt{s}$ results in a rise of (maximal) initial energy density ϵ and hence of (maximal) initial pressure $p_{\max}$. At the same time the initial transverse acceleration $a = \mathrm{grad}(p)/\epsilon \propto p_{\max}/\epsilon$ does not change, if EoS of the primary thermalized matter is the same. Then, as it is easy to check, the variations of initial energy density in the same initial volume do not change the space-time distribution of hydrodynamic velocities, only re-scale the energy. Thus, an increase of intensive flow with a growth of the energy in initially the same space-time region is possible only due to three factors.

(i) The first one is obvious: an increase of the time of hydro-evolution that the system needs to reach the same (or less) freeze-out energy density or temperature at higher initial density. Then the intensity of flow (13) $I = v'_{\mathrm{T}}\overline{R}_{\mathrm{T}}$ grows due to a growth of transverse area $\overline{R}_{\mathrm{T}}$ occupied by decaying fireball (this area is defined by the conditions of freeze-out) and an increase of the average transverse velocity gradient. All that is confirmed by numerical hydrodynamic calculations presented in Fig. 4. According to the approximate formula (15) it should result in an increase of the R_{L} interferometry radii with energy. However, apparently, relativistic hydrodynamic picture with fixed equation of state overestimates the increase of the longitudinal interferometry radii as compared to the experimental data.

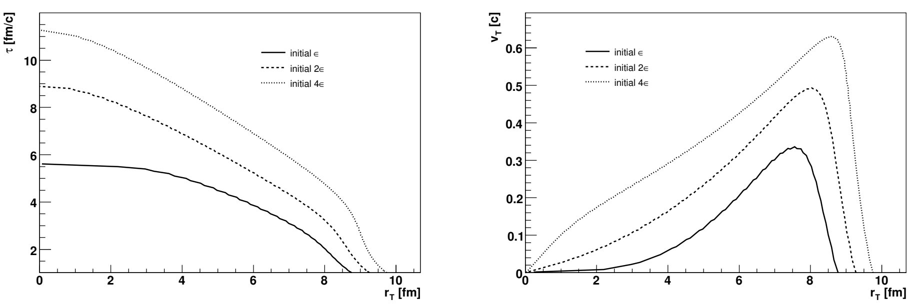

Fig. 4. Left: The typical freeze-out hypersurfaces with the *fixed* f.o. energy density presented in τ–r plane for Bjorken-like azimuthally symmetric hydrodynamic expansion with equation of state $P = \epsilon/3$ and zero initial transverse flow. The curves correspond to the different initial energy densities $\epsilon_i(\tau_i, y, r)$=$\epsilon_0(\tau_i, y, r)$, $2\epsilon_0(\tau_i, y, r)$, $4\epsilon_0(\tau_i, y, r)$ distributed in r-plane according to the Woods–Saxon formula. The initial proper time is $\tau_i = 1$ fm$/c$. Right: The transverse velocity distributions on freeze-out for different initial energy densities correspondingly to the left panel [37].

(ii) Secondly, a transition from *initially* soft equation of state (EoS) to the hard one can also give increase of I. Indeed, let us suppose the thermal system that initially has longitudinal flow (say, Bjorken-like in mid-rapidity) always passes through a "mixed phase" stage where there is the softness point in EoS. First, it is clear that at the same initial energy density for initially soft EoS it takes more time to reach the fixed "final" temperature than from initial state with hard EoS. In the former case a smaller portion of initial energy of particles/partons will transform into transverse kinetic energy than in the latter one. Then, if a transition to higher collision energies is accompanied by a transition from initially soft EoS to initially hard one, the existence of a region with the soft equation of state in the thermodynamic trajectory of the system is now not so important for total expansion time as in the previous case of *initially* soft EoS. The evolving system meets this "soft stage" with well developed transverse flow that allows it to reach the hadronic phase in relatively short time, not sacrificing thereby the transverse energy in favor of the longitudinal one. In this scenario one can hope to moderate the serious increase of R_{L} between SPS and RHIC typically expected for the fixed EoS.

(iii) One more factor that can play role in this drama is the initial transverse velocity which may develop at the pre-thermal partonic stage and obviously has an influence on the time of evolution and intensity of transverse flow at freeze-out. Moreover, what is essentially important, this factor has the direct connection to the second component of HBT puzzle: the unexpectedly small ratio of outward to sideward interferometry radii. We will consider the influence of this factor in the following sections where we will analyze the experimental data quantitatively.

5. The new class of freeze-out parameterizations and its dynamical realization

Now we will realize some ideas discussed previously. In Sec. 2 we stressed that real process of particle liberation, or freeze-out, is quite complicated because the particles escape from the system during the whole period of its evolution. We do not consider now the whole complexity of the freeze-out process within hydro-kinetic picture (such work is in progress now) but use instead the idea of approximate duality (see Subsec. 2.2) and apply generalized Landau/Cooper-Frye prescription. The hypersurface of constant temperature (an isotherm) or a constant energy density for the finite system expanding into vacuum should be enclosed in Minkowski space (together with initial hypersurface where the initial conditions are settled). Such an enclosed hypersurface contains, typically (see, *e.g.*, Fig. 4), space-like sectors of the volume emission and non-space-like ones (with space-like normal) of the surface emission. The contribution to the particle spectra from the surface emission is not small, especially at fairly large $p_{\rm T}$ as was discussed in Subsec. 2.2. Note, in addition, that at each moment of time the periphery of an expanding system is more dilute, the matter there is in the hadronic phase and hadrons can easily escape from this layer into the surrounding vacuum despite the relatively high (hadronization) temperature; this was argued in detail in Ref. [34].

It is well known that protracted surface emission may lead to the ratio of the outward to sideward interferometry radii, $R_{\rm out}/R_{\rm side}$, much bigger than unity for typical scenarios of phase transition in heavy ion collisions [30]. As we mentioned already, these expectations are in contradiction with current experimental data from RHIC where $R_{\rm out}/R_{\rm side} \simeq 1$ and this discrepancy is a component of the so-called "HBT puzzle".

Naively, one might conclude from that observation that the duration of emission is very short. However, the $R_{\rm out}/R_{\rm side}$ ratio is also quite sensitive to the shape of freeze-out hypersurface because, unlike $R_{\rm side}$, $R_{\rm out}$ is a mixture of the *out*- width, time spread of emission, and the $r_{\rm out} - t$ correlations. Indeed, in the Gaussian approximation for the correlation function the interferometry radii can be expressed in terms of the space-time variances:

$$R_i^2(p) = \langle(\triangle r_i - v_i\triangle t)^2\rangle_p = \langle\triangle r_i^2\rangle_p + v_i^2\langle\triangle t^2\rangle_p - 2v_i\langle\triangle r_i\triangle t\rangle_p, \qquad (22)$$

where $v_i = p_i/p_0$, p_i, r_i (i =out, side, long) are the Cartesian components of the vectors $\boldsymbol{v}$, $\boldsymbol{p}$, and $\boldsymbol{r}$, respectively, and $p^\mu = (p_0, p_{\rm out}, 0, p_{\rm long})$ is the mean 4-momentum of the two registered particles. Here $\triangle r_i = r_i - \langle r_i\rangle_p$, $\triangle t = t - \langle t\rangle_p$, and $\langle ...\rangle_p$ denotes the averaged (over the distribution function) value taken at some momentum p. Note that in the Bertsch–Pratt frame [35] $p_{\rm side} = 0$ and therefore $p_{\rm out} = p_{\rm T}$, where $p_{\rm T}$ is the absolute value of the transverse component of the vector $\boldsymbol{p}$. It is easy to see from Eq. (22) that *positive* $r_{\rm out}$–t correlations, $\langle\triangle r_{\rm out}\triangle t\rangle_p > 0$, give a *negative* contribution to $R_{\rm out}$ reducing thereby the $R_{\rm out}/R_{\rm side}$ ratio.[3] Therefore one can conclude that relatively small $R_{\rm out}/R_{\rm side} \simeq 1$ ratio in a case of a prolonged surface emission can take place if there are positive $r_{\rm out}$–t correlations in the corresponding sector of the freeze-out hypersurface.

Such type of a parametrization is used in Ref. [6] to analyze RHIC data. The only fit with positive $r_{\rm out} - t$ correlations, as it is presented in Fig. 5, results in good description of the spectra pions, kaons and protons and pion interferometry data, including $R_{\rm side}$ and $R_{\rm out}$, see Fig. 6. All details are

[3] This mechanism of reduction of the $R_{\rm out}/R_{\rm side}$ ratio is realized in a multiphase transport (AMPT) model [36].

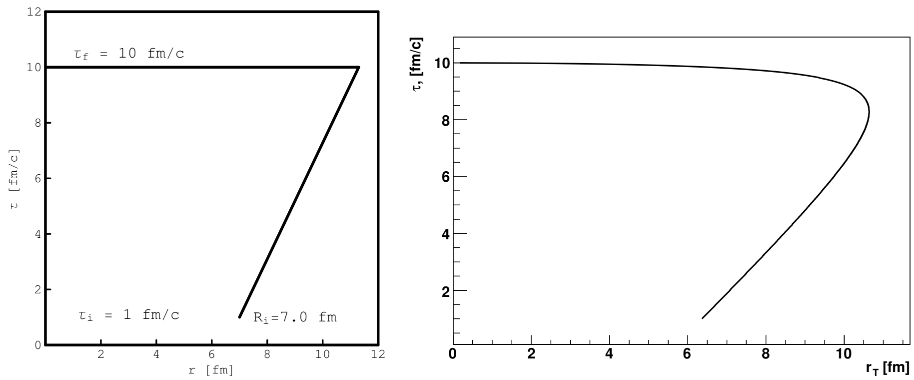

Fig. 5. Left: The parametrization of enclosed freeze-out hypersurface $\tau(r)$ with positive $r-\tau$ correlations. The initial proper time is $\tau_i = 1$ fm/c, initial collective velocity is maximal at the system border (at transverse radius $r = R_i = 7$ fm) and is $v(R_i,\tau_i) \approx 0.34$. Right: The dynamical realization of the freeze-out with positive $r-\tau$ correlations at constant energy density [37] based on $(3+1)D$ the exact analytical solutions of relativistic hydrodynamics [38].

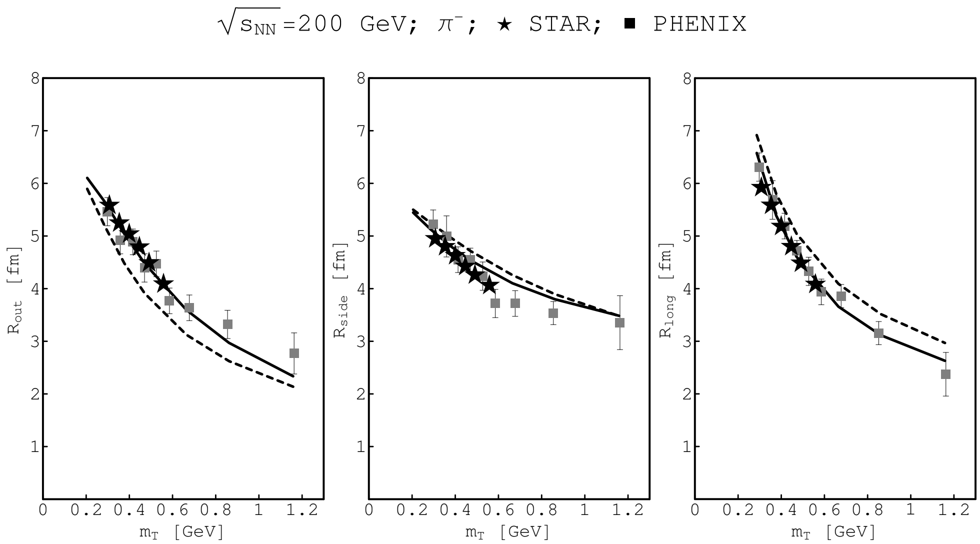

Fig. 6. Comparison of the pion source $R_{\rm out}$, $R_{\rm side}$, $R_{\rm long}$ radii measured by the STAR (stars) and PHENIX (boxes) Collaborations with model calculations performed for whole enclosed hypersurface (solid lines) and for shelf-like part only (dashed lines). The PHENIX data are adjusted for most central bin (see the text for details).

presented in Ref. [6]. In this model hadron emission can be separated in two parts. First one corresponds to "volume" radiation at $\tau = 10\,\mathrm{fm}/c$ and is similar to "blast wave" parametrization [33] but needs smaller transverse flow to describe the data because there is another source of emission[4]. The second one is the "surface" radiation from non space-like sectors of the enclosed freeze-out hypersurface which is fairly long: 9 fm/c and has the temperature $T = 150$ MeV close to hadronization temperature. This protracted surface emission is compensated in outward interferometry radii by positive r_{out}–t correlations that are the result of an intensive transverse expansion. It is noteworthy, that the best fit is achieved when the system already at $\tau_i = 1$ fm/c expands in the transverse direction with a non-zero collective velocity. It may indicate that a noticeable contribution into buildup of transverse flow at RHIC energies is given at very small times, possibly at partonic pre-thermal stage of the Glasma [39] evolution. Note that the hydrodynamic simulations [40] of heavy ion collisions at RHIC energies also indicate the necessity of an initial (pre-hydrodynamic) transverse flow to better account for slopes of the observed spectra.

What is very important in this picture that the positive r_{out}–t correlation at freeze-out can be realized only if the hydrodynamic system has developed flows at initial moment $\tau = 1$ fm/c. The very typical situation in relativistic hydrodynamics is presented at the right panels of Fig. 4 and Fig. 5. While the former case with negative r_{out}–t correlations at freeze-out corresponds to Bjorken-like expansion without initial transverse flow, in the latter case (Fig. 5) the system has quite developed initial flows. The exact analytical solution which illustrates the possibility of the dynamical realization of the parametrization Fig. 5 (right panel) is slightly deviated from Bjorken-type and belong to the new class of exact solutions of relativistic hydrodynamics with constant or negligible pressure. This class was found in Ref. [38]. The enthalpy in that particular solution has the form:

$$\epsilon + p = \frac{c_\epsilon \epsilon \left(1 + \frac{d}{\cosh\eta}\right)^2}{\tau\left(\tau + \frac{a}{\cosh\eta}\right)^2} \exp\left(-\frac{b^2}{1 - \lambda^2 r_\perp^2 \left(\frac{\cosh\eta}{\tau\cosh\eta + T}\right)^2}\right). \tag{23}$$

Here $a, b, \lambda, c_\epsilon$ are constants, η is fluid longitudinal rapidity. The similar expression with $c_\epsilon \to c_n$ takes place for particle number density $n(t, x)$.

[4] In blast wave parametrization the surface radiation is ignored and just because it gives no contribution to R_{out}. The use instead of this just smearing (proper) time factor for l.eq. distribution function to wash out the sharp freeze-out, first, contradict to strong r–t correlation of particle radiation at earlier times of evolution, and, second, is incorrect since the emission function cannot be proportional to the thermal distribution function as was discussed in Subsec. 2.1.

It is also clear from numerical hydrodynamic calculations that dynamical realization of prolonged surface emission with positive r–t correlations is possible if the system has initially developed collective transverse flow. Therefore, this factor is important to explain the RHIC HBT puzzle, at least, $R_{\rm out}$ to $R_{\rm side}$ ratio within the realistic picture of particle freeze-out.

6. Initial conditions for hydrodynamic expansion in $A + A$ collisions

A problem of formation of the initial transverse velocity at pre-thermal partonic stage leads inevitably to the complex matter of the initial stage in ultrarelativistic $A + A$ collisions and the problem of thermalization. Within the lecture it is impossible to review a huge number of studies devoted to this topic. Without all the details, we will keep in mind very naive picture and apply it phenomenologically.

Let us imagine a box (with size L) that have the ideally reflecting walls and contains the standing (electromagnetic) waves inside. Then collide two such boxes with the energy that allows to crush them completely. Standing waves then will be destroyed due to a stochastisation that is accompanied by the crashing *processes* [41]. In other words, strong correlations between phases of traveling "backward" and "forward" waves, with discrete momenta, say, $2\pi/L$ and $-2\pi/L$, caused by ideal reflections from the opposite walls, will vanish and instead the random phases $\exp(\alpha_{p_i})$ will appear:

$$\sin\frac{2\pi x}{L} = \frac{1}{2}\left(\exp\frac{i2\pi x_{\rm T}}{L} - \exp\frac{-i2\pi x_{\rm T}}{L}\right) \Rightarrow \sum \rho_{p_i}\exp(\alpha_{p_i})\exp(ip_i x_{\rm T}).$$

In the case of a very week field we will see then, say, two incoherent photons traveling, for instance, in transversal plane in opposite directions.

It is easy to make analogy now with high energy nucleus–nucleus collisions by imaging them as the collisions of the two "boxes" containing many "small boxes" — nucleons. Due to the non-commutativity of the gluon number operator with the operator of Lorentz boost, there is a huge number of coherent partons in the fast moving box — this state probably can be represented within the Color Glass Condensate (CGC) approach [43]. Correspondingly, after collision there will be not just two gluons but the classical color field (because of large occupation number) expanding into vacuum. When occupation number reduces, one can see the picture of expanding system of incoherent partons. It looks like a "partonic explosion" when many hidden degrees of freedom, associated with incoherent partons and carried significant transverse momentum, are liberated almost suddenly. An estimate of the thermalization time for this gas is a rather complicated problem and we do no touch it here. It seems that partons interact weakly enough and instability mechanism [42] works not so fast, as necessary to reach very small time of momentum symmetrization (thermalization?), less than 1 fm/c, required by hydrodynamics models to describe elliptic flows.

Let us simplify the problem again and consider the development of transverse velocity at pre-thermal partonic stage in an approximation of free streaming for these weekly interacting particles.

We start from the non-relativistic example. Let us put the initial momentum distribution of particles with mass m to be spherically symmetric Gaussian with the width corresponding to thermal Boltzmann distribution with uniform temperature T_0, no flows: $\boldsymbol{u}(t=0,\boldsymbol{r})=0$, and also spherically symmetric Gaussian profile (with radius R_0) for particle density. Let the particles just be a free stream. Then according to [19] the collective velocities, which can be defined at any time t according to Eckart:

$$u^i=\int\frac{d^3p}{m^4}p^if(t,\boldsymbol{x};p)$$

are

$$\boldsymbol{u}(t,\boldsymbol{r})=\boldsymbol{r}\frac{tT_0}{mR_0^2+T_0t^2}\,.$$

As one can see the collective velocities at free streaming grow with decrease of particle mass, grow with initial parameter T_0 for $m\neq 0$, and are independent of the initial "temperature" at $m=0$. Qualitatively, the same happens for relativistic partonic gas.

Let us consider the initial momentum distribution in relativistic partonic gas at Bjorken proper time τ= 1 fm/c that corresponds to "transverse momentum" Fourier components in the color field in the CGC picture found in Ref. [44]. Suppose that after collision the similar transverse spectrum will appear for incoherent partons. As for the longitudinal ones we will use also quasi-thermal distribution as it was proposed in Ref. [45] based on the Schwinger/Hawking mechanism of the partonic production. Let us use the boost-invariant approximation in mid-rapidity and the Woods–Saxon initial profile for energy density in transverse plane. Then the partonic distribution function at the initial proper time $\tau=\tau_0$ is:

$$f_0=\frac{1}{\exp\frac{m_{\mathrm{T}}}{T}\cosh\theta-1}\,\frac{1}{\exp\frac{1}{\delta}\left(r_{\mathrm{T}}-R\right)+1}\,, \tag{24}$$

where θ is the difference between particle and fluid rapidities. The main parameters of the distribution is agreed with Refs. [44, 45]: $T=0.465\varLambda_s$, $\delta=0.67$ fm, $\varLambda_s=1.3$ GeV; $\tau_0=1$ fm, $R=7.3$ fm, partonic mass is taken to be equal to $m=m_0=0.0385\varLambda_s$. The evolution of this function is defined by the equation for free streaming

$$p^{\mu}\frac{\partial f}{\partial x^{\mu}} = 0\,. \tag{25}$$

The solution of this equation describes the distribution function at any hypersurface τ = const. by the use of the following substitution in the arguments of the function f_0 related to the initial proper time τ_0=1 fm/c:

$$\boldsymbol{r}_{\mathrm{T}} \;\to\; \boldsymbol{r}_{\mathrm{T}} - \frac{\boldsymbol{p}_{\mathrm{T}}}{m_{\mathrm{T}}}\left(\tau\cosh\theta - \sqrt{\tau_0^2 + \tau^2\sinh^2\theta}\right)\,, \tag{26}$$

$$\theta \;\to\; \operatorname{arcsinh}\left(\frac{\tau}{\tau_0}\sinh\theta\right)\,. \tag{27}$$

The development of transverse velocities in such a system is presented in Fig. 7. Unlike the non-relativistic case discussed above, where the initial isotropy in local rest frames of the distribution function is preserved during further evolution, this ultrarelativistic evolution is not locally equilibrated, the distribution function and energy-momentum tensor are anisotropic in the local rest frames, and thus the developing of the transverse velocities is not associated with hydrodynamics of ultrarelativistic gas.

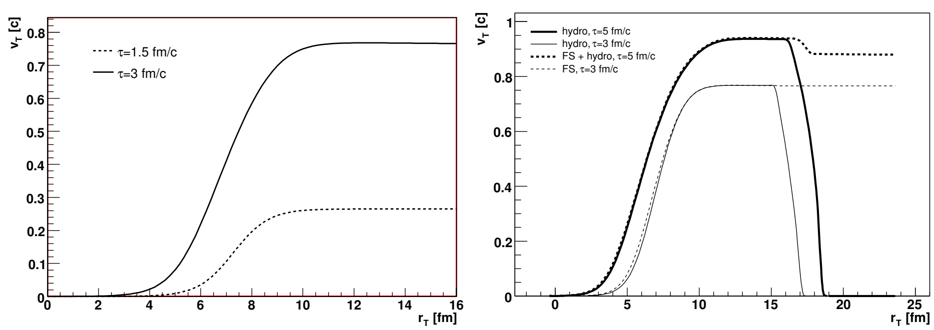

Fig. 7. Left: The collective transverse velocities (according to Ekkart ≈ Landau–Lifshitz) in weakly interacting partonic system in the approximation of free streaming. The initial state at 1 fm/c is supposed to be quasi-thermal and corresponds to the distribution (24). Dashed curve corresponds to τ=1.5 fm/c, solid line — 3 fm/c. Right: The simulation of the transverse collective velocities of the quasi-free and almost massless partons within ideal hydrodynamics with the same initial conditions as for partonic system. The velocity is good approximated by such a hydro-evolution with extra-hard EoS $P = 0.45\epsilon$. Dashed curve corresponds to a weakly interacting partonic system at τ=3 fm/c and 5 fm/c, solid line — to hydro-evolution at corresponding proper times. [7].

As we demonstrate in Fig. 7, right panel, such development of transverse velocities can be, nevertheless, approximated by the hydrodynamic expansion with abnormal hard EoS: $P = 0.45\epsilon$ ("normal" upper limit $P = \epsilon/3$ has the ultrarelativistic gas). It means that the effect of formation of the initial transverse velocity at pre-thermal stage is non-trivial and should be taken into account in a description of the experimental date.

The first important conclusion is that the short thermalization time is not necessary for developing of observed radial flows. They can be developed, and even more productive, at the pre-thermal or quasi-thermal stage. The natural objection against such a scenario might me the problem of not radial but the elliptic flows. They need earlier thermalization in order that the initial geometrical asymmetry in transverse plane transforms more effectively into momentum asymmetry. The pre-thermal transverse flows can smear out the asymmetry in momenta obtained due to the asymmetry in pressure gradients.

The solution of the problem could be an account for the residual — after the exclusion of non-participants — a transversely directed angular momentum which the system of participants has just after the collision due to a shift of the center of masses of colliding nuclei in reaction plane, that is associated with non-zero impact parameter. Then, as it is shown in Ref. [46], the corresponding tilt in the major axis of longitudinal expansion gives positive contribution to the asymmetry of the particle momenta in transverse to beam plane, or in v_2 coefficient. The account for an interplay between the initial pre-thermal transverse velocity and the angular momentum which the system of participants obtains in non-central collisions can open the new way of understanding the problem of matter evolution in nucleus–nucleus collisions.

7. Conclusion

1. There is the rough duality between hydrodynamics and kinetic approaches. The hydrodynamics with Landau/Cooper-Frye prescription on freeze-out hypersurface σ, generalized in the way that $\sigma(x) \rightarrow \sigma(x,p)$, can be applied despite the particle liberation process in fact happens in 4D volume and cannot be approximated by the sudden freeze-out. Typically, the corresponding hypersurface σ is bordered with a region of applicability of hydrodynamics. The next improvement can be reached in the hydrokinetic approach [6, 11].

2. In ultrarelativistic $A + A$ collisions the effective temperature of the transverse spectra does not change much during the system evolution since heat energy transforms into collective flows of expansion [3, 5]. The interferometry volumes, if measured at any time of the expansion of hadron-resonance gas, also do not grow significantly during the system evolution even if initial conditions are quite asymmetric [4, 5].

3. The average phase-space density APSD is an approximate integral of motion upon a chemically frozen evolution [3]. Therefore its value is associated directly with hadronization/chemical freeze-out stage. Then the plateau in APSD $\sqrt{s}$-behavior can be associated with deconfinement phase transition at low SPS energies; a saturation of this quantity at RHIC energies indicates, apparently, the limiting Hagedorn temperature for hadronic matter [4].

4. Also, an experimental behavior of the interferometry volumes at different energies (RHIC HBT puzzle) and for different nuclei can be explained based on points 2, 3 [4].

5. In realistic hydro-inspired parametrization the freeze-out hypersurface should be enclosed, and to be successful, *e.g.* to describe $R_{\rm out}$ to $R_{\rm side}$ ratio, it should have a positive $r-t$ correlation between radial r coordinates and time t for emitting particles, and also include initial transverse flows [6]. On the other hand, typically the $r-t$ correlation at the freeze-out hypersurface, according to equations of relativistic hydrodynamics, can be predominantly positive only if the system has at initial moment a developed transverse flow.

6. Intensive transverse flows can be developed at the very early pre-thermal stage just after the "partonic explosion" when many hidden degrees of freedom, associated with incoherent partons and carried significant transverse momentum, are liberated. The interplay of those flows and angular momenta which the system get in non-central collisions could lead to new scenario of the matter evolution and help to describe the experimental data in central and non-central collisions.

I want to thank to Peter Braun-Munzinger, Miklos Gyulassy, Dima Kharzeev, Yogiro Hama, Stanislav Mashkevich, Larry McLerran, Raju Venugopalan, Kacper Zalewski and also my co-authors for useful discussions on the subject of this paper. I would like to thank very much to Tetsufumi Hirano who gave me the possibility to use the hydrodynamic code to clarify some features of relativistic solutions. The research described in this publication was made possible in part by Award No. UKP1-2613-KV-04 of the U.S. Civilian Research & Development Foundation for the Independent States of the Former Soviet Union (CRDF). Also, the research carried out within the scope of the ERG (GDRE): Heavy ions at ultrarelativistic energies — a European Research Group comprising IN2P3/CNRS, EMN, Universite de Nantes, Warsaw University of Technology, JINR Dubna, ITEP Moscow and Bogolyubov Institute for Theoretical Physics NAS of Ukraine.

REFERENCES

[1] W. Heisenberg, *Z. Phys.* **101**, 533 (1936); G. Watagin, *Phys. Rev.* **63**, 137 (1943); E. Fermi, *Prog. Theor. Phys.* **5**, 570 (1950).

[2] L.D. Landau, *Izv. Akad. Nauk SSSR, Ser. Fiz.* **17**, 51 (1953).

[3] S.V. Akkelin, Yu.M. Sinyukov, *Phys. Rev.* **C70**, 064901 (2004).

[4] S.V. Akkelin, Yu.M. Sinyukov, *Phys. Rev.* **C73**, 034908 (2006); *Nucl. Phys.* **A774**, 647 (2006).

[5] N.S. Amelin, R. Lednicky, L.V. Malinina, T.A. Pocheptsov, Yu.M. Sinyukov, *Phys. Rev.* **C73**, 044909 (2006).

[6] M.S. Borysova, Yu.M. Sinyukov, S.V. Akkelin, B. Erazmus, Iu.A. Karpenko, *Phys. Rev.* **C73**, 024903 (2006).

[7] M. Gulassy, Yu. Karpenko, A. Nazarenko, Yu. Sinyukov, in preparation.

[8] Yu. Sinyukov, S. Akkelin, N. Xu, *Phys. Rev.* **C59**, 3437 (1999).

[9] P.F. Kolb, P. Huovinen, U. Heinz, H. Heiselberg, *Phys. Lett.* **B500**, 232 (2001).

[10] L.V. Bravina, K. Tywoniuk, E.E. Zabrodin, *J. Phys. G* **31**, S989 (2005).

[11] Yu.M. Sinyukov, S.V. Akkelin, Y. Hama, *Phys. Rev. Lett.* **89**, 052301 (2002).

[12] S.V. Akkelin, M.S. Borysova, Yu.M. Sinyukov, *Acta Phys. Hung. A — Heavy Ion Phys.* **22**, 165 (2005).

[13] U.A. Wiedemann, U. Heinz, *Phys. Rep.* **319**, 145 (1999).

[14] E.V. Shuryak, I. Zahed, *Phys. Rev.* **C70**,021901 (2004) [hep-ph/0307267]; G.E. Brown *et al.*, *Nucl. Phys.* **A740**, 171 (2004) [hep-ph/0312175]; E. Shuryak, *Prog. Part. Nucl. Phys.* **53**, 273 (2004) [hep-ph/0312227].

[15] D. Teaney, nucl-th/0204023.

[16] G.E. Uhlenbeck, G.W. Ford, *Lectures in Statistical Mechanics*, American Mathematical Society, Providence, Rhode Island, 1963; M.N. Kogan, *Dynamics of Rarefied Gases*, Nauka, Moskow, 1967 (in Russian); P. Csizmadia, T. Csörgő, B. Lukács, *Phys. Lett.* **B443**, 21 (1998).

[17] B. Tomasik, U.A. Wiedemann, *Phys. Rev.* **C68**, 034905 (2003).

[18] Yu.M. Sinyukov, *Nucl. Phys.* **A566**, 589c (1994); *Acta Phys. Hung. A — Heavy Ion Phys.* **10**, 113 (1999).

[19] S.V. Akkelin, P. Braun-Munzinger, Yu.M. Sinyukov, *Nucl. Phys.* **A710**, 439 (2002).

[20] E. Schnedermann, J. Sollfrank, U. Heinz, *Phys. Rev.* **C48**, 2462 (1993).

[21] T. Csorgo, J. Zimanyi, *Heavy Ion Phys.* **17**, 281 (2003).

[22] G.F. Bertsch, *Phys. Rev. Lett.* **72**, 2349 (1994); **77**, 789 (1996).

[23] D. Adamova *et al.* (The CERES Collaboration), *Phys. Rev. Lett.* **90**, 022301 (2003).

[24] M. Gazdzicki, M.I. Gorenstein, *Acta Phys. Pol. B* **30**, 2705 (1999).

[25] P. Danielewicz, M. Gyulassy, *Phys. Rev.* **D31**, 53 (1985).

[26] M. Stephanov, K. Rajagopal, E. Shuryak, *Phys. Rev. Lett.* **81**, 4816 (1998); E. Shuryak, `hep-ph/0504048`.

[27] C. Adler *et al.*, (STAR Collaboration), *Phys. Rev. Lett.* **87**, 082301 (2001); K. Adcox *et al.* (PHENIX Collaboration), *Phys. Rev. Lett.* **88**, 192302 (2002); S. Kniege *et al.* (NA49 Collaboration), *J. Phys. G* **30**, S1073 (2004).

[28] U. Heinz, P.F. Kolb, [`hep-ph/0204061`]; U. Heinz, *Nucl. Phys.* **A721**, 30 (2003); S. Pratt, *Nucl. Phys.* **A715**, 389c (2003); S. Soff, S. Bass, D. Hardtke, S. Panitkin, *Nucl. Phys.* **A715**, 801c (2003).

[29] S.S. Adler *et al.* (PHENIX Collaboration), *Phys. Rev. Lett.* **93**, 152302-1 (2004); H. Appelshauser, *J. Phys.* **G30**, S935 (2004).

[30] D.H. Rischke, M. Gyulassy, *Nucl. Phys.* **A608**, 479 (1996).

[31] M.A. Lisa, U. Heinz, U.A. Wiedemann, *Phys. Lett.* **B489**, 287 (2000); M.A. Lisa, for the STAR Collaboration, `nucl-ex/0301005`; M. Lopez-Noriega, for the STAR Collaboration, *Nucl. Phys.* **A715**, 623 (2003).

[32] D. Teaney, J. Lauret, E.V. Shuryak, *Phys. Rev. Lett.* **86**, 4783 (2001); `nucl-th/0110037`.

[33] F. Retiere, M. Lisa, *Phys. Rev.* **C70**, 044907 (2004); B. Tomasik, *Nucl. Phys.* **A749**, 209 (2005).

[34] Yu.M. Sinyukov, *Z. Phys.* **C43**, 401 (1989).

[35] S. Pratt, *Phys. Rev. Lett.* **53**, 1219 (1984); G.F. Bertsch, M. Gong, M. Tohyama, *Phys. Rev.* **C37**, 1896 (1988).

[36] Z.W. Lin, C.M. Ko, S. Pal, *Phys. Rev. Lett.* **89**, 152301 (2002); Z.W. Lin, C.M. Ko, B.A. Li, B. Zhang, S. Pal, *Phys. Rev.* **C72**, 064901 (2005) [`nucl-th/0411110`].

[37] Iu.A. Karpenko, Master Thesis, National University of Kiev, 2006.

[38] Yu.M. Sinyukov, Iu.A. Karpenko, *Acta Phys. Hung. A — Heavy Ion Phys.* **25/1**, 141 (2006).

[39] T. Lappia, L. McLerran, *Nucl. Phys.* **A772**, 200 (2006) [`hep-ph/0602189`].

[40] P.F. Kolb, R. Rapp, *Phys. Rev.* **C67**, 044903 (2003).

[41] S. Mashkevich, privite communication.

[42] St. Mrowczynski, *Phys. Rev.* **D39**, 1940 (1989).

[43] L. McLerran, R. Venugopalan, *Phys. Rev.* **D49**, 2233, 3352 (1994); **D50**, 2225 (1994); **D53**, 458 (1996); **D59**, 09400 (1999).

[44] A. Krasnitz, Y. Nara, R. Venugopalan, *Nucl. Phys.* **A727**, 427 (2003), [`hep-ph/0305112`].

[45] D. Kharzeev, E. Levin, K. Tuchin, `hep-ph/0602063`.

[46] T. Csörgő, S.V. Akkelin, Y. Hama, B. Lukacs, Yu.M. Sinyukov, *Phys. Rev.* **C67** 034904 (2003).