LATTICE STUDY OF GLUON VISCOSITIES — A STEP TOWARDS RHIC PHYSICS*

Atsushi Nakamura

Research Institute for Information Science and Education
Hiroshima University
Higashi-Hiroshima, 739-8521, Japan

Sunao Sakai

Faculty of Education, Yamagata University
Yamagata 990-8560, Japan

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After a brief overview of the high energy heavy ion collisions and lattice QCD simulations at finite temperature and density, we report our lattice study of transport coefficients of quark–gluon plasma (QGP) in the lattice quench approximation. We discuss why the transport coefficients are important and interesting in QGP physics.

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1. Introduction — QCD as a function of T and μ

One of the authors (A.N.) is very happy to come back to Zakopane after twenty years for the occasion of Prof. Andrzej Bialas' seventieth birthday. When he was a post-doc in Europe, Andrzej kindly asked him to give a talk at Zakopane on lattice QCD and his first finite temperature and density lattice simulation results [1]. During his stay in Cracow, he had an opportunity to discuss his work with Andrzej and Larry McLerran. They also read his draft and encouraged him very strongly to pursue this direction. This paper is now considered as the first lattice simulation of finite density QCD [2]. His talk today is a report to Andrzej after twenty years.

QCD is now a well established theory for the quark–gluon dynamics at the zero-temperature and zero-baryon density. The theory is very simple: it

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consists of only two fields, gluon as a gauge field and quark as Dirac fermion field, and we can write down the Lagrangian in one line. Nevertheless its behavior is very non-trivial due to the non-linear character of gluons and the approximate chiral symmetry. At large distance, the confinement of quarks and gluons emerges as a non-perturbative feature, that is unique among all theories describing the real world in the history of physics.

It is straightforward to formulate the statistical physics of QCD (at least in the case of equilibrium systems),

$$\begin{aligned} Z &= \mathrm{Tr}(\mathrm{e}^{-\frac{1}{kT}(H-\mu N)}) = \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left\{ -\int_0^{1/kT} d\tau \int d^3x (L+\mu n) \right\} \\ &= \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi \mathrm{e}^{-(S_G+\bar{\psi}\Delta\psi)} = \int \mathcal{D}U \det\Delta \mathrm{e}^{-S_G}, \end{aligned} \tag{1}$$

where S_G stands for the gluon action, and k is the Boltzmann constant, which is set to be one in the following. Here we express the formula in the lattice notation, *i.e.*, lattice link fields $U_\mu(x)$ are related to the gauge potential as $U_\mu(x) = \exp(iaA_\mu(x))$, and we do not include the gauge fixing term. Using this formula, we can calculate expectation values of physical quantities in QCD non-perturbatively by lattice simulation.

We can apply this well formulated theory, QCD, to predict phenomena that can be tested by experiments or observations. This is a great pleasure of physicists. Already more than twenty years ago, Gross, Pisarski and Yaffe pointed out that there are places where one might look for the effects of high temperature and/or large baryon density, *i.e.*, *(i)* the interior of neutron stars, *(ii)* during the collision of heavy ions at very high energy per nucleon, and *(iii)* about 10^{-5} sec after the big bang [3].

Our dream comes true. Now QCD at finite density and temperature is not anymore a theorist's imagination: experimentalists are creating high temperature and density state in heavy ion collisions on the earth. Braun-Munzinger, Redlich and Stachel have demonstrated this fact by estimating temperature and chemical potential at RHIC, SPS, AGS and SIS from produced hadron experimental data and by comparing them with lattice data [4]. In Fig. 1, we show results of analysis in Ref. [5] and recent lattice QCD results. Observed hadrons are expected to path through these points in (T, μ_B) plain. When they are created, the temperature and the density are probably even higher. Phase transition lines estimated by lattice QCD calculations are also shown in the figure. Ref. [6] is a nice review on QCD phase diagram and the critical point. Ref. [7] is a comprehensive new review of QGP.

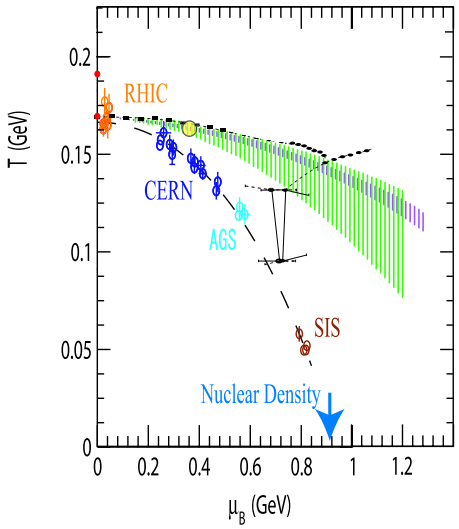

Fig. 1. A compilation of chemical freeze-out parameters together with the lattice gauge theory results. The nuclear density corresponds $\mu = 923$ MeV.

From this figure, we see that in RHIC the confinement/deconfinement transition temperature is exceeded. In this school, we hear from Peter Seyboth that they survey regions near a possible end critical point [8].

Two decades ago, using lattice simulations, three groups predicted that at high temperature there is a phase transition from the confinement to the deconfinement phase [9–11]. Since then, there have been many lattice studies on the phase transition point at $T > 0$ and $\mu = 0$. See Ref. [12] for lattice studies at finite temperature.

At large baryon density regions, many theoretical works have revealed that QCD has rich phase structure there. A remarkable feature is the existence of the color super conductivity phase, which were pointed out in Refs. [13–15]. It was thought, however, that this phenomena would be easily washed out at relatively low temperature due to the small gap energy and could not be observed. In late ninetieth, using instanton type modeling of the attractive force, Alford *et al.* [16] and Rapp *et al.* [17] argued that the gap energy is of the order of 100 MeV, and therefore the transition temperature, which should be approximately equal to the gap energy, is relatively high. The color super conductivity may be realized in neutron stars. After these papers, the field has become very active and many new possible phases were found. See Refs. [19, 20].

2. QGP produced in heavy ion collisions

Since 2000, RHIC has brought us many experimental informations in high energy heavy ion collisions. The news is very surprising: The produced matter does not look as a quasi-free gas, as naively expected, but rather is well described as a fluid. In SPS energy regions, the hydro-model describes well one-particle distributions, HBT *etc.*, but fails to describe the elliptic flow data. The success of the hydro-model may not be so surprising. Fifty years ago, Landau criticized Fermi's statistical model [21], and noticed 'owing to high density of the particles and to strong interaction between them, one cannot really speak of their number' and proposed his relativistic hydro-dynamical model [22]. The first quantum field theoretical analysis of the applicability conditions of the Landau hydro-dynamical model was reported in Ref. [23].

In three-dimensional hydro-dynamical calculations to analyze RHIC data, it is assumed that the matter produced is a perfect fluid, *i.e.*, *its viscosity is zero.* This assumption is supported by several phenomenological analyses.

This also suggests that the new state of matter produced at RHIC should be treated as a strongly coupled system. In order to get the point, let us consider the lowest perturbative calculation result,

$$\eta = \frac{\eta_1 T^3}{g^4 \ln(\mu^*/gT)} \,. \tag{2}$$

The viscosity is small when g is large. See Fig. 2. This is contrary to the pressure,

$$P = \frac{\pi^2}{90} T^4 \left(1 - \frac{15}{8}\left(\frac{g}{\pi}\right)^2 + \cdots\right). \tag{3}$$

Here the deviation from the free case can be controlled as a corrections of order g^2, g^4 *etc.* It might look strange that the viscosity becomes small at large coupling. There should be, however, sufficient frequent momentum exchange to realize a perfect fluid. Indeed

Free Gas	$\Longleftrightarrow$	Perfect Fluid
(weak interaction limit)		(strong interaction limit)

Policastro, Son and Starinets have shown an example of a strongly coupled theory in which the viscosity is indeed very small, *i.e.*, $\eta/s = 1/4\pi$ [24, 25]. They stressed that this is much smaller than that of ordinary matter, such as water or liquid helium, and conjectured that this is the lowest bound (KSS bound) [26].

It is thus important now to calculate the transport coefficients from QCD, non-perturbatively.

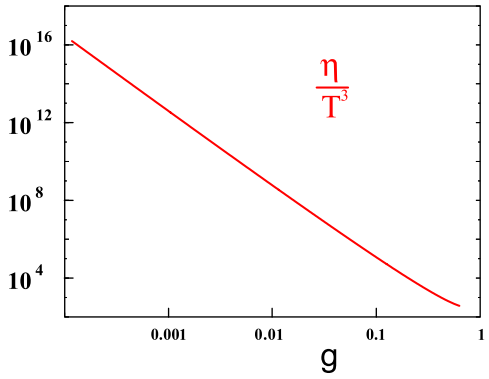

Fig. 2. η/T^3 in the case of the lowest perturbation theory.

3. Transport coefficients on lattice

On the lattice, the calculation of the transport coefficients is formulated in the framework of the linear response theory [27, 28].

$$\eta = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{\epsilon(t_1-t)} \int_{-\infty}^{t_1} dt' \langle T_{12}(\vec{x},t) T_{12}(\vec{x'},t')\rangle_{\rm ret} \,. \tag{4}$$

Here, $\langle T_{\mu\nu} T_{\rho\sigma}\rangle_{ret}$ is the retarded Green's function of energy momentum tensors $T_{\mu\nu}$ at a given temperature. In the quenched approximation, the energy momentum tensors are constructed from only gluonic field strength terms. Bulk viscosity is defined in a similar manner.

Shear viscosity in Eq. (4) is also expressed using the spectral function ρ of the retarded Green's function $\rho(\omega)$ [28] as

$$\eta = \pi \lim_{\omega\to 0} \frac{\rho(\omega)}{\omega} = \pi \lim_{\omega\to 0} \frac{d\rho(\omega)}{d\omega} \,, \tag{5}$$

i.e., the viscosity is controlled by the shape of the spectral function near $\omega = 0$.

For evaluating $\rho(\omega)$, we use a fact that the spectral function of the retarded Green's function at temperature T is the same as that of Matsubara–Green's function, *i.e.*, Abrikosov–Gorkov–Dzyalosinski–Fradkin Theorem [30]. Therefore, our target is to calculate Matsubara–Green's function($G_\beta(t_n)$) on a lattice and determine ρ from it [31].

To determine the spectral function $\rho(\omega)$ from $G_\beta(t_n)$, we adopt the simplest non-trivial ansatz, *i.e.*, a Breit–Wigner type ansatz proposed by Karsch and Wyld [32],

$$\rho_{\rm BW}(\omega) = \frac{A}{\pi}\left(\frac{\gamma}{(m-\omega)^2+\gamma^2} - \frac{\gamma}{(m+\omega)^2+\gamma^2}\right). \tag{6}$$

As this formula has already 3 parameters, to determine them, the lattice size in temperature direction(N_T) should be $N_T \geq 8$. Thus, the minimum lattice size should be $24^3 \times 8$, to obtain non trivial results.

Simulations are carried out using the Iwasaki's improved action and standard Wilson action. Use of the improved action reduces noise of the Monte Carlo simulation at intermediate coupling regions. This suggests that our observables the energy-momentum, suffer finite lattice spacing artifact.

The simulations are performed at $\beta = 3.05$, 3.3, 4.5 and 5.5 for the improved action and at β=7.5 and 8.5 for Wilson action. With roughly 10^6 MC measurements at each β, we determine Matsubara–Green's functions $G_\beta(t_n)$. The errors of G_β are still large in the large t region, however, we fit them with the spectral function ρ given by Eq. (6).

The bulk viscosity is equal to zero within the range of error bars, whereas the shear viscosity remains finite.

4. Conclusions

Because we adopt an ansatz for the spectral function, our study is not a first-principle calculation, yet, although this is best that one can do now. Therefore it is important to investigate reliability of the ansatz.

We may compare our results with the perturbation results of η in rather high temperature regions. In perturbation, bulk viscosity becomes zero [28,29], whereas shear viscosity in the next-to-leading log is given by Eq. (2). As seen in Fig. 3 (bottom), in low-T regions, the perturbative calculation becomes inapplicable. At very high temperature, lattice and perturbative results are satisfactorily consistent with each other. Although our result depends on the assumption regarding $\rho_{\rm BW}$ given in Eq. (6), it may be a reasonable approximation of $d\rho/d\omega$ at $\omega = 0$.

Aarts and Resco has proposed an another form of ρ as [34]

$$\rho(\omega) = \rho^{\rm low}(\omega) + \rho^{\rm high}(\omega)\,, \tag{7}$$

$$\rho^{\rm igh}(\omega) = \theta(\omega - 4m_{\rm th}^2)\frac{d_A(\omega^2 - 4m_{\rm th}^2)^{5/2}}{80\pi^2\omega}\left[n\left(\frac{\omega}{2}\right) + 0.5\right]\,, \tag{8}$$

where $d_A = N_c^2 - 1$ and $n(\omega) = 1/(\exp(\omega/T) - 1)$. $\rho^{\rm low}(\omega)$ is a rational function with coefficients as parameter.

In order to study the effect of $\rho^{\rm high}$ on the shear viscosity, η, we assume that ρ is given by $\rho = \rho_{\rm BW} + \rho^{\rm high}$, where $\rho_{\rm BW}$ is given by Eq. (6). By changing $m_{\rm th}$, the change in η is studied at $\beta = 3.3$ of improved action. When $\rho^{\rm high}$ is absent($m_{\rm th} = \infty$), ηa^3=0.00225(201). If m_{th} is set to be 5.0, 3.0 and 2.0, ηa^3 becomes 0.00223(0.00191), 0.00194(0.00194) and 0.00126(0.00204), respectively. At $m_{\rm th} = 1.8$, the contribution from $\rho^{\rm high}$ becomes larger than

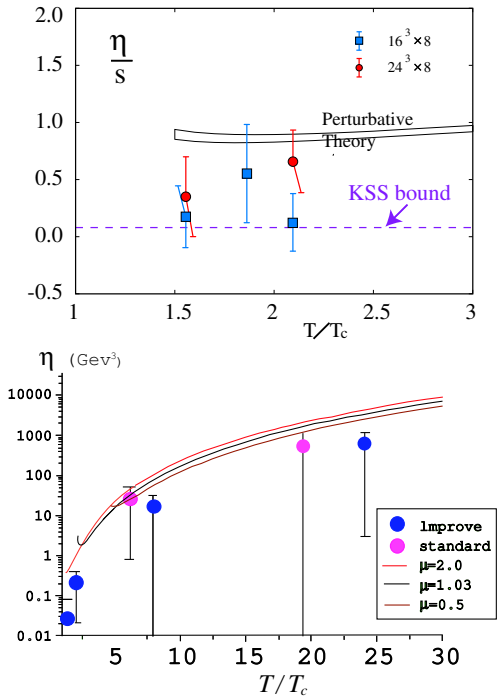

Fig. 3. Shear viscosity as a function of temperature. Its ratio to entropy density in RHIC temperature regions (top) and data in physical unit in wide temperature regions together with perturbative calculations (bottom).

$G_\beta(t_n)$ of simulation at $t = 1$, that fit could not be done. Generally, as $m_{\rm th}$ decreases, the contribution from $\rho^{\rm high}$ increases and ρ in the small ω region is suppressed. In this case, it results in a decrease in η.

We have calculated Matsubara–Green's function and determine the shear viscosity of gluon plasma. In the high-temperature region, the agreement of the lattice and perturbative calculation is satisfactory. The lattice result of η/s in $T/T_{\rm c} \leq 3$ is smaller than that obtained by the extrapolation of the perturbative calculation and satisfies the KSS bound. From the well known relation between the mean free path and viscosity, our results also suggest that gluon plasma is strongly interactive.

Although our results depend on the form of the spectral function $\rho_{\rm BW}$ given by Eq. (6), we think that the qualitative features will not change, because as discussed, our results are stable if the high frequency part of the spectral function is included. We think that η and η/s will not reach 10 times of the present value when more accurate determination of the transport coefficients is carried out.

However, it is important to carry out a more reliable and accurate calculation of transport coefficients, independent of the assumption regarding the spectral function. To this purpose, we are starting simulations on an anisotropic lattice, to apply maximum entropy method, and simulations with improved energy-momentum operators where we use the clover-type operators.

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