QCD NEAR SATURATION: A SPIN-GLASS STRUCTURE*

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When quarks and gluons tend to form a dense medium, like in high energy or/and heavy-ion collisions, it is interesting to ask the question which are the relevant degrees of freedom that Quantum Chromodynamics predicts. The present notes correspond to two lectures given at Zakopane in the (rainy) summer of 2006, where this question is adressed concretely in two cases, one in the QCD regime of weak coupling, the other one at strong coupling. (The second Lecture is published on page 0000 of this volume.) Each case corresponds to the study of an elusive but dynamically important transient phase of quarks and gluons expected to appear from Quantum Chromodynamics during high energy collisions. Below, we examine the dynamical phase space of gluon transverse momenta near the so-called "saturation" phase including its fluctuation pattern. "Saturation" is expected to appear when the density of gluons emitted during the collision reaches the limit when recombination effects cannot be neglected, even in the perturbative QCD regime. We demonstrate that the gluon momenta exhibit a nontrivial clustering structure, analoguous to "hot spots", whose distributions are derived using an interesting matching with the thermodynamics of directed polymers on a tree with disorder and its "spin-glass" phase.

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1. Introduction

Saturation in QCD is expected to occur when parton densities inside a hadronic target are so high that their wave-functions overlap. This is expected from the rapidity $Y = \log(W^2)$ evolution of deep-inelastic scattering

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amplitudes governed by the Balitsky Fadin Kuraev Lipatov (BFKL) kernel [1]. The BFKL evolution equation is such that the number of gluons of fixed size increases exponentially and would lead without modification to a violation of unitarity. By contrast, the renormalisation-group evolution equations following Dokshitzer, Gribov and Lipatov, Altarelli and Parisi (DGLAP) [2] explain the evolution at fixed Y as a function of the hard scale Q^2. They lead to a dilute system of asymptotically free partons. As schematized in Fig. 1, the transition to saturation [3,4] is characterized by a typical transverse momentum scale $Q_s(Y)$, depending on the overal rapidity of the reaction, when the unitarity bound is reached by the BFKL evolution of the amplitude. The two-dimensional plot showing the two QCD evolution schemes and the transition boundary to saturation are represented in Fig. 1.

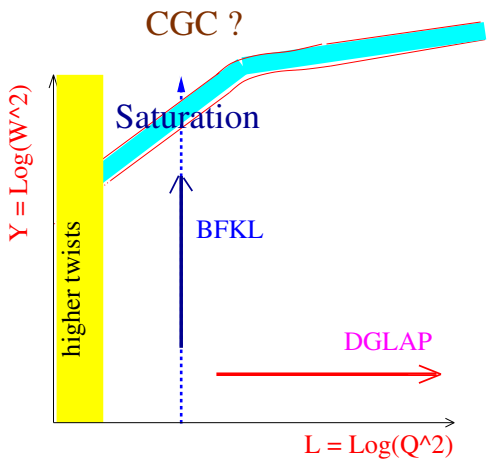

Fig. 1. Schematic view of the transition region to saturation. The DGLAP and BFKL evolution ranges are displayed, together with the saturation region where the density bounds are reached.

The problem we address here is the characterization of the gluon momentum distribution *near saturation*. Our aim is to understand the transverse-momenta spectrum of the gluons which are generated by the BFKL evolution in rapidity, *i.e.* characterising a transient QCD phase structure near saturation *as a whole*. The new material contained in this lecture comes from Ref. [5].

As a guide for the further developments, the basic structure underlying the transition to saturation can be understood in terms of *traveling waves*. If at first one neglects the fluctuations (in the mean-field approximation), the effect of saturation on a dipole-target amplitude is described by the non-linear Balitsky–Kovchegov [6] (BK) equation, where a nonlinear damping term adds to the BFKL equation. As shown in [7], this equation falls into

the universality class of the Fisher and Kolmogorov, Petrovsky, Piscounov (F-KPP) nonlinear equation [8] which admits asymptotic traveling wave solutions. The exponential behaviour of the BFKL evolution quickly enhances the effects of the tail towards a region where finally the nonlinear damping regulates both the traveling wave propagation and structure.

Indeed, one of the major recent challenges in QCD saturation is the problem of taking into account the rôle of fluctuations, *i.e.* the structure of gluon momenta beyond the average. In these conditions it was realized for traveling waves [9] and thus in the QCD case [10], that the fluctuations may have a surprisingly large effect on the overall solution of the nonlinear equations of saturation. Indeed, a fluctuation in the dilute regime may grow exponentially and thus modify its contribution to the overall amplitude. Hence, in order to enlarge our understanding of the QCD evolution with rapidity, it seems important to give a quantitative description of the pattern of momenta generated by the BFKL evolution equations for the set of cascading dipoles (or, equivalently gluons) *near saturation*, which is the subject of the lecture.

Technically speaking, we shall work in the leading order in $1/N_{\rm c}$, where the QCD dipole framework is valid [11]. Moreover we will use the diffusive approximation of the 1-dimensional BFKL kernel. In fact, the phase structure appears quite rich already within this approximation scheme. Many aspects we will obtain show "universality" features and thus are expected to be valid beyond the approximations.

2. Rapidity evolution of cascading QCD dipoles

Let us start by briefly describing the QCD evolution of the dipole distributions [11–13].

The structure of BFKL cascading describes a 2-dimensional tree structure of dipoles in transverse position space evolving with rapidity. Let us, for instance, focus on the rapidity evolution starting from one massive $q\bar{q}$ pair or onium [11], see Fig. 2. At each branching vertex, the wave function of the onium-projectile is described by a collection of color dipoles. The dipoles split with a probability per unit of rapidity defined by the BFKL kernel [1]

$$\mathcal{K}(\boldsymbol{v},\boldsymbol{w};\boldsymbol{z}) = \frac{\alpha_{\rm s} N_{\rm c}}{\pi} \frac{(\boldsymbol{v}-\boldsymbol{w})^2}{(\boldsymbol{v}-\boldsymbol{z})^2\,(\boldsymbol{z}-\boldsymbol{w})^2} \tag{1}$$

describing the dissociation vertex of one dipole $(\boldsymbol{v},\boldsymbol{w})$ into two dipoles at $(\boldsymbol{v},\boldsymbol{z})$ and $(\boldsymbol{z},\boldsymbol{w})$, where $\boldsymbol{v},\boldsymbol{w},\boldsymbol{z}$ are arbitrary 2-dimensional transverse space coordinates.

As an approximation of the 2-dimensional formulation of the BFKL kernel (1) obtained when one neglects the impact-parameter dependence, we

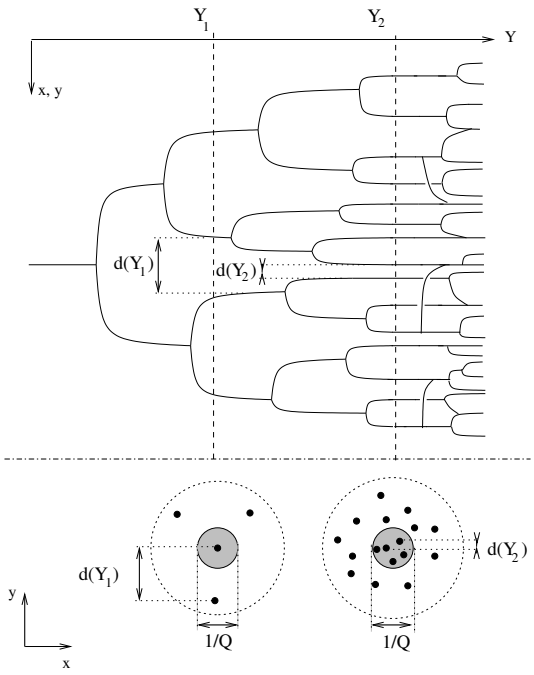

Fig. 2. BFKL cascading and saturation. The QCD branching process in the BFKL regime and beyond is represented along the rapidity axis (upper part). Its 2-dimensional counterpart in transverse position space is displayed at two different rapidities (lower part). The interaction region is represented by a shaded disk of size $1/Q$. At rapidity Y_1, the interaction still probes individual dipoles (or gluons), which corresponds to the exponential BFKL regime. There exists a smooth transition to a regime where the interaction only probes groups of dipoles or gluons, *e.g.* at rapidity Y_2. This gives a description of the *near-saturation* region corresponding to a mean-field approximation, where correlations can be neglected. Further in rapidity, $Y > Y_2$, other dynamical effects, such as merging and correlations appear.

shall restrict our analysis in the present paper to the 1-dimensional reduction of the problem to the transverse-momenta moduli k_i of the cascading gluons. After Fourier transforming to the transverse-momentum space, the leading-order BFKL kernel [1] defining the rapidity evolution in the 1-dimensional approximation is known [6] to act in transverse momentum space as a differential operator of infinite order

$$\chi(-\partial_l) \equiv 2\psi(1) - \psi(-\partial_l) - \psi(1+\partial_l)\,, \tag{2}$$

where $l = \log k^2$ and Y is the rapidity in units of the fixed coupling constant $\alpha_{\rm s} N_{\rm c}/\pi$.

In the sequel, we shall restrict further our analysis to the diffusive approximation of the BFKL kernel. We thus expand the BFKL kernel to second order around some value γ_c

$$\chi(\gamma) \sim \chi_c + \chi'_c(\gamma - \gamma_c) + \tfrac{1}{2}\chi''_c(\gamma - \gamma_c)^2 = A_0 - A_1\gamma + A_2\gamma^2\,, \qquad (3)$$

where γ_c will be defined in such a way to be relevant for the *near-saturation* region of the BFKL regime.

Within this diffusive approximation, it is easy to realize that the first term (A_0) is responsible for the exponential increase of the BFKL regime while the third term (A_2) is a typical diffusion term. The second term (A_1) is a "shift" term since it amounts to a rapidity-dependent redefinition of the kinematic variables, as we shall see.

In Eq. (3), γ_c is chosen in order to ensure the validity of the kernel (3) in the transition region from the BFKL regime towards saturation. Indeed, the derivation of asymptotic solutions of the BK equation [7] leads to the condition

$$\chi(\gamma_c) = \gamma_c\ \chi'(\gamma_c) \qquad (4)$$

whose solution determines γ_c.

This condition applied to the kernel formula (2) gives $\gamma_c = \sqrt{A_0/A_2} \approx 0.6275...$ and $\{A_0, A_1, A_2\} \approx \{9.55, 25.56, 24.26\}$. These numbers may appear anecdotic, but they fully characterize the critical parameters of the saturation transition, as we will realize later. For different kernels, *e.g.* including the next-to-leading log effects [14], they could be different, of course. But, then the traveling wave solution will be in a different "universality class" in mathematical terms.

3. Mapping to thermodynamics of directed polymers

From the properties in transverse-momentum space and within the 1-dimensional diffusive approximation (3), we already noticed that the BFKL kernel models boils down to a branching, shift and diffusion operator acting in the gluon transverse-momentum-squared space. Hence the cascade of gluons can be put in correspondence with a continuous branching, velocity-shift and diffusion probabilistic process, see Fig. 2, whose probability by unit of rapidity is defined by the coefficients A_i of (3).

Let us first introduce the notion of gluon momenta "histories" $k_i(y)$. They register the evolution of the gluon momenta starting from the unique initial gluon momentum $k(0)$ and terminating with the specific i-th momentum k_i, after successive branchings. They define a random function of the *running rapidity* y, with $0 \leq y \leq Y$, the final rapidity range when the evolution ends up (say, for a given total energy). It is obvious that two different histories

$k_i(y)$ and $k_j(y)$ are equal before the rapidity when they branch away from their common ancestor.

In order to fromulate precisely the mapping to the polymer problem, we then introduce random paths $x_i(t)$ using formal *space* and *time* coordinates $x, t\ ,\ 0 \leq t \leq \Delta t$ which we relate to gluon momenta "histories" as follows:

$$y = \frac{t}{A_0}\,; \qquad \log k_i^2(y) \equiv -\beta\ (x_i(t) - x(0)) + (A_0 - A_1)y\,, \tag{5}$$

where $(A_0 - A_1)y$ is a conveniently chosen and deterministic "drift term", $x(0)$ is an arbitrarily fixed origin of the unique initial gluon and thus the same for all subsequent random paths. The random paths $x_i(t)$ are generated by a continuous branching and Brownian diffusion process in space-time (*cf.* Fig. 3).

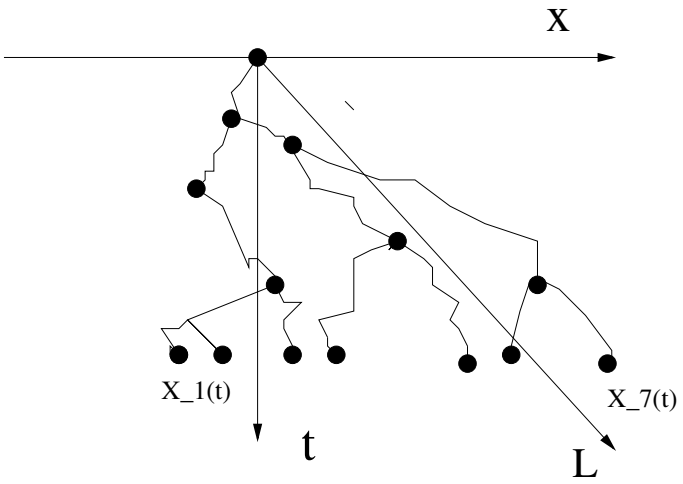

Fig. 3. Branching diffusion model for polymers. The coordinates $x_1(t) \cdots x_7(t)$ correspond to the random paths along the tree in the x, t phase-space. The oblique axis is for $L = \beta x + A_1/A_0\ t$, which takes into account the "time-drift" in the mapping to the QCD problem.

As we shall determine later, the important parameter β, which plays the rôle of an inverse of the temperature T for the Brownian movements of the polymer process, is given by

$$\frac{1}{T} \equiv \beta = \sqrt{2A_2/A_0}\,. \tag{6}$$

In fact, the relation (6) will be required by the condition that the stochastic process of random paths describes the BFKL regime of QCD *near saturation*. Another choice of β would eventually describe the same branching

process but in other conditions. Hence the condition (6) will be crucial to determine the QCD phase at saturation (within the diffusive approximation).

Let now introduce the tree-by-tree random function defined as the partition function of the random paths system

$$Z(t) \equiv \sum_{i=1}^{n} e^{-\beta x_i(t)} = e^{-\beta x_0 + A_1 y} \times \frac{1}{n} \sum_{i=1}^{n=e^{A_0 y}} k_i^2(y) \;\propto\; e^{A_1 y} \times \bar{k}^2(y) \;, \quad (7)$$

where $\frac{1}{n}\sum_{i=1}^{n} k_i^2(y) \equiv \bar{k}^2(y)$ is the *event-by-event* average over gluon momenta at rapidity y. Note that one has to distinguish $\overline{\cdots}$ *i.e.* the average made over only *one* event from $\langle \cdots \rangle$, which denotes the average over samples (or events).

$Z(t)$ is an event-by-event random function. The physical properties are obtained by averaging various observables over the events. Note that the distinction between averaging over one event and the sample-to-sample averaging appears naturally in the statistical physics problems in terms of "quenched" disorder: the time scale associated with the averaging over one random tree structure is much shorter than the one corresponding to the averaging over random trees.

With these definitions, Z appears to be nothing else than the partition function for the model of directed polymers on a random tree [15].

Let us now justify the connection of the directed-polymer properties with the description of the gluon momentum phase *near saturation* by rederiving the known saturation features from the statistical model point-of-view. Using the known properties [15] of the partition function of the polymer problem, one finds

$$\begin{aligned} \log Q_s^2 \;&\equiv\; \langle\, \log \bar{k}^2 \,\rangle \equiv \langle\, \log Z \,\rangle - A_1 Y \\ &=\; \left[(2\sqrt{A_2 A_0} - A_1) Y - \frac{3}{2} \sqrt{\frac{A_0}{A_2}} \log Y \right] + \mathcal{O}(1) \end{aligned} \quad (8)$$

which in fact matches exactly the asymptotic expansion found in [7] for the saturation scale.

In the same way, the solution of the statistical physics problem allows to derive the event-by-event *free energy* spectrum of the $\log Z(t)$ of the system around its average. From (7), one gets

$$\mathcal{N}(\bar{k}^2, Y) \sim \mathcal{P}\left(\log Z - \langle \log Z \rangle\right) \;\propto\; \log\left[\frac{\bar{k}^2}{Q_s^2}\right] \;\exp\left\{ -\sqrt{\frac{A_0}{A_2}} \log\left[\frac{\bar{k}^2}{Q_s^2}\right] \right\} \;, \quad (9)$$

which is the well-known geometrical scaling property, empirically found in Ref. [17] and theoretically derived in [7] from the BK equation for the dipole amplitude $\mathcal{N}(\bar{k}^2, Y)$.

Both properties (8), (9) prove the consistency of the model with the properties expected from saturation. We shall then look for other properties of the cascading gluon model. It is important to realize that this consistency fails for a choice of the parameter β different from (6). This justifies a-posteriori the identification of the equivalent temperature of the system in the gluon/polymer mapping framework.

4. The spin-glass phase of gluons

Let us now come to the main new results concerning the determination of structure of the gluon momentum phase at saturation.

The striking property of the directed polymer problem on a random tree is the spin-glass structure of the low temperature phase. As we shall see this will translate directly into a specific *clustering* structure of gluon transverse momenta in their phase near the "unitarity limit", see Fig. 4.

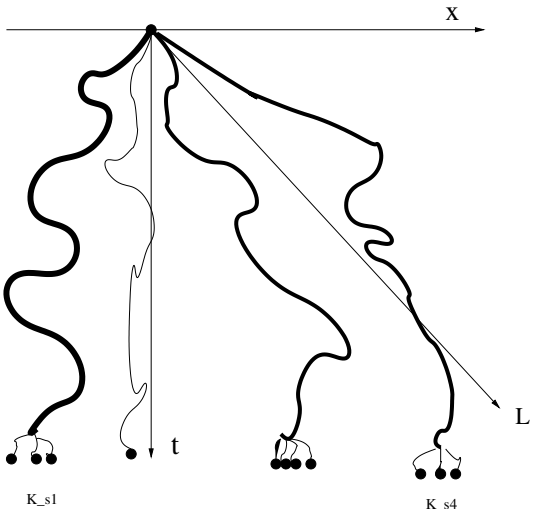

Fig. 4. The clustering structure of gluons near saturation. The drawing represents the $s_1 \cdots s_4$ clusters near momenta $k_{s1} \cdots k_{s4}$. They branch either near $t \ll 1$ or $(\Delta t - t) \ll 1$, where Δt is the total amount of time evolution.

Following the relation (6), one is led to consider the polymer system at temperature T with

$$\frac{T_{\rm c} - T}{T_{\rm c}} \equiv 1 - \frac{\beta_{\rm c}}{\beta} = 1 - \gamma_{\rm c} \ , \tag{10}$$

where γ_c is the critical exponent defined by (4). We are thus naturally led to consider the low-temperature phase ($T < T_c$), at some distance $T_c - T$ from the critical temperature $T_c = 1/\sqrt{2}$. In the language of traveling waves [16], this corresponds to a "pulled-front" condition with "frozen"and "universal" velocity and front profile.

As derived in [15], the phase space of the polymer problem is structured in "valleys" which are in the same universality class as those of the Random Energy Model (REM) [19] and of the infinite range Sherrington-Kirkpatrick (SK) model [20].

Translating these results in terms of gluon momenta moduli, the phase space landscape consists in event-by-event distribution of clusters of momenta around some values $k_{si}^2 \equiv 1/(n_i) \sum_{i \in si} k_i^2$, where n_i is the cluster multiplicity. The probability weights to find a cluster s_i after the whole evolution range Y is defined by

$$W_{si} = \frac{\sum_{i \in si} k_i^2}{\sum_i k_i^2}, \tag{11}$$

where the summation in the numerator is over the momenta of gluons within the s_i^{th} cluster (see Fig. 4). The normalized distribution of weights W_{si} thus allows one to study the probability distribution of clusters. The clustering tree structure, (called "ultrametric" in statistical mechanics) is the most prominent feature of spin-glass systems [21].

Note again that, for the QCD problem, this property is proved for momenta in the region of the "unitarity limit", or more concretely in the momentum region around the saturation scale. This means that the cluster average-momentum is also such that $k_{si}^2 = \mathcal{O}(Q_s^2)$. Hence the clustering structure is expected to appear in the range which belongs to the traveling wave front [7] or, equivalently, of clustering with finite fluctuations around the saturation scale.

In order to quantify the cluster structure, one may introduce a well-known "overlap function" in statistical physics of spin-glasses [21]. Translating the definitions (*cf.* [15]) in terms of the QCD problem, one introduces an event-by-event indicator of the strength of clustering which is built in from the weights (11), namely $\mathcal{Y} = \sum_{si} W_{si}^2$. The non-trivial probability distribution of overlaps $\Pi(\mathcal{Y})$ possesses some universality features, since it is identical to the one of the REM and SK models and shares many qualitative similarities with other systems possessing a spin-glass phase [22, 23]. Examples are given in Fig. 5.

The rather involved probability distribution $\Pi(\mathcal{Y})$ is quite intringuing. It possesses a priori an infinite number of singularities at $\mathcal{Y} = 1/n, n$ integer. It can be seen when the temperature is significantly lower from the critical value, see for instance the curve for $1 - T/T_c = 0.7$ in Fig. 5. However, the

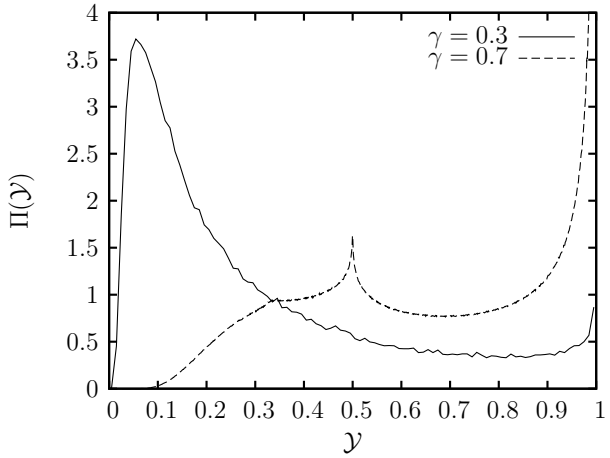

Fig. 5. The probability distribution of overlaps $\Pi(\mathcal{Y})$. The figure (the simulation is by courtesy from [18], using the method of Ref. [23]) is drawn both for the theoretical QCD value $\gamma_c = 0.7$, and for $\gamma_c = 0.3$ for comparison. The statistics used for the simulation is 10^8 events in 10^3 bins for $\gamma_c = 0.3$ and 3.25 10^6 events in 10^2 bins for $\gamma_c = 0.7$.

predicted curve for the QCD value $1 - T/T_c \sim 0.3$ is smoother and shows only a final cusp at $W_s = 1$ within the considered statistics. It thus seems that configurations with only one cluster can be more prominent than the otherwise smooth generic landscape. However, it is also a "fuzzy" landscape since many clusters of various sizes seem to coexist in general.

5. Summary

We investigated the landscape of transverse momenta in gluon cascading around the saturation scale at asymptotic rapidity. Limiting our study to a diffusive 1-dimensional modelization of the BFKL regime of gluon cascading, we make use of a mapping on a statistical physics model for directed polymers propagating along random tree structures at fixed temperature. We then focus our study on the region near the unitarity limit where information can be obtained on saturation, at least in the mean-field approximation. Our main result is to find a low-temperature spin-glass structure of phase space, characterized by event-by-event clustering of gluon transverse momenta (in modulus) in the vicinity of the rapidity-dependent saturation scale. The weight distribution of clusters and the probability of momenta overlap during the rapidity evolution are derived.

Interestingly enough the clusters at asymptotic rapidity are branching either near the beginning ("overlap 0" or $y/Y \ll 1$) or near the end ("overlap 1" or $1 - y/Y \ll 1$) of the cascading event. The probability distribution of overlaps is derived and shows a rich singularity structure.

On a phenomenological ground, it is remarkable that saturation density effects are not equally spread out on the event-by-event set of gluons; our study suggests that there exists random spots of higher density whose distribution may possess some universality properties. In fact it is natural to expect this clustering property to be present not only in momentum modulus (as we could demonstrate) but also in momentum azimuth-angle. This is reminiscent of the "hot spots" which were some time ago [24] advocated from the production of forward jets in deep-inelastic scattering at high energy (small-x). The observability of the cluster distribution through the properties of "hot spots" is an interesting possibility.

Many aspects depicted in this lecture come from constant collaboration and discussion inside (and outside) our "Saturation Team" in Saclay 2006, in particular Edmond Iancu, Cyrille Marquet, Gregory Soyez.

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