Vol. 37 (2006) ACTA PHYSICA POLONICA B No 12

A PERFECT FLUID FROM STRING/GAUGE DUALITY*

Robi Peschanski

Service de Physique Théorique, CEA/Saclay
91191 Gif-sur-Yvette cedex, France†
pesch@spht.saclay.cea.fr

(Received October 13, 2006)

We discuss the non-perturbative regime of QCD, which is supposed to be relevant for the description of the transient phase as quark–gluon plasma formed during heavy-ion collisions at very high energies. Since there is not yet an available field-theoretical scheme for non perturbative QCD in those conditions, we study the dynamics of strongly interacting gauge-theory matter (modelling quark–gluon plasma) using the AdS/CFT duality between gauge field theory at strong coupling and a gravitational background in Anti-de Sitter space. The relevant gauge theory is a-priori equipped with $\mathcal{N}=4$ supersymmetries, but qualitative results may give lessons on this issue. As an explicit example, we show that perfect fluid hydrodynamics emerges at large times as the unique nonsingular asymptotic solution of the nonlinear Einstein equations in the bulk. The gravity dual can be interpreted as a black hole moving off in the fifth dimension.

PACS numbers: 11.25.Tq, 12.38.Mh

1. Introduction

From the first years of the running of heavy-ion collisions at RHIC, it has been advocated that various observables are in good agreement with models based on hydrodynamics [1] and with quark–gluon plasma (QGP) in a strongly coupled regime [2]. To a large extent it seems that the QGP behaves approximately as a perfect fluid as was first considered in [3]. A schematic view of the theoretical expectations in given in Fig. 1. It is a challenge of QCD to derive from first principles the properties of the dynamics of a strongly interacting plasma formed in heavy-ion collisions and in particular to understand why the perfect-fluid hydrodynamic equations appear to be relevant.

---

* Lecture II presented at the XLVI Cracow School of Theoretical Physics, Zakopane, Poland, May 27–June 5, 2006.

† URA 2306, Unité de Recherche Associée au CNRS.

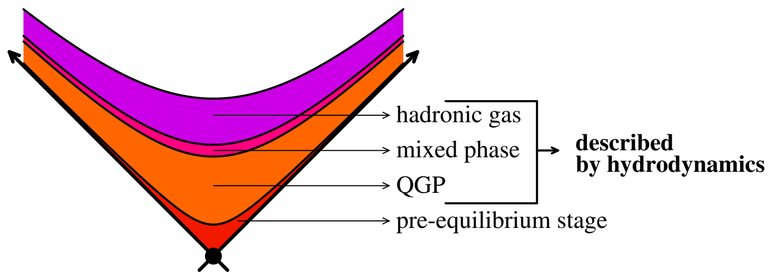

Fig. 1. Scenario for the quark–gluon plasma (QGP) formation. After a pre-equilibrium stage in a heavy-ion collision, probably governed by a weak-coupling but dense regime, the QGP is formed with local equilibrium and hydrodynamic properties.

Even if the experimental situation is still developing and rather complex, it is worth simplifying the problem in order to be able to attack it with appropriate theoretical tools. Recently the AdS/CFT correspondence [4, 5] emerged as a new approach to study strongly coupled gauge theories. This has been largely worked out in the supersymmetric case, in particular for the conformal case of $\mathcal{N} = 4$ super Yang–Mills theory (SYM). Interestingly enough, since the QGP is a deconfined and strongly interacting phase of QCD, we could expect that results for the nonconfining $\mathcal{N} = 4$ theory may be relevant or at least informative as far as the unknown strong coupling QCD problem is concerned. We will start from this assumption in our work.

In this lecture we focus on the spacetime evolution of the gauge theory (4d) energy-momentum tensor, and derive its asymptotic behaviour from the solutions of the nonlinear Einstein equations of the gravity dual.

Imposing the absence of curvature singularities in the gravity dual, we will show that, in the boost invariant setting (as in [3]), perfect fluid hydrodynamics emerges from the AdS/CFT solution at large times. The new material contained here comes from Refs. [6].

2. String/Gauge fields Duality

As an introduction to our lecture, let us briefly recall some aspects of the String/Gauge Duality. The AdS/CFT correspondence [4] has many interesting formal and physical facets. Concerning the aspects which are of interest for our problem, it allows one to find relations between gauge field theories at strong coupling and string gravity at weak coupling in the limit of large number of colours ($N_c \to \infty$). It can be examined quite precisely in the AdS_5/CFT_4 case which conformal field theory corresponds to $SU(N)$ gauge theory with $\mathcal{N}=4$ supersymmetries.

Let us recall the canonical derivation leading to the AdS_5 background, see Fig. 2. One starts from the (super)gravity classical solution of a system of N D_3-branes in a $10{-}d$ space of the (type IIB) superstrings. The metric solution of the (super)Einstein equations read

$$ds^2 = \frac{1}{\sqrt{f}}\left(-dt^2 + \sum_{1-3} dx_i^2\right) + \sqrt{f}(dr^2 + r^2 d\Omega_5)\,, \tag{1}$$

where the first four coordinates are on the brane and r corresponds to the coordinate along the normal to the branes. In formula (1), one defines

$$f = 1 + \frac{R^4}{r^4}\,; \qquad R = 4\pi g_{\mathrm{YM}}^2 \alpha'^2 N\,, \tag{2}$$

where $g_{\mathrm{YM}}^2 N$ is the 't Hooft–Yang-Mills coupling and α' the string tension. One considers the limiting behaviour considered by Maldacena, where one zooms on the neighbourhood of the branes while in the same time going to the limit of weak string slope α'. The near-by space-time is thus distorted

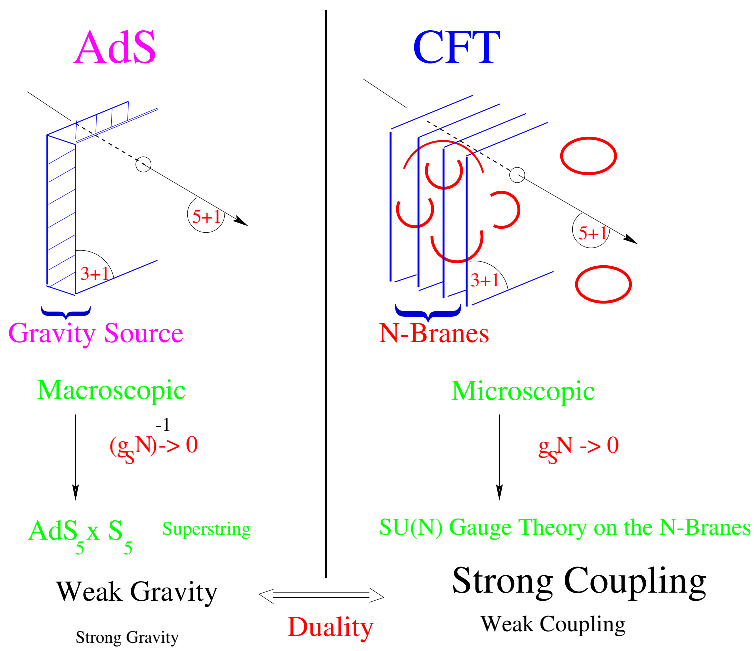

Fig. 2. Schematic view of the Gauge/String Duality. Left: The string background is the Anti-de Sitter space (AdS); Right: the gauge theory is a Conformal field theory (CFT) on the 4-d N-branes.

due to the (super) gravitational field of the branes. One goes to the limit where

$$R \text{ fixed}\,;\ \frac{\alpha'(\to 0)}{r(\to 0)} \to z \text{ fixed}\,. \tag{3}$$

This, from the second equation of (2) obviously implies

$$\alpha' \to 0\ ,\ g^2_{\rm YM} N \sim \frac{1}{\alpha'^2} \to \infty\,, \tag{4}$$

i.e. both a weak coupling limit for the string theory and a strong coupling limit for the dual gauge field theory. By reorganizing the two parts of the metrics one obtains

$$ds^2 = \frac{1}{z^2}\left(-dt^2 + \sum_{1-3} dx_i^2 + dz^2\right) + R^2 d\Omega_5\,, \tag{5}$$

which corresponds to the $\mathrm{AdS}_5 \times \mathrm{S}_5$ background structure, S_5 being the 5-sphere. More detailed analysis shows that the isometry group of the 5-sphere is the geometrical dual of the $\mathcal{N}=4$ supersymmetries. More intricate is the quantum number dual to $N_{\rm c}$, the number of colours, which is the invariant charge carried by the Ramond–Ramond form field.

3. Bjorken hydrodynamics

Coming back to the physical world, a model of the central rapidity region of heavy-ion reactions based on hydrodynamics was pioneered in [3] and involved the assumption of boost invariance. Our goal is to study the dynamics of strongly interacting gauge-theory matter assuming boost invariance.

We will be interested in the spacetime evolution of the energy-momentum tensor $T_{\mu\nu}$ of the gauge-theory matter. It is convenient to introduce the proper-time (τ) and space-rapidity (y) coordinates in the longitudinal position plane:

$$x^0 = \tau \cosh y \qquad\qquad x^1 = \tau \sinh y\,. \tag{6}$$

In these coordinates, all components of the energy momentum tensor can be expressed (see [6]) in terms of a *single* function $f(\tau)$:

$$T_{\mu\nu} = \begin{pmatrix} f(\tau) & 0 & 0 & 0 \\ 0 & -\tau^3 \frac{d}{d\tau} f(\tau) - \tau^2 f(\tau) & 0 & 0 \\ 0 & 0 & f(\tau) + \frac{1}{2}\tau \frac{d}{d\tau} f(\tau) & 0 \\ 0 & 0 & 0 & f(\tau) + \frac{1}{2}\tau \frac{d}{d\tau} f(\tau) \end{pmatrix} \tag{7}$$

where the matrix $T_{\mu\nu}$ is expressed in (τ, y, x_1, x_2) coordinates.

Furthermore the function $f(\tau)$ is constrained to verify

$$f(\tau) \geq 0\,, \qquad f'(\tau) \leq 0\,, \qquad \tau f'(\tau) \geq -4f(\tau)\,. \tag{8}$$

The dynamics of the gauge theory should pick a specific $f(\tau)$. A perfect fluid or a fluid with nonzero viscosity and/or other transport coefficients will lead to different choices of $f(\tau)$.

We thus address the problem of determination of the function $f(\tau)$ from the AdS/CFT correspondence. Let us first describe two distinct cases of physical interest:

For a perfect fluid (Bjorken hydrodynamics) $f(\tau) \sim 1/\tau^{\frac{4}{3}}$, while for a "free streaming case" expected just at the beginning of the interaction [11] , $f(\tau) \sim 1/\tau$. In the following we introduce a family of $f(\tau)$ with the large τ behaviour of the form

$$f(\tau) \sim \tau^{-s}. \tag{9}$$

4. Boost-invariant geometries

The most general form of the bulk metric respecting boost-invariance can be written

$$ds^2 = \frac{-e^{a(\tau,z)}d\tau^2 + \tau^2 e^{b(\tau,z)}dy^2 + e^{c(\tau,z)}dx_\perp^2}{z^2} + \frac{dz^2}{z^2}\;. \tag{10}$$

The three coefficient functions can be (non trivially) derived from the Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - 6\,g_{\mu\nu} = 0\;, \tag{11}$$

in the asymptotic limit where $\tau \to \infty$. Interestingly enough, they depend only on the scaling variable $v = z/\tau^{s/4}$, where s labels the one-parameter family of solutions (9).

After quite painful calculations, the solution reads [6]

$$a(v) = A(v) - 2m(v)\,, b(v) = A(v) + (2s-2)m(v)\,, c(v) = A(v) + (2-s)m(v)\,, \tag{12}$$

where

$$\begin{aligned} A(v) &= \frac{1}{2}\left(\log(1+\Delta(s)\,v^4) + \log(1-\Delta(s)\,v^4)\right) \\ m(v) &= \frac{1}{4\Delta(s)}\left(\log(1+\Delta(s)\,v^4) - \log(1-\Delta(s)\,v^4)\right) \end{aligned} \tag{13}$$

with

$$\Delta(s) = \sqrt{\frac{3s^2-8s+8}{24}}\,. \tag{14}$$

Specializing first to the perfect fluid case, this gives rise to the following *asymptotic* geometry

$$z^2\ ds^2 = -\frac{\left(1-\frac{e_0}{3}\frac{z^4}{\tau^{4/3}}\right)^2}{1+\frac{e_0}{3}\frac{z^4}{\tau^{4/3}}}d\tau^2 + \left(1+\frac{e_0}{3}\frac{z^4}{\tau^{4/3}}\right)(\tau^2 dy^2 + dx_\perp^2) + dz^2 \,. \quad (15)$$

Remarkably enough this geometry can be identified (in a suitable metric) to be a *moving* Black Hole, which evolves in the fifth dimension z.

For the free streaming case, one finds

$$\begin{aligned} z^2\ ds^2 = &- \left(1+\frac{v^4}{\sqrt{8}}\right)^{\frac{1-2\sqrt{2}}{2}}\left(1-\frac{v^4}{\sqrt{8}}\right)^{\frac{1+2\sqrt{2}}{2}} dt^2 \\ &+ \left(1+\frac{v^4}{\sqrt{8}}\right)^{\frac{1}{2}}\left(1-\frac{v^4}{\sqrt{8}}\right)^{\frac{1}{2}} \tau^2 dy^2 \\ &+ \left(1+\frac{v^4}{\sqrt{8}}\right)^{\frac{1+\sqrt{2}}{2}}\left(1-\frac{v^4}{\sqrt{8}}\right)^{\frac{1-\sqrt{2}}{2}} dx_\perp^2 + dz^2 \,, \end{aligned} \quad (16)$$

which is qualitatively different from the perfect fluid case, in particular it displays singularities or zeroes at $v^4 = \sqrt{8}$ in all coefficients. More generally, it is possible to show [6] that the perfect fluid case is the only one free of physical singularities, namely singularities which cannot be removed by a change of coordinates. In order to check this feature, we considered the metric-invariant curvature scalar

$$\mathfrak{R}^2 = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}\ . \quad (17)$$

As an illustration, we represent this property in Fig. 3, where the value of $\mathfrak{R}^2$ is studied as a function of the distance from the horizon, for s values at the perfect fluid point and very near-by values.

Let us add some comments on the specific features of our approach and results. We concentrate on looking for solutions of the full nonlinear Einstein equations. It would be interesting to confront this approach with the linearization methods of Refs. [7]. In particular viscosity terms are expected to appear in the study of subasymptotic terms Ref. [9]. Note that the possibility of black hole formation in the *dual* geometry has been argued in Ref. [8]. More specifically, the geometry of a brane moving w.r.t. a black hole background has been advocated in Ref. [10] for the dual description of the cooling and expansion of a quark–gluon plasma. In our case we could interpret the solution (15) as a kind of 'mirror' situation in terms of a black hole moving off from the AdS boundary.

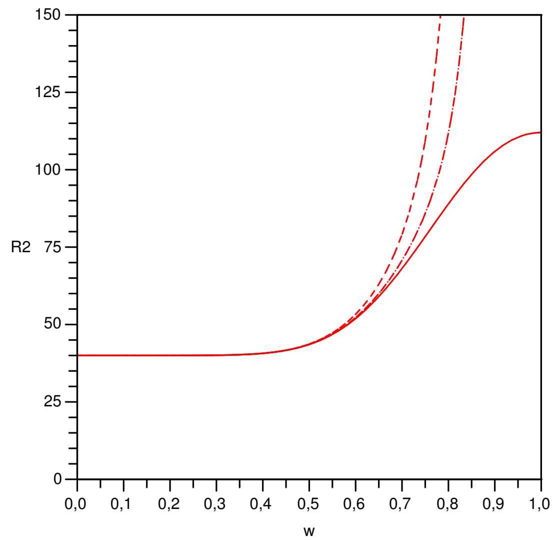

Fig. 3. The curvature scalar $\mathfrak{R}^2$. $\mathfrak{R}^2$ is calculated as a function of $w = v/\Delta(s)^{1/4}$ for the perfect fluid case $s = 4/3$ (solid line), $s = 4/3 - 0.1$ (dotted line) and $s = 4/3 + 0.2$ (dashed line).

5. Summary

We have introduced a general framework for studying the dynamics of matter (plasma) in strongly coupled gauge theory using the AdS/CFT correspondence for the $\mathcal{N} = 4$ SYM theory. We constructed dual geometries for given 4-dimensional gauge theory energy-momentum tensor profiles. Further imposing boost-invariant dynamics inspired by the Bjorken hydrodynamic picture, we have found the corresponding asymptotic solutions of the nonlinear Einstein equations. Among the family of asymptotic solutions, the only one with bounded curvature scalars is the gravity dual of a perfect fluid through its energy-momentum tensor profile. This selected nonsingular solution, given by the metric (15), corresponds to a black hole moving off in the 5th dimension as a function of the physical proper time. As an application of this framework, we can obtain [6] the thermalization time of the perfect fluid, which describes the decay back to equilibrium of a scalar excitation of the perfect fluid out of equilibrium, by computation of the quasi-normal modes of the moving Black Hole. In some sense, the moving Black Hole is a quite *stable* geometric configuration. We conjecture that it may represent, through the Gauge/Gravity duality, a powerful "attractor" for the QGP evolution, or even perhaps for more general evolution of a strongly coupled system of quarks and gluons *e.g.* in a high energy hadron–hadron reaction.

I warmly thank Romuald Janik for his major contribution in the fruitful collaboration whose results are discussed in this lecture.

REFERENCES

[1] P.F. Kolb, U.W. Heinz, nucl-th/0305084.

[2] see *e.g.* E.V. Shuryak, *Nucl. Phys.* **A750**, 64 (2005) [hep-ph/0405066].

[3] J.D. Bjorken, *Phys. Rev.* **D27**, 140 (1983). Note that the pionneering work of L.D. Landau on hydrodynamics for particle production, *On the multiparticle production in high-energy collisions*, *Izv. Akad. Nauk Ser. Fiz.* **17**, 51 (1953) did not lead to boost-invariant solutions.

[4] J.M. Maldacena, *Adv. Theor. Math. Phys.* **2**, 231 (1998) [*Int. J. Theor. Phys.* **38**, 1113 (1999)] [hep-th/9711200]; S.S. Gubser, I.R. Klebanov, A.M. Polyakov, *Phys. Lett.* **B428**, 105 (1998) [hep-th/9802109]; E. Witten, *Adv. Theor. Math. Phys.* **2**, 253 (1998) [hep-th/9802150].

[5] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri, Y. Oz, *Phys. Rep.* **323**, 183 (2000) [hep-th/9905111].

[6] R.A. Janik, R. Peschanski, *Phys. Rev.* **D73**, 045013 (2006) [hep-th/0512162]; *Phys. Rev.* **D74**, 046007 (2006) [hep-th/0606149].

[7] G. Policastro, D.T. Son, A.O. Starinets, *Phys. Rev. Lett.* **87**, 081601 (2001) [hep-th/0104066];

[8] H. Nastase, hep-th/0501068.

[9] R.A. Janik, hep-th/0610144.

[10] E. Shuryak, S.J. Sin, I. Zahed, hep-th/0511199.

[11] Y.V. Kovchegov, *Nucl. Phys.* **A774**, 869 (2006) [*Eur. Phys. J.* **A29**, 43 (2006)] [hep-ph/0510232].