ENERGY CONSERVATION AND POMERON LOOPS IN HIGH ENERGY EVOLUTION*

Emil Avsar

Department of Theoretical Physics, Lund University
Sölvegatan 14A, S-223 62 Lund, Sweden
emil@thep.lu.se

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We present a formalism which modifies the Mueller Dipole Model such that it incorporates energy-momentum conservation and also important colour suppressed effects. We implement our formalism in a Monte Carlo simulation and compare the results to inclusive data from HERA and the Tevatron, where we see that there is a good agreement between the data and our model.

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1. Introduction

A convenient approach to small-x evolution in QCD is the dipole picture which was formulated by Mueller a decade ago [1–3]. Mueller's formalism leads to the BFKL equation but it also goes beyond the BFKL formalism, since here it is possible to take into account unitarisation effects due to multiple scatterings (multiple pomeron exchanges). In Mueller's model dipoles in the same cascade (wavefunction) are, however, not allowed to interact and the model is therefore, valid only in a limited range in rapidity, where this type of saturation effects can be ignored. Saturation effects are included in the so called Colour Glass Condensate (CGC) formalism [4, 5], which is an effective theory for QCD valid at high gluon densities. The master equation within the CGC formalism is called the JIMWLK equation which in the weak field limit reduces to the BFKL equation.

In this paper we continue to develop our model, introduced in [6], by adding N_c suppressed effects in the dipole evolution. We also improve our proton model and use it to study inclusive data from $\gamma^* p$ and pp collisions.

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2. The Mueller dipole model

In Mueller's model we start with a $q\bar{q}$ pair, heavy enough for perturbative calculations to be applicable, and calculate the probability to emit a soft gluon from this pair. Here the quark and the antiquark are assumed to follow light-cone trajectories and the emission of the gluon is calculated in the eikonal approximation, in which the emitters do not suffer any recoil. Adding the contributions to the emission from the quark and the antiquark, including the interference, one obtains the result (for notations, see Fig. 1)

$$\frac{d\mathcal{P}}{dY} = \frac{\bar{\alpha}}{2\pi}\, d^2\boldsymbol{z} \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2} \equiv \frac{\bar{\alpha}}{2\pi}\, d^2\boldsymbol{z}\mathcal{M}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}). \tag{1}$$

Here $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$ are two-dimensional vectors in transverse coordinate space and $Y = \log(1/x)$ denotes the rapidity, which acts as the time variable in the evolution process.

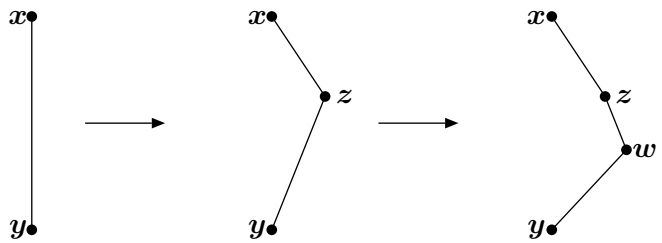

Fig. 1. The evolution of the dipole cascade. At each step, a dipole can split into two new dipoles with decay probability given by (1).

This formula can be interpreted as the emission probability from a dipole located at $(\boldsymbol{x},\boldsymbol{y})$. In the large N_c limit the gluon can be seen as a quark-antiquark pair and the formula above can then be interpreted as the decay of the original dipole $(\boldsymbol{x},\boldsymbol{y})$ into two new dipoles, $(\boldsymbol{x},\boldsymbol{z})$ and $(\boldsymbol{z},\boldsymbol{y})$. In the same limit further emissions factorize, so that at each step one has a chain of dipoles where each dipole can decay into two new dipoles with the decay probability given by (1). In this way one obtains a cascade of dipoles which evolve through dipole splittings, and the number of dipoles grows exponentially with Y.

3. Energy momentum conservation

We note that the expression above has non-integrable singularities at $\boldsymbol{z} = \boldsymbol{x}$ and $\boldsymbol{z} = \boldsymbol{y}$. In numerical calculations it is therefore necessary to introduce a cutoff, ρ, such that $(\boldsymbol{x}-\boldsymbol{z})^2, (\boldsymbol{z}-\boldsymbol{y})^2 \geq \rho^2$. Even though this cutoff does not show up in the cross section (the divergence is canceled by

virtual corrections, and $\sigma_{\rm tot}$ approaches a constant when $\rho \to 0$) it must still be kept in a Monte Carlo program. A small value of ρ, which is needed in order to simulate the physics with a good accuracy, will imply that we get very many small dipoles in the cascade.

A small dipole means that we have two well localized gluons in the transverse plane, and these gluons must then have a correspondingly large transverse momentum of the order of the inverse dipole size, $p_\perp \sim 1/r$. Thus if these small dipoles are interpreted as corresponding to real emissions with $p_\perp \sim 1/r$, then the diverging number of such dipoles would imply the violation of energy-momentum conservation. This suggest that these dipoles should be interpreted as virtual fluctuations, which means that the dipole cascade will not correspond to the production of exclusive final states.

In [6] we presented a formalism to take into account energy-momentum conservation in Mueller's model, using the Linked Dipole Chain (LDC) model [7] as a guidance. We used this formalism in a MC program and it was found that a result of energy-momentum conservation is that the number of dipoles grows much more slowly, and the onset of saturation is delayed. In fact it is found that in DIS the unitarity effects become quite small within the HERA energy regime.

An important consequence of energy-momentum conservation is that it implies a *dynamical cutoff*, $\rho(\Delta y)$, which is large for small steps in rapidity, Δy, but gets smaller for larger Δy. (Alternatively it can be described as a cutoff for Δy which depends on r. Here we want to emphasize that y is the true rapidity and not $\log(1/x)$.) Besides its physical effects, energy-momentum conservation also simplifies the MC treatment, since large numerical complications in a MC without energy conservation, as discussed in [8], are not present.

4. Evolution equations in high energy QCD and pomeron loops

We now consider a scattering process where we have a dipole (or a collection of dipoles) impinging on some arbitrary target. We denote the scattering amplitude for a single dipole by $\langle T \rangle$, and the scattering amplitude for k dipoles is for simplicity denoted by $\langle T^k \rangle$. The brackets here denote an averaging over different events. The high energy evolution equations for these amplitudes are determined by the so called Balitsky–JIMWLK (B–JIMWLK) equations [9–13]. In the large $N_{\rm c}$ limit, the equation for $\langle T^k \rangle$ can be written as

$$\partial_Y \langle T^k \rangle = \mathcal{K} \otimes \langle T^k \rangle - \mathcal{M} \otimes \langle T^{k+1} \rangle \,. \tag{2}$$

Here we see that we have an infinite set of coupled equations for the amplitudes. We also note that $\langle T^k \rangle$ gets contributions from all $\langle T^n \rangle$ with $n \geq k$.

If the rapidity increment is given to the projectile, the appearance of $\langle T^{k+1}\rangle$ in (2) simply means that one of the k dipoles can split into two new dipoles with probability density given by (1), and that both of these scatter off the target, so that there are in total $k+1$ dipoles interacting with the target. This term therefore, corresponds to pomeron splitting, since there is an exchange of $k+1$ pomerons instead of k pomerons. Thus dipole splittings in the projectile corresponds to pomeron splittings. On the other hand one can think that the target is evolved rather than the projectile. In this case the $\langle T^{k+1}\rangle$ contribution to (2) corresponds to pomeron merging inside the target, since there are only k dipoles in the projectile which couple to k pomerons.

We thus see that the B–JIMWLK equations describe either pomeron mergings, when the target is evolved, or pomeron splittings, in case the projectile is evolved, but not both. Ever since it was realized that the B–JIMWLK equations are not complete, there has been a lot of effort to construct a model which contains both pomeron mergings and splittings, and, through iterations, pomeron loops. This has been done in the large $N_{\rm c}$ limit [14,15] where the dipole model has been used to add pomeron splittings to the B–JIMWLK equations in the dilute region. The extension to the dense region is then obtained by simply adding the remaining terms arising from the large $N_{\rm c}$ version of the B–JIMWLK hierarchy. The main principle is that the two kinds of pomeron interactions (splittings and mergings) are important in different, well separated, kinematical regions.

The new equation for $\langle T^k\rangle$ receives a contribution also from $\langle T^{k-1}\rangle$ and it can be written as

$$\partial_Y \langle T^k\rangle = \mathcal{K}\otimes\langle T^k\rangle - \mathcal{M}\otimes\langle T^{k+1}\rangle + \mathscr{F}\otimes\langle T^{k-1}\rangle\,, \tag{3}$$

where $\mathscr{F}$ is a quite complicated expression describing the fluctuations in the target (or saturation effects in the projectile).

5. Finite $N_{\rm c}$ effects in dipole language

In the dipole model saturation effects are included in the collisions between two cascades, but not in the evolution of each cascade separately. Multiple collisions are formally colour suppressed, and in the Lorentz frame where the collisions are studied they lead to the formation of pomeron loops. As the evolution is only leading in $N_{\rm c}$ such loops cannot be formed during the evolution itself. If one *e.g.* studies the collision in a very asymmetric frame, where one of the onia carries almost all the available energy and the other is almost at rest, then the possibility to have multiple collisions is strongly reduced. Only those pomeron loops are included, which are cut in the specific Lorentz frame used for the calculations, which obviously only

forms a limited set of all possible pomeron loops. It implies that the dipole model is not frame independent, and the preferred Lorentz frame is the one where the two colliding systems have approximately the same density of dipoles. It is apparent that a frame independent formulation must include colour suppressed interactions between the dipoles during the evolution of the cascades, but so far it has not been possible to formulate such a model, which includes saturation effects and is explicitely frame independent[1].

As described above, equation (2) can be understood in terms of dipole evolution. The kernel of the evolution is simply given by the dipole kernel $\mathcal{M}$ in (1). Since pomeron splittings can be interpreted as dipole splittings it has been hoped that pomeron mergins also can be interpreted in terms of dipole reactions, such as dipole mergings, *i.e.* that one can interpret equation (3) in terms of a system of dipoles which can either split or merge. However, it was shown in [16] that this is not quite possible. Formally this can be done, but the problem is that the would-be dipole merging vertex has no fixed sign, as would have been required by a proper probabilistic formalism. Before going on we note that one can generate pomeron mergings without the process of dipole mergings. For example, any $2 \to n$ dipole vertex, where $n \geq 2$, also leads to pomeron mergings since there is always the possibility that only one of the new dipoles interact with the target, while the rest are spectators. Of course, such a transition leads to all possible $2 \to m$ ($m = 1, \ldots, n$) pomeron transitions.

We also want to emphasize that in our formalism the dipoles have a specified direction, going from colour to anticolour, and the cascade forms a connected and directed chain (see Fig. 2). In this picture one cannot simply take two arbitrary dipoles and merge them into one dipole (as this would not lead to an allowed colour structure), as opposed to the effective picture described above. Such an effective picture is adequate if one just wants to calculate the total inclusive cross section to leading order. However, since we demand energy-momentum conservation and have the ambition to include final state radiation and hadronization in future work, it is absolutely necessary to keep track of this orientation of the dipoles.

Another problem related to the finite number of colour charges is the over-completeness of the dipole basis. Beyond large N_c more complicated colour structures will appear, and the charges in the cascade cannot be represented in a unique way in terms of dipoles. The nonleading corrections in N_c make the colour structure of the gluon cascade really non-trivial, and one loses the picture of a system of dipoles evolving through dipole splittings in a stochastic process. However, it may still be possible to find a working approximation using only dipoles. If, for example, the charges of a quadrupole are well separated, then it is possible to view such a quadrupole

[1] Except for the toy model in which transverse dimensions are neglected.

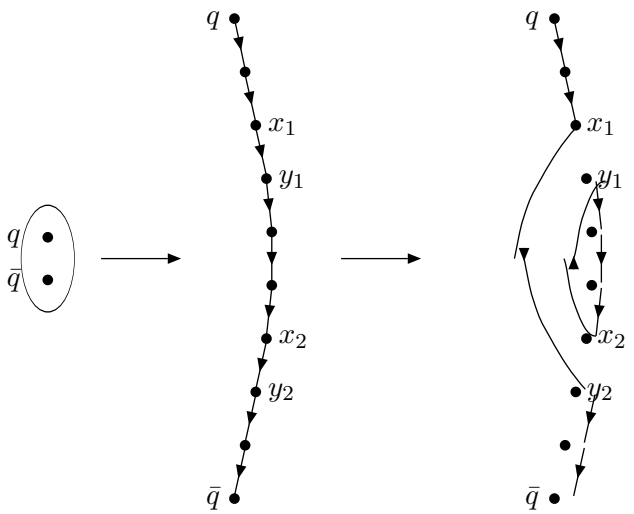

Fig. 2. Schematic picture of a colour recoupling, or dipole swing. The two dipoles $(\boldsymbol{x}_1, \boldsymbol{y}_1)$ and $(\boldsymbol{x}_2, \boldsymbol{y}_2)$ are transformed into two new dipoles $(\boldsymbol{x}_1, \boldsymbol{y}_2)$ and $(\boldsymbol{x}_2, \boldsymbol{y}_1)$ after a recoupling of the colour flow. The initial chain of dipoles is replaced by a new chain stretching between the original $q\bar{q}$ pair, and a loop of dipoles.

as two dipoles formed by the nearby colour charges. Such an approximative scheme can be realized by introducing a $2 \to 2$ transition vertex in the dipole evolution.

We choose this $2 \to 2$ vertex such that it favours the formation of dipoles by nearby colour charges. Initially we have two dipoles $(\boldsymbol{x}_1, \boldsymbol{y}_1)$ and $(\boldsymbol{x}_2, \boldsymbol{y}_2)$ which might then "swing" into two new dipoles, $(\boldsymbol{x}_1, \boldsymbol{y}_2)$ and $(\boldsymbol{x}_2, \boldsymbol{y}_1)$, as shown in Fig. 2. The weight for this process is chosen to be

$$\mathcal{W} = \frac{\lambda}{N_c^2 - 1} \frac{(\boldsymbol{x}_1 - \boldsymbol{y}_1)^2 (\boldsymbol{x}_2 - \boldsymbol{y}_2)^2}{(\boldsymbol{x}_1 - \boldsymbol{y}_2)^2 (\boldsymbol{x}_2 - \boldsymbol{y}_1)^2}\,, \tag{4}$$

where λ is a phenomenological parameter. Note that, with this choice of $\mathcal{W}$, the total weight for a dipole chain is $\propto \prod_i \frac{1}{r_i^2}$ where r_i is the size of the "remaining dipoles" in the cascade. Strictly speaking, if seen as a way to approximate more complicated colour structures, this process should happen instantly, rather than being proportional to dY. However, the value we choose for the phenomenological parameter, $\lambda = 1$, turn out be quite interesting because increasing it does not change the cross section [17], so that the $2 \to 2$ process can indeed be seen as happening instantly (for details see [17]). Actually this process could also be seen as the result of a gluon exchange between $(\boldsymbol{x}_1, \boldsymbol{y}_1)$ and $(\boldsymbol{x}_2, \boldsymbol{y}_2)$, which would also result in the same colour structure.

6. Results

In this section we present our results for $\gamma^* p$ and pp scattering, which we obtain from our MC. For now, we only calculate total cross sections and all calculations are performed using a running coupling constant α_s. In $\gamma^* p$ scattering the γ^* fluctuates into a $q\bar{q}$ pair, which initiates the dipole cascade. This process can be described using perturbative methods and the results for the γ^* longitudinal and transverse wave functions, $|\psi_L(z,r)|^2$ and $|\psi_T(z,r)|^2$ respectively, are well known. These wave functions give the probability for the γ^* to fluctuate into a $q\bar{q}$ pair, separated by transverse distance $\boldsymbol{r}$ and where the quark carries a fraction z of the γ^* longitudinal momentum.

The proton end of the cascade is more complicated. The easiest way to picture the proton would be to use a toy model which simply consists of a collection of uncorrelated dipoles. However, such a model fails to describe the energy dependence, and the impact parameter profile (which is not black enough at $\boldsymbol{b} = 0$ and has a too long tail for large $\boldsymbol{b}$), of the pp cross section and it is seen that the dipoles inside the proton need to be more tightly correlated. Our main assumption is that a proton at rest mainly consists of its three valence quarks. It is then natural to think that these quarks are the endpoints of three dipoles. Actually, one can try to calculate the emission probability of a soft gluon from these quarks. It turns out that they indeed form a system of three dipoles[2] [18] so that the topology of the system equals that of a triangle, see Fig. 3. The problem is however that further emissions do not factorize, even in the large N_c limit[3], as they do in the onium case. Nevertheless, as an approximation, we use the Δ topology for the initial system in the proton and we see that this works very well.

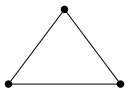

Fig. 3. The Δ shaped topology for the proton.

For the proton model described above we choose the initial dipole sizes to be distributed as Gaussians with average size $\sim 3.1\text{GeV}^{-1}$ as this value agrees best with data. Obviously confinement must somehow suppress the formation of too large dipoles, and it is natural to choose a maximum allowed dipole size. We choose this value to be the same as the average size of the initial dipoles in the proton as this corresponds to the nonperturbative input

[2] With the emission probability suppressed by a factor 2, since we have 3 dipoles formed by 3 quarks and not 3 gluons.

[3] I would like to thank M. Praszalowicz for pointing this out.

of the problem. Each new emission, giving a dipole of size r, is then allowed with a probability $\exp(-r^2/r_{\max}^2)$ where $r_{\max} = 3.1\,\mathrm{GeV}^{-1}$.

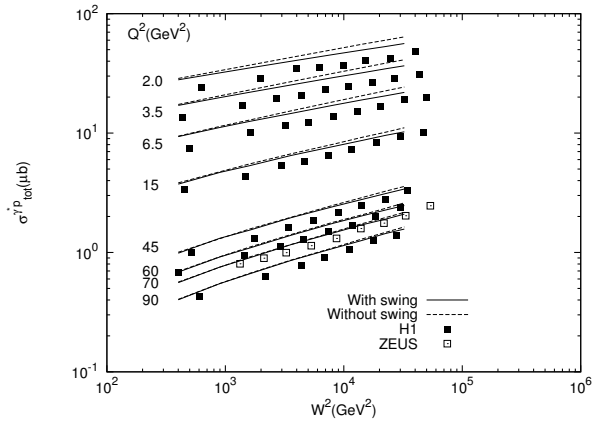

Fig. 4. The $\gamma^* p$ total cross section shown for different Q^2. The solid lines include the dipole swing while the dashed lines are without dipole swing. W denotes the cms energy. Data points are taken from [19, 20].

In Fig. 4 we show the results for the $\gamma^* p$ total cross section. As we can see the results are in quite good agreement with data except for the fact that the normalization is around 10–15% too high. We also see that the effects of dipole swing are quite small, visible for $Q^2 \lesssim 15\,\mathrm{GeV}^2$. In Fig. 5 we show the results for the logarithmic slope $\lambda_{\mathrm{eff}} = d\,(\log\sigma)/d\,(\log 1/x)$. We see that there is a good agreement with data for all points lying in the interval $1\mathrm{GeV}^2 \lesssim Q^2 \lesssim 100\ \mathrm{GeV}^2$. Here the slope is determined within the same energy interval from which the experimental points are determined.

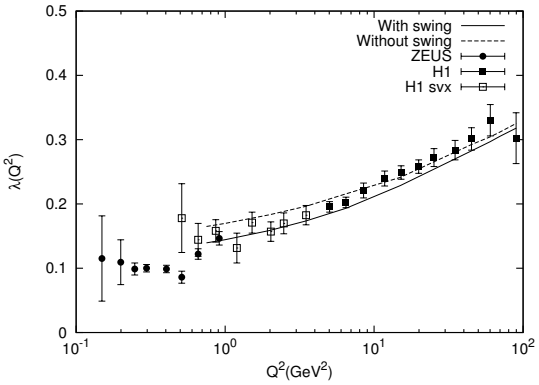

Fig. 5. The effective slope measured at different Q^2. The solid line is our result including the dipole swing while the dashed line is without. Filled circles are data from ZEUS [20] while filled [19] and open [21] squares are data from H1.

Fig. 6 shows the results for the *pp* total cross section. Here it is seen that the dipole swing has a rather large effect, as expected. Here we also show the results in the one pomeron approximation and one can see the large effects from unitarisation. These results are calculated in the center of mass frame where the colliding protons share the energy equally. In the figure we also show the result obtained in the "lab" frame where one of the protons carries almost all avaliable energy while the other one is essentially at rest. Due to the fact that the Monte Carlo simulation becomes very inefficient in such a frame (since the energetic proton has to be boosted to quite high rapidities) we have evaluated $\sigma_{\rm tot}$ only up to $\sqrt{s} = 1.8\,\text{TeV}$. Although the result is not exactly frame independent we see that the frame dependence is reduced, and now very small. It is our intention to try to find a formalism where frame independence is explicit. It also turns out that the impact parameter profile of the cross section agrees well with data [17].

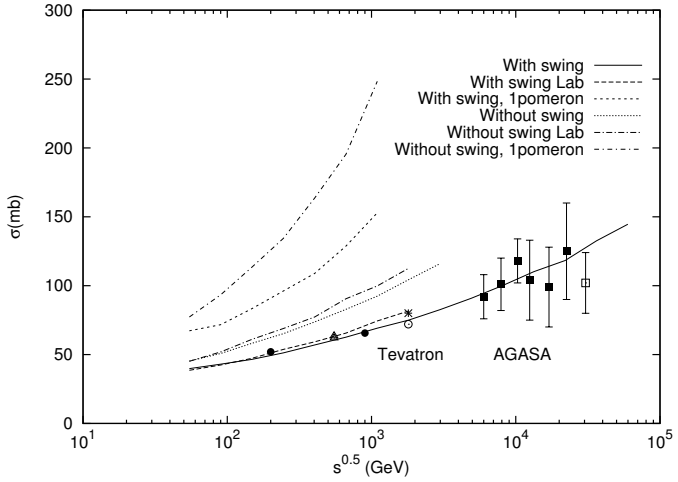

Fig. 6. The total cross section for *pp* scattering as a function of the cms energy $\sqrt{s}$. Here results are shown for evolution with and without the colour recouplings. The results for the one pomeron cross sections are also shown.

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REFERENCES

[1] A.H. Mueller, *Nucl. Phys.* **B415**, 373 (1994).

[2] A.H. Mueller, B. Patel, *Nucl. Phys.* **B425**, 471 (1994).

[3] A.H. Mueller, *Nucl. Phys.* **B437**, 107 (1995).

[4] E. Iancu, A. Leonidov, L. McLerran, `hep-ph/0202270`.

[5] E. Iancu, R. Venugopalan, `hep-ph/0303204`.

[6] E. Avsar, G. Gustafson, L. Lonnblad, *J. High Energy Phys.* **07**, 062 (2005).

[7] B. Andersson, G. Gustafson, J. Samuelsson, *Nucl. Phys.* **B467**, 443 (1996).

[8] G.P. Salam, *Nucl. Phys.* **B461**, 512 (1996).

[9] I. Balitsky, *Nucl. Phys.* **B463**, 99 (1996).

[10] J. Jalilian-Marian, A. Kovner, A. Leonidov, H. Weigert, *Nucl. Phys.* **B504**, 415 (1997).

[11] J. Jalilian-Marian, A. Kovner, A. Leonidov, H. Weigert, *Phys. Rev.* **D59**, 014014 (1999).

[12] E. Iancu, A. Leonidov, L.D. McLerran, *Phys. Lett.* **B510**, 133 (2001).

[13] H. Weigert, *Nucl. Phys.* **A703**, 823 (2002).

[14] E. Iancu, D.N. Triantafyllopoulos, *Nucl. Phys.* **A756**, 419 (2005).

[15] A.H. Mueller, A.I. Shoshi, S.M.H. Wong, *Nucl. Phys.* **B715**, 440 (2005).

[16] E. Iancu, G. Soyez, D.N. Triantafyllopoulos, *Nucl. Phys.* **A768**, 194 (2006).

[17] E. Avsar, G. Gustafsson, L. Lönnblad, `hep-ph/0610157`.

[18] M. Praszalowicz, A. Rostworowski, *Acta Phys. Pol. B* **29**, 745 (1998).

[19] H1 Collaboration, C. Adloff *et al.*, *Eur. Phys. J.* **C21**, 33 (2001).

[20] ZEUS Collaboration, J. Breitweg *et al.*, *Phys. Lett.* **B487**, 53 (2000).

[21] A. Petrukhin, prepared for 12th International Workshop on Deep Inelastic Scattering (DIS 2004), Strbske Pleso, Slovakia, April 14–18, 2004.