FINAL STATE OF HAWKING RADIATION IN QUANTUM GENERAL RELATIVITY*

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We use a new approach to the UV behavior of quantum general relativity, together with some recent results from the phenomenological asymptotic safety analysis of the theory, to discuss the final state of the Hawking radiation for an originally very massive black hole solution of Einstein's theory. We find that, after the black hole evaporates to the Planck mass size, its horizon is obviated by quantum loop effects, rendering the entire mass of the originally massive black hole accessible to our Universe.

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Given the many successes of Einstein's classical theory of general relativity [1–3], the fact that the only accepted complete treatment of quantum general relativity, superstring theory [4, 5], involves[1] many hitherto unseen degrees of freedom, some at masses well-beyond the Planck mass, is even more of an acute issue, as we have to wonder if such degrees of freedom are anything more than a mathematical artifact? The situation is reminiscent of the old string theory [7] of hadrons, which was ultimately superseded by the fundamental point particle field theory of QCD [8].

Accordingly, in the recent literature, several authors have attempted to apply well-tested methods from the Standard Model [8,9] (SM) physics arena to quantum gravitational physics: in Refs. [10], the famous low energy expansion technique from chiral perturbation theory for QCD has been used to address quantum gravitational effects in the large distance regime, in Refs. [11] renormalization group methods in curved space-time have been

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[1] Recently, the loop quantum gravity approach [6] has been advocated by several authors, but it has still unresolved theoretical issues of principle, unlike the superstring theory. Like the superstring theory, loop quantum gravity introduces a fundamental length, the Planck length, as the smallest distance in the theory. This is a basic modification of Einstein's theory.

used to address astrophysical and cosmological (low energy) effects and in Refs. [12–14] the asymptotic safety fixed-point approach of Weinberg [15] has been used to address the bad UV behavior of quantum general relativity whereas in Refs. [16–19] the new resummed quantum gravity approach (RQG) has also been used to address the bad UV behavior of quantum general relativity (QGR). The ultimate check on these developments, which are not mutually exclusive, will be the confrontation with experimental data. In this vein, we focus in the following on an important issue that arises when semi-classical arguments are applied to massive black hole solutions of Einstein's theory.

More precisely, Hawking [20] has pointed-out that a massive black hole emits thermal radiation with a temperature known as the Bekenstein–Hawking temperature [20, 21]. This result is well accepted by now. This raises the question as to what is the final state of the Hawking evaporation process? In Ref. [13], it was shown that an originally massive black hole emits Hawking radiation until its mass reaches a critical mass $M_{\rm cr} \sim M_{\rm Pl}$, at which the Bekenstein–Hawking temperature vanishes and the evaporation process stops. Here, $M_{\rm Pl}$ is the Planck mass, 1.22×10^{19} GeV. This would in principle leave a Planck scale remnant as the final state of the Hawking process.

Specifically, in Ref. [13], the running Newton constant was found to be

$$G(r) = \frac{G_{\rm N} r^3}{r^3 + \tilde{\omega} G_{\rm N} \left[r + \gamma G_{\rm N} M\right]} \tag{1}$$

for a central body of mass M where γ is a phenomenological parameter [13] satisfying $0 \leq \gamma \leq \frac{9}{2}$, $\tilde{\omega} = \frac{118}{15\pi}$ and $G_{\rm N}$ is the Newton constant at zero momentum transfer. The respective lapse function in the metric class

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2 d\Omega^2 \tag{2}$$

is then taken to be

$$f(r) = 1 - \frac{2G(r)M}{r} = \frac{B(x)}{B(x) + 2x^2}\Big|_{x=\frac{r}{G_{\rm N}M}}\,, \tag{3}$$

where

$$B(x) = x^3 - 2x^2 + \Omega x + \gamma\Omega \tag{4}$$

for

$$\Omega = \frac{\tilde{\omega}}{G_{\rm N} M^2} = \frac{\tilde{\omega} M_{\rm Pl}^2}{M^2}\,. \tag{5}$$

This leads to the conclusions that [13] for $M < M_{\rm cr}$ there is no horizon in the metric in the system and that for $M \downarrow M_{\rm cr}$ the Bekenstein–Hawking temperature vanishes, leaving a Planck scale remnant, where

$$M_{\rm cr} = \left[\frac{\tilde{\omega}}{\Omega_{\rm cr} G_{\rm N}}\right]^{\frac{1}{2}} \tag{6}$$

for

$$\Omega_{\rm cr} = \frac{1}{8}(9\gamma+2)\sqrt{\gamma+2}\sqrt{9\gamma+2} - \frac{27}{8}\gamma^2 - \frac{9}{2}\gamma + \frac{1}{2}\,. \tag{7}$$

For reference, we see that for the range $0 < \gamma < \frac{9}{2}$ from Ref. [13] we have $1 > \Omega_{\rm cr} \gtrsim .2$.

The source of these results can be seen to be the fixed-point behavior for the running Newton constant in momentum space found in Ref. [13],

$$G(k) = \frac{G_{\rm N}}{1 + \omega G_{\rm N} k^2}\,, \tag{8}$$

where $\omega \sim 1$ depends on the precise details of the IR momentum cut-off in the blocking procedure used in Ref. [13]. As we have shown in Refs. [16–18], our RQG theory gives the same fixed-point behavior for $G(k)$ so that we would naively conclude that we should have the same black hole physics phenomenology as that described above for Ref. [13]. Indeed, we have shown [17, 18] that for massive elementary particles, the classical conclusion that they should be black holes is obviated by our rigorous quantum loop effects, which do not contain any unknown phenomenological parameters. We note as well that the results in Ref. [22], obtained in a simple toy model using loop quantum gravity methods [6], also support the conclusion that, for masses below a critical value, black holes do not form; the authors in Ref. [22] are unable to specify the precise value of this critical mass.

However, as we have shown in Ref. [16–18], for elementary massive particles, and this Bonnano–Reuter Planck scale remnant would indeed be such a massive object with a mass smaller than many of the fundamental excitations in the superstring theory for example, quantum loop effects, resummed to all orders in $\kappa = \sqrt{8\pi G_{\rm N}}$, lead to the Newton potential

$$\Phi_{\rm N}(r) = -\frac{G_{\rm N} M_{\rm cr}}{r}(1 - e^{-ar})\,, \tag{9}$$

where the constant a depends on the masses of the fundamental particles in the Universe. We take here the latter particles to be those in the SM and its extension as suggested by the theory of electroweak symmetry breaking [23] and the theory of grand unification [24]. For the upper bound on a we use, we will not need to speculate about what particles may exist beyond those in the SM; and for the SM particles we use the known rest masses [25,26][2] as

[2] For the neutrinos, we use the estimate $m_\nu \sim 3$eV [27].

well as the value $m_H \cong 120$ GeV for the mass of the physical Higgs particle — the latter is known to be greater than 114.4 GeV with 95% CL [28].

More precisely, when the graphs Figs. 1 and 2 are computed in our resummed quantum gravity theory as presented in Refs. [16–18], the coefficient

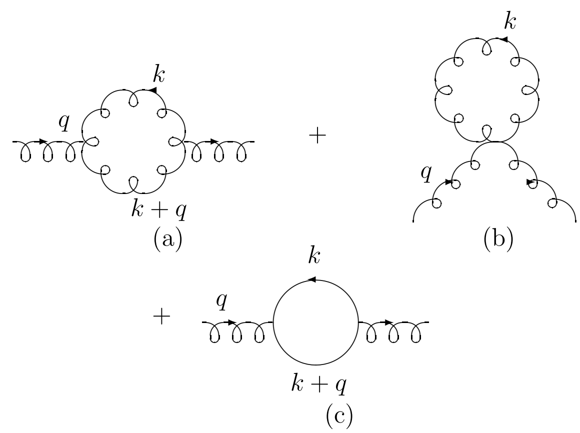

Fig. 1. The graviton ((a), (b)) and its ghost ((c)) one-loop contributions to the graviton propagator. q is the 4-momentum of the graviton.

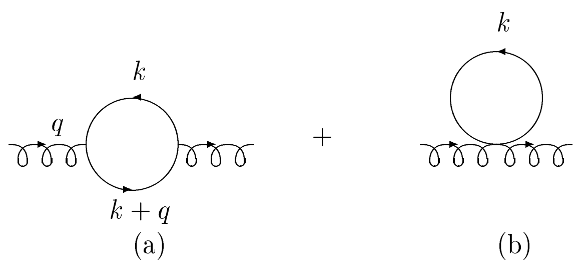

Fig. 2. The scalar one-loop contribution to the graviton propagator. q is the 4-momentum of the graviton.

$c_{2,\mathrm{eff}}$ in Eq. (12) of Ref. [18] becomes here, summing over the SM particles in the presence of the recently measured small cosmological constant [29], which implies the gravitational infrared cut-off of $m_g \cong 3.1 \times 10^{-33}$eV,

$$c_{2,\mathrm{eff}} = \sum_j n_j I(\lambda_c(j)) \,, \tag{10}$$

where we define [18] n_j as the effective number of degrees of freedom for particle j and the integral I is given by

$$I(\lambda_c) \cong \int_0^\infty dx x^3 (1+x)^{-4-\lambda_c x} \tag{11}$$

with the further definition $\lambda_c(j) = \frac{2m_j^2}{\pi M_{\rm Pl}^2}$ where the value of m_j is the rest mass of particle j when that is nonzero. When the rest mass of particle j is zero, the value of m_j turns-out to be [30] $\sqrt{2}$ times the gravitational infrared cut-off mass [29]. We further note that, from the exact one-loop analysis of Ref. [31], it also follows that the value of n_j for the graviton and its attendant ghost is 42. For $\lambda_c \to 0$, we have found the approximate representation

$$I(\lambda_c) \cong \ln\frac{1}{\lambda_c} - \ln\ln\frac{1}{\lambda_c} - \frac{\ln\ln\frac{1}{\lambda_c}}{\ln\frac{1}{\lambda_c} - \ln\ln\frac{1}{\lambda_c}} - \frac{11}{6}\,. \tag{12}$$

We wish to combine our result in (9) with the result for $G(r)$ in (1) from Ref. [13]. We do this by omitting from the $c_{2,\rm eff}$ the contributions from the graviton and its ghost, as these are presumably already taken into account in $G(r)$ in (1), and by replacing $G_{\rm N}$ in (9) with the running result $G(r)$ from (1). Thus our improved Newton potential reads

$$\Phi_{\rm N}(r) = -\frac{G(r)M_{\rm cr}}{r}(1-e^{-ar})\,, \tag{13}$$

where now, with

$$c_{2,\rm eff} \cong 1.41 \times 10^4 \tag{14}$$

and, from Eq. (8) in Ref. [18],

$$a \cong \left(\frac{360\pi M_{\rm Pl}^2}{c_{2,\rm eff}}\right)^{\frac{1}{2}} \tag{15}$$

we have that

$$a \cong 0.283 M_{\rm Pl}\,. \tag{16}$$

Since the result from Ref. [13] for $G(r)$ is based on analyzing the pure Einstein theory with no matter, it only contains the effects of pure gravity loops whereas, if we omit the graviton and its ghost loops from our result for $c_{2,\rm eff}$, our result for a in (13) only contains matter loops. Hence, there really is no double counting of effects in (13).

As we have explained elsewhere [18], if we use the connection between k and r that is employed in Ref. [13] and restrict our result for $c_{2,\rm eff}$ to pure graviton and its ghost loops, we recover the results of Ref. [13] for $G(k)$ and $G(r)$ with the similar value of the coefficient of k^2 in the denominator of $G(k)$, for example. Thus, we can arrive at our result in (13) independent of the exact renormalization group equation (ERGE) arguments in Ref. [13]. A more detailed version of such an analysis will appear elsewhere [30].

We also stress here that, when one uses the ERGE for a theory, one obtains the flow of the coupling parameters in the theory. To get to the exact S-matrix, and derive a formula for the Newton potential for example, one then has to employ the corresponding improved Feynman rules for example. Thus, in the analysis in Ref. [13] and in Ref. [32], which extends the ERGE analysis of Ref. [13] to include matter fields, one finds the results for the behavior of the couplings at the analyzed asymptotically safe fixed point. The corresponding computation of the S-matrix near the fixed point with the attendant improved running couplings is fully consistent with our results in (9), (13) [30].

At the critical value $M_{\rm cr}$, the function $B(x)+2x^2 = x^3+\Omega_{\rm cr}x+\gamma\Omega_{\rm cr}$ just equals $2x^2$ at $x = x_{\rm cr}$, producing there a double zero of $B(x)$ and of the lapse function $f(r) = 1+2\Phi_{\rm N}$. When we introduce our improvement into the lapse function via $G(r) \to G(r)(1 - e^{-ar})$, the effect is to reduce the size of the coefficient of $-2x^2$ in $B(x)$ to $-2\xi x^2$ where $\xi = \xi(x) = 1 - e^{-aG_{\rm N}M_{\rm cr}x} < 1$ and thereby to remove the double zero at $x_{\rm cr}$. The respective monotone behaviors of the polynomials $x^3 + \Omega_{\rm cr}x + \gamma\Omega_{\rm cr}$ and $2\xi(x_{\rm cr})x^2$ then allow us to conclude that the lapse function remains positive and does not vanish as $x \downarrow 0$, *i.e.*, our quantum loop effects have obviated the horizon of the would-be Planck scale remnant so that the entire mass of the would-be Planck scale remnant is made accessible to our Universe by our quantum loop effects. This result holds for all choices of the parameter γ in the range specified by Ref. [13].

We note the nature of the way the results in Ref. [13] and our result in (13) are to be combined: first one carries out the analysis in Ref. [13] and shows that the originally massive black hole evaporates by Hawking radiation down to the critical mass $M_{\rm cr}$; then, in this regime of masses, the Schwarzschild radius is in the Planck scale regime, wherein the calculation in (13) is applicable to show that the horizon at $M_{\rm cr}$ is in fact absent. One cannot simply use the result in (13) for all values of M because it is only valid in the deep UV. Above, we have used a step function at $x = x_{\rm cr}$ to turn-on our improvement for $x \leq x_{\rm cr}$.

This is still only a rather approximate way of combining our result in (13) and the result (3) of Ref. [13] and it leaves open the question as to the sensitivity of our conclusions to the nature of the approximation. In principle each result is a representation of the quantum loop effects on the lapse function if we interpret these effects in terms their manifestations on the effective value of Newton's constant as it has been done in Ref. [13]:

$$f(r) = 1 - \frac{2G_{\text{eff}}(r)M}{r}\,, \tag{17}$$

where we take either $G_{\text{eff}}(r)$ from (1) or we use (13) to get $G_{\text{eff}} = G_{\text{N}}(1 - e^{-ar})$ where now we must set $a = 0.210M_{\text{Pl}}$ to reflect the effects of pure gravity loops. The former choice is valid for very large r, so it applies to very massive black holes outside of their horizons whereas the latter choice should be applicable to the deep UV at or below the Planck scale. A better approximation is then, after the originally very massive black hole has, via the analysis of Ref. [13], Hawking radiated down to a size approaching the Planck size, to join the two continuously at some intermediate value of r by determining the outermost solution, $r_>$, of the equation

$$1 - \frac{2G(r)M}{r} = 1 - \frac{2G_{\text{N}}(1 - e^{-ar})M}{r}\,, \tag{18}$$

where $G(r)$ is given above by (1), and to use the RHS of the latter equation for $f(r)$ for $r < r_>$. For example, for $\Omega = 0.2$, we find $r_> \cong 27.1/M_{\text{Pl}}$, so that, whereas the result (3) would give an outer horizon at $x_+ \cong 1.89$ for $\gamma = 0$ [3], we get $x_+ \cong 1.15$ when we do this continuous combination; moreover, the inner horizon implied by (3) at $x_- \cong 0.106$ moves to negative values of x so that it ceases to exist. The Bekenstein–Hawking temperature in this continuous combination remains positive for all $x > 0$ because the two equations

$$\begin{aligned} 0 &= x - 2 + 2e^{-\frac{yx}{2}}, \\ 0 &= 1 - ye^{-\frac{yx}{2}}, \end{aligned} \tag{19}$$

where $y = 2\sigma/\Omega^{\frac{1}{2}}$ for $\sigma = (a/M_{\text{Pl}})\sqrt{\tilde{\omega}}$, require as well

$$1 = ye^{-(y-1)}. \tag{20}$$

The expression on the RHS of this latter equation has a maximum for $y \geq 0$ at $y = 1$ and this maximum is just 1, the constant on the LHS of the same equation. The only positive solution to (20) is then $y = 1$, or $\Omega = 4\sigma^2$. This corresponds to $x = 0$ in the (19), which contradicts the assumption therein that $x > 0$. Hence, we see that the outer horizon just approaches $x = 0$ for $\Omega \to 4\sigma^2 \equiv \Omega'_{\text{cr}}$ and that, at this point, the derivative of the lapse function is positive. The mass M'_{cr} implied by Ω'_{cr} is 2.38 M_{Pl}. In other words,

[3] We follow Ref. [13] and ask for self-consistency in the determination of γ and this leads us to the choice $\gamma = 0$ here.

originally massive black holes emit Hawking radiation until they reach the point $M'_{\rm cr} \sim M_{\rm Pl}$ at which their horizon vanishes, in complete agreement with our more approximate treatment above.

The intriguing question is that, after reaching the final mass $M'_{\rm cr}$, which is now made accessible by quantum loops, how will that mass manifest itself? Depending on the value of its baryon number, we can expect that there is non-zero probability for its decay into just two body final states, such as two nucleons, resulting in cosmic rays with energy $E = \frac{1}{2}M'_{\rm cr} \cong 1.2M_{\rm Pl}$. More complicated decays would populate the cosmic ray spectrum with energy $E < \frac{1}{2}M'_{\rm cr} \cong 1.2M_{\rm Pl}$. Such cosmic rays may help to explain the current data [33, 34] on cosmic rays with energies exceeding 10^{19}eV.

In sum, all of the mass of the originally very massive black hole is ultimately made accessible to our Universe by quantum loop effects. This conclusion agrees with some recent results by Hawking [35].

Note added

We point out that the map given in Ref. [13] for the phenomenological distance correlation for the respective infrared cut-off k is based on standard arguments from quantum mechanics and the parameter γ encodes a large part of the phenomenological aspects of that correlation. In Refs. [16–18], the variable k is the Fourier conjugate of the position 4-vector x so that the connection from function space of $\vec{k}$ space to that of $\vec{r}$ space is given by standard Fourier transformation with no phenomenological parameters,*i.e.*, the result in (9) does not have any sensitivity to parameters such as γ. This underscores the correctness of (13) and the main conclusion we draw from it: for all choices of γ, the Planck scale remnant has its horizon obviated by quantum loop effects.

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