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HIGH-ENERGY LIMIT OF NEUTRINO QUASIELASTIC CROSS SECTION

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It is a common knowledge that the quasielastic neutrino–neutron and antineutrino–proton cross sections tend to the same constant as (anti)neutrino energy becomes high. In this paper we calculate the exact expression of the limit in terms of the parameters describing quasielastic scattering. We check that even at very high energies only small absolute values of the four-momentum transfer contribute to the cross section, hence the Fermi theory can be applied. The dipole approximation of the form factors allows to perform analytic calculations. Obtained results are neutrino-flavour independent.

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1. Introduction

Quasielastic neutrino scattering plays a dominant role in neutrino–nucleon reactions at energies below 1 GeV. When neutrino energy increases another channels open and quasielastic processes become less important. At high energy the total cross section for neutrino scattering is approximately proportional to the value of the energy while the quasielastic cross section is roughly constant. The latter behaviour is known on the basis of numerical computations but as far as we know, it has not been shown analytically yet.

The quasielastic cross section is usually calculated within the Fermi theory. At low energies four-momentum transfer is understood to fulfil the condition $|q^2| \ll M_W^2$, where $M_W = 80.4\,\mathrm{GeV}$ is W boson mass. It will be shown in Sec. 3 that in fact even for very-high-energy neutrinos overwhelming contribution to the cross section satisfies such constraint, therefore the use of the Fermi theory is well justified.

Radiative corrections are not taken into account, but in Sec. 3 we estimate that they can be neglected.

In the theoretical description of the neutrino–nucleon interaction the hadronic current is expressed in terms of the four form factors due to Lorentz invariance and assumption that there are no second-class currents. The form factors can be expressed in various ways, see [1]. We consider dipole form factors because of their simplicity in analytic calculations.

The quasielastic cross section for neutrino–neutron scattering can be written as [2]

$$\sigma = \frac{M^2 G_{\mathrm{F}}^2 \cos^2\theta_C}{8\pi {E_\nu}^2} \int dq^2 \left[A(q^2) - B(q^2)\frac{(s-u)}{M^2} + C(q^2)\frac{(s-u)^2}{M^4} \right], \quad (1)$$

where $M = (m_n + m_p)/2$ is the average nucleon mass and

$$\begin{aligned} A(q^2) &= \frac{m_l^2 - q^2}{4M^2} \Bigg[|F_{\mathrm{A}}|^2 \left(4 - \frac{q^2}{M^2}\right) - \left|F_{\mathrm{V}}^1\right|^2 \left(4 + \frac{q^2}{M^2}\right) \\ &\quad - \frac{q^2}{M^2} \left|\xi F_{\mathrm{V}}^2\right|^2 \left(1 + \frac{q^2}{4M^2}\right) - \frac{4q^2}{M^2} \Re\big(F_{\mathrm{V}}^1 (\xi F_{\mathrm{V}}^2)^*\big) \\ &\quad - \frac{m_l^2}{M^2} \left(\left|F_{\mathrm{V}}^1 + \xi F_{\mathrm{V}}^2\right|^2 + |F_{\mathrm{A}}|^2 + 4\Re(F_{\mathrm{A}} F_{\mathrm{P}}^*) + \frac{q^2}{M^2} |F_{\mathrm{P}}|^2 \right) \Bigg], \\ B(q^2) &= -\frac{q^2}{M^2} \Re\left((F_{\mathrm{V}}^1 + \xi F_{\mathrm{V}}^2) F_{\mathrm{A}}^* \right), \\ C(q^2) &= \frac{1}{4} \Big(\left|F_{\mathrm{V}}^1\right|^2 - \frac{q^2}{4M^2} \left|\xi F_{\mathrm{V}}^2\right|^2 + |F_{\mathrm{A}}|^2 \Big). \end{aligned}$$

In above formulae m_l is charged-lepton mass, E_ν neutrino energy and $\xi = \mu_p - \mu_n - 1$, where μ_p and μ_n are the proton and neutron magnetic moments, respectively. In the case of antineutrino–proton scattering $-B(q^2)$ in Eq. (1) should be replaced by $+B(q^2)$. We also need to know the interval of integration $\big[(q^2)_A, (q^2)_B\big]$:

$$\begin{aligned} (q^2)_A &= \frac{m_l^2(E_\nu + M) - 2ME_\nu^2 - \sqrt{\Delta}}{2E_\nu + M}, \\ (q^2)_B &= \frac{m_l^4 M}{m_l^2(E_\nu + M) - 2ME_\nu^2 - \sqrt{\Delta}}, \end{aligned} \quad (2)$$

with $\Delta = (2ME_\nu^2 - m_l^2 E_\nu)^2 - 4m_l^2 M^2 E_\nu^2$.

As it was mentioned before, in this paper we will consider dipole form factors. Using the Sachs form factors

$$G_{\mathrm{E}}^{\mathrm{V}}(q^2) = \frac{1}{(1 - q^2/M_{\mathrm{V}}^2)^2}, \qquad G_{\mathrm{M}}^{\mathrm{V}}(q^2) = \frac{1+\xi}{(1 - q^2/M_{\mathrm{V}}^2)^2},$$

the vector form factors can be expressed in the following way:

$$F_{\rm V}^1(q^2) = \Big(1 - \frac{q^2}{4M^2}\Big)^{-1}\Big[G_{\rm E}^{\rm V}(q^2) - \frac{q^2}{4M^2}G_{\rm M}^{\rm V}(q^2)\Big]\,,$$

$$\xi F_{\rm V}^2(q^2) = \Big(1 - \frac{q^2}{4M^2}\Big)^{-1}\Big[-G_{\rm E}^{\rm V}(q^2) + G_{\rm M}^{\rm V}(q^2)\Big]\,,$$

whereas the pseudoscalar form factor $F_{\rm P}$ is related to the axial one due to PCAC hypothesis:

$$F_{\rm A}(q^2) = \frac{g_{\rm A}}{(1-q^2/M_{\rm A}^2)^2}\,, \qquad F_{\rm P}(q^2) = \frac{2M^2F_{\rm A}(q^2)}{m_\pi^2 - q^2}\,.$$

By m_π we denoted the pion mass.

2. High-energy limit

If neutrino energy E_ν is high enough to fulfil the condition $M^{\rm max}/E_\nu \ll 1$, where $M^{\rm max} = \max\{m_l, M, M_{\rm V}, M_{\rm A}\}$, one can write

$$\varDelta = 4M^2E_\nu^2\Big[E_\nu^2 - \frac{m_l^2}{M}E_\nu + \frac{m_l^4}{4M^2} - m_l^2\Big] \to 4M^2E_\nu^4\,,$$

what results in

$$\begin{aligned}(q^2)_A &\to -2ME_\nu\,,\\ (q^2)_B &\to 0\,.\end{aligned} \tag{3}$$

The cross section Eq. (1) is the sum of terms

$$\begin{aligned}\alpha &\doteq \frac{\mathcal{G}}{4E_\nu^2}\int dq^2 M^2A(q^2)\,,\\ -\beta &\doteq \frac{\mathcal{G}}{4E_\nu^2}\int dq^2 B(q^2)\,(s-u)\,,\\ \kappa &\doteq \frac{\mathcal{G}}{4E_\nu^2}\int dq^2 C(q^2)\frac{(s-u)^2}{M^2}\,,\end{aligned} \tag{4}$$

where we have introduced the compact notation for the constant factor

$$\mathcal{G} = \frac{G_{\rm F}^2\cos^2\theta_C}{2\pi}\,.$$

The first term, that is α, tends to zero as neutrino energy becomes infinite. We will show it in Appendix A.

Next, in Appendix B it is calculated directly that in the discussed limit β also approaches zero.

Thus only κ gives a nonzero contribution to the high-energy (anti)neutrino quasielastic cross section:

$$\sigma_\infty \doteq \lim_{E_\nu\to\infty} \sigma = \lim_{E_\nu\to\infty} \kappa\,.$$

Our main result can be written in the form

$$\sigma_\infty \;=\; \frac{G_{\mathrm{F}}^2\cos^2\theta_C}{6\pi}\Bigg[M_{\mathrm{V}}^2 + g_{\mathrm{A}}^2 M_{\mathrm{A}}^2 + \frac{2\xi(\xi+2)M_{\mathrm{V}}^4}{(4M^2-M_{\mathrm{V}}^2)^2}\left(M^2 - M_{\mathrm{V}}^2\right) + \frac{3\xi(\xi+2)M_{\mathrm{V}}^8}{(4M^2-M_{\mathrm{V}}^2)^3}\left(\frac{4M^2}{4M^2-M_{\mathrm{V}}^2}\ln\frac{4M^2}{M_{\mathrm{V}}^2} - 1\right)\Bigg].$$

Detailed calculations are presented in Appendix C. Introducing notation $\rho = 4M^2/M_{\mathrm{V}}^2$ and $\mu = \xi + 1$ we can write it in the more compact way:

$$\sigma_\infty \;=\; \frac{G_{\mathrm{F}}^2\cos^2\theta_C}{6\pi}\Bigg[M_{\mathrm{V}}^2 + g_{\mathrm{A}}^2 M_{\mathrm{A}}^2 + 2M^2\frac{\mu^2-1}{(\rho-1)^2}\left(1-\frac{4}{\rho}\right) + 3M_{\mathrm{V}}^2\frac{\mu^2-1}{(\rho-1)^3}\left(\frac{\rho}{\rho-1}\ln(\rho) - 1\right)\Bigg]. \tag{5}$$

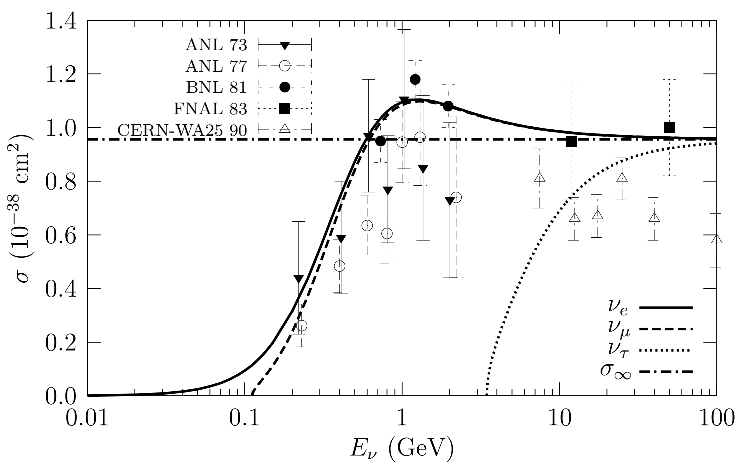

Fig. 1. The cross sections' dependence on neutrino energy. σ_∞ stands for the high-energy limit of σ calculated in this paper. Experimental data for quasielastic ν_μ scattering from D_2 target are taken from ANL 1973 [3], ANL 1977 [4], BNL 1981 [5], FNAL 1983 [6] and CERN–WA25 1990 [7].

The above expression does not depend on the charged-lepton mass, therefore, the $E_\nu \to \infty$ limit of the cross section is equal for all the neutrinos and antineutrinos, see also Fig. 1.

3. Discussion

To obtain numerical value of the limit we assume values of the constants for dipole form factors as in [1], see Table I. Note the corrected value of the axial mass: $M_{\rm A} = 1.001 \pm 0.020\,{\rm GeV}$. Then

$$\sigma_\infty = 0.956 \times 10^{-38}{\rm cm}^2\,.$$

We observe next that none of the four terms in Eq. (5) can be neglected. Contribution of the term with the axial form factor is equal to about 46%. The dependence of σ_∞ on the value of the axial mass is shown in Fig. 2.

TABLE I

The values of the constants used in numerical calculations.

$G_{\rm F}$	1.1803	$10^{-5}/{\rm GeV}^2$
$\cos\theta_C$	0.9740	
$g_{\rm A}$	-1.267	
ξ	3.7059	μ_N
$M_{\rm A}$	1.001	GeV
$M_{\rm V}^2$	0.71	${\rm GeV}^2$

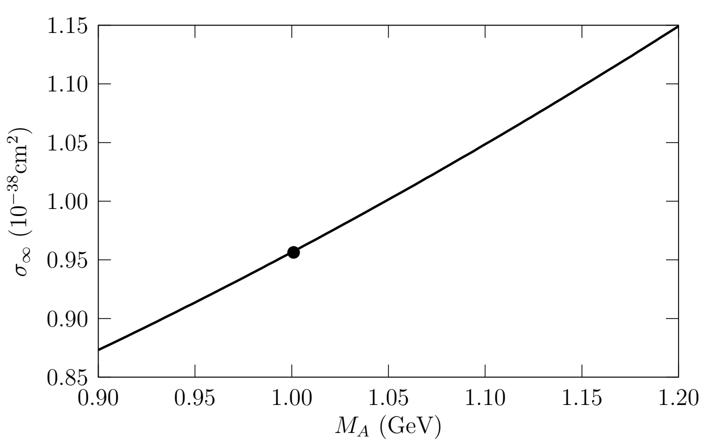

Fig. 2. The dependence of the high-energy limit of the cross section σ_∞ on the axial mass. Marked point represents the value of $M_{\rm A}$ as in [1].

It is necessary to check if our approach based on the Fermi theory is consistent. We do it numerically by computing the cross section with the W boson propagator σ_W and comparing the result with the cross section within the Fermi theory σ. Fig. 3 presents the dependence of the ratio

$$R = \frac{\sigma_W - \sigma}{\sigma_W}\,,$$

on neutrino energy. When $E_\nu \geq 50$ GeV the ratio R is roughly constant and less than 0.01% (for each flavour). It means that only small four-momentum transfers $|q^2|$ contribute to the quasielastic cross section, thus calculations within the Fermi theory are reasonable even for very high neutrino energies.

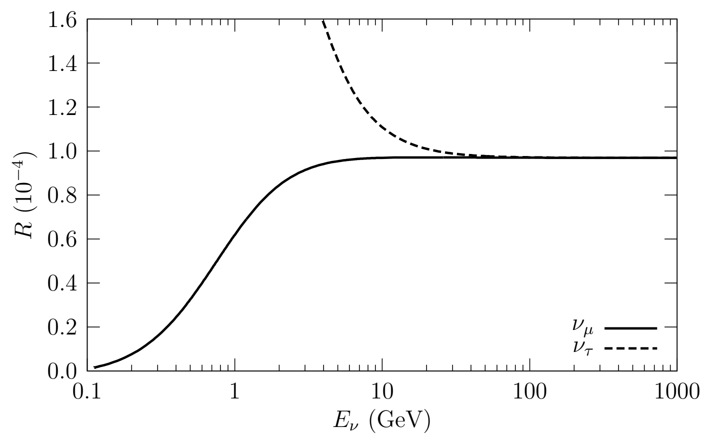

Fig. 3. The ratio of the difference between the cross section σ_W with the W boson propagator and the Fermi theory cross section normalised with respect to σ_W itself.

In calculations of the limit of the cross section no radiative corrections were taken into account. We guess that corrections to quasielastic scattering are of the same order of magnitude as to deep inelastic scattering, *i.e.* they are roughly constant and of the order of half a percent [8] (the value refers to the corrections which come from bremsstrahlung of the charged lepton, W boson and quarks). If the hypothesis is true, it makes them of low importance unless experiments reach very high precision.

More important improvements could come from the non-dipole form factors as in [1]. Presented there figures suggest that they would yield the value of the limit 3% smaller with respect to our result, but unfortunately "BBA-2003 Form Factors" are practically unapplicable to analytic calculations.

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Appendix A

Why α tends to zero

The α term defined in Eq. (4) is an integral of rational function of q^2 divided by neutrino energy squared. As $E_\nu \to \infty$, α would not tend to zero only if the integral rose at least as E_ν^2. The form of the limits Eq. (3) implies that the lower one always gives zero and only the upper one could produce nonzero terms, if the integrand is of the order of at least one in q^2. Let us write explicitly the term of the highest order for each form factor, keeping in memory that $F_{\rm P}$ can be expressed by $F_{\rm A}$:

$$\frac{1}{4}\Big(\frac{q^2}{M^2}\Big)^2|F_{\rm A}|^2 = \frac{g_{\rm A}^2 M_{\rm A}^8}{4M^4}\frac{(q^2)^2}{(M_{\rm A}^2-q^2)^4}\,,$$

$$\frac{1}{4}\Big(\frac{q^2}{M^2}\Big)^2|F_{\rm V}^1|^2 = \frac{M_{\rm V}^8}{4M^4}\frac{\big(q^2\big)^2}{\big(4M^2-q^2\big)^2}\frac{\big(4M^2-q^2(\xi+1)\big)^2}{\big(M_{\rm V}^2-q^2\big)^4}\,,$$

$$\frac{1}{16}\Big(\frac{q^2}{M^2}\Big)^3|\xi F_{\rm V}^2|^2 = \frac{\xi^2 M_{\rm V}^8}{M^2}\frac{1}{(4M^2-q^2)^2}\frac{(q^2)^3}{(M_{\rm V}^2-q^2)^4}\,.$$

We can see that each one of them is a proper fraction, so as neutrino energy becomes infinite α tends to zero. To illustrate this, let us perform the calculation for the second of above expressions. We can obtain easy-to-integrate form by decomposing it into partial fractions:

$$\frac{1}{4}\Big(\frac{q^2}{M^2}\Big)^2|F_{\rm V}^1|^2 = \frac{M_{\rm V}^8}{4M^4}\Big[\frac{c}{(4M^2-q^2)} - \frac{c}{(M_{\rm V}^2-q^2)} + \mathcal{O}(q^{-4})\Big]\,,$$

where c is a constant and $\mathcal{O}(q^{-4})$ denotes terms of lower order in q^2. As neutrino energy becomes high the limits of integration are given by Eq. (3) hence

$$\frac{1}{4}\int dq^2\Big(\frac{q^2}{M^2}\Big)^2|F_{\rm V}^1|^2 \to \frac{M_{\rm V}^8}{4M^4}\Big[2c\ln\frac{M_{\rm V}}{2M} + \text{other constants}\Big]\,.$$

The above integral tends to a constant as $E_\nu \to \infty$. Only higher order term in q^2 could give result increasing with E_ν but there is no such term in α. Since that for the whole expression holds true that

$$\alpha \to \frac{\text{const}}{E_\nu^2} \to 0\,.$$

Appendix B

Why β tends to zero

In the frame in which target nucleon is at rest $(s-u)=(4ME_\nu - m_l^2+q^2)$, so the quantity defined in Eq. (4) can be explicitly written as

$$\beta = \frac{\mu g_{\rm A}\mathcal{G}(M_{\rm A}M_{\rm V})^4}{4M^2E_\nu^2}\int dq^2\big(\mathcal{B}(4ME_\nu - m_l^2)+\mathcal{B}q^2\big)\,,$$

where

$$\mathcal{B} = \frac{q^2}{(M_{\rm A}^2-q^2)^2(M_{\rm V}^2-q^2)^2}\,.$$

To perform the integration one needs to decompose the integrand into partial fractions:

$$\mathcal{B} = \frac{1}{\mathcal{R}_{\rm A}^2}\left[\frac{M_{\rm A}^2}{(M_{\rm A}^2-q^2)^2}+\frac{M_{\rm V}^2}{(M_{\rm V}^2-q^2)^2}+\frac{M_{\rm A}^2+M_{\rm V}^2}{\mathcal{R}_{\rm A}}\Big(\frac{1}{M_{\rm A}^2-q^2}-\frac{1}{M_{\rm V}^2-q^2}\Big)\right],$$

$$\mathcal{B}q^2 = \frac{1}{\mathcal{R}_{\rm A}^2}\left[\frac{M_{\rm A}^4}{(M_{\rm A}^2-q^2)^2}+\frac{M_{\rm V}^4}{(M_{\rm V}^2-q^2)^2}+\frac{2M_{\rm A}^2M_{\rm V}^2}{\mathcal{R}_{\rm A}}\Big(\frac{1}{M_{\rm A}^2-q^2}-\frac{1}{M_{\rm V}^2-q^2}\Big)\right],$$

where $\mathcal{R}_{\rm A} = M_{\rm A}^2 - M_{\rm V}^2$. As $M^{\rm max}/E_\nu \ll 1$, after integrating in the limits Eq. (3) we obtain

$$\beta \to \frac{2\mu g_{\rm A}\mathcal{G}(M_{\rm A}M_{\rm V})^4}{ME_\nu\mathcal{R}_{\rm A}^2}\left(1+\frac{M_{\rm A}^2+M_{\rm V}^2}{\mathcal{R}_{\rm A}}\ln\frac{M_{\rm V}}{M_{\rm A}}\right)\to 0\,.$$

Appendix C

Why κ tends to constant

The last term in Eq. (4) expressed by the form factors is

$$\kappa = \frac{\mathcal{G}}{(4ME_\nu)^2}\int dq^2\Big[|F_{\rm V}^1|^2-\frac{q^2}{4M^2}|\xi F_{\rm V}^2|^2+|F_{\rm A}|^2\Big](s-u)^2\,.$$

For convenience we separate the axial part from the vector one:

$$\kappa_{\rm A} \doteq \frac{\mathcal{G}}{(4ME_\nu)^2}\int dq^2\,|F_{\rm A}|^2(s-u)^2\,,$$
$$\kappa_{\rm V} \doteq \frac{\mathcal{G}}{(4ME_\nu)^2}\int dq^2\Big[|F_{\rm V}^1|^2-\frac{q^2}{4M^2}|\xi F_{\rm V}^2|^2\Big](s-u)^2\,.$$

To evaluate the integral

$$\kappa_{\rm A} = \frac{g_{\rm A}^2 \mathcal{G} M_{\rm A}^8}{(4ME_\nu)^2} \int dq^2 \frac{(s-u)^2}{(M_{\rm A}^2 - q^2)^4}$$

one needs to know decomposition of the integrand. If we add $M_{\rm A}$ and $-M_{\rm A}$ to $(s-u)$ and square it in the following way

$$(s-u)^2 = (4ME_\nu - m_l^2 + M_{\rm A}^2)^2 - 2(4ME_\nu - m_l^2 + M_{\rm A}^2)(M_{\rm A}^2 - q^2) + (M_{\rm A}^2 - q^2)^2,$$

we will get

$$\frac{(s-u)^2}{(M_{\rm A}^2 - q^2)^4} = \frac{(4ME_\nu - m_l^2 + M_{\rm A}^2)^2}{(M_{\rm A}^2 - q^2)^4} - \frac{2(4ME_\nu - m_l^2 + M_{\rm A}^2)}{(M_{\rm A}^2 - q^2)^3} + \frac{1}{(M_{\rm A}^2 - q^2)^2}.$$

It means that as neutrino energy fulfils condition $M^{\max}/E_\nu \ll 1$, integration in the limits Eq. (3) leads to

$$\kappa_{\rm A} \to \frac{g_{\rm A}^2 \mathcal{G} M_{\rm A}^2}{3} \left(1 - \frac{2m_l^2 + M_{\rm A}^2}{4ME_\nu}\right)$$

and

$$\lim_{E_\nu \to \infty} \kappa_{\rm A} = \mathcal{G}\, \frac{g_{\rm A}^2 M_{\rm A}^2}{3}.$$

The integrand in definition of $\kappa_{\rm V}$, *i.e.*

$$|F_{\rm V}^1|^2 - \frac{q^2}{4M^2}|\xi F_{\rm V}^2|^2 = \left(1 - \frac{q^2}{4M^2}\right)^{-1} \left(1 - \frac{q^2}{M_{\rm V}^2}\right)^{-4} \left[\mu^2 \left(1 - \frac{q^2}{4M^2}\right) + 1 - \mu^2\right],$$

with $\mu = \xi + 1$, can be written as

$$|F_{\rm V}^1|^2 - \frac{q^2}{4M^2}|\xi F_{\rm V}^2|^2 = \frac{\mu^2 M_{\rm V}^8}{(M_{\rm V}^2 - q^2)^4} - \frac{4M^2 M_{\rm V}^8(\mu^2 - 1)}{(4M^2 - q^2)(M_{\rm V}^2 - q^2)^4}\,.$$

Let us denote the last-fraction's numerator as $\mathcal{K} = 4M^2 M_{\rm V}^8(\mu^2 - 1)$. Above expression decomposed into partial fractions is

$$|F_{\rm V}^1|^2 - \frac{q^2}{4M^2}|\xi F_{\rm V}^2|^2 = \frac{\mathcal{K}}{\mathcal{R}_{\rm V}^4}\left(\frac{1}{M_{\rm V}^2 - q^2} - \frac{1}{4M^2 - q^2}\right) + \frac{\mathcal{K}}{\mathcal{R}_{\rm V}^3 (M_{\rm V}^2 - q^2)^2} + \frac{\mathcal{K}}{\mathcal{R}_{\rm V}^2 (M_{\rm V}^2 - q^2)^3} + \frac{\mathcal{K}_\mu}{\mathcal{R}_{\rm V} (M_{\rm V}^2 - q^2)^4}\,,$$

where $\mathcal{R}_{\rm V} = 4M^2 - M_{\rm V}^2$ and $\mathcal{K}_\mu = M_{\rm V}^8(4M^2 - \mu^2 M_{\rm V}^2)$. By repeating the trick made during the computation of $\kappa_{\rm A}$ we obtain

$$\Big[|F_{\rm V}^1|^2 - \frac{q^2}{4M^2}|\xi F_{\rm V}^2|^2\Big](s-u)^2 = \frac{c_1}{M_{\rm V}^2 - q^2} - \frac{c_1}{4M^2 - q^2} + \frac{c_2}{(M_{\rm V}^2 - q^2)^2} + \frac{c_3}{(M_{\rm V}^2 - q^2)^3} + \frac{c_4}{(M_{\rm V}^2 - q^2)^4}\,,$$

where coefficients are:

$$\begin{aligned}
c_1 &= \frac{\mathcal{K}}{\mathcal{R}_{\rm V}^4}\left(4ME_\nu - m_l^2 + 4M^2\right)^2,\\
c_2 &= \frac{\mathcal{K}}{\mathcal{R}_{\rm V}^3}\Big[\frac{\mu^2 M_{\rm V}^8 \mathcal{R}_{\rm V}^3}{\mathcal{K}} - (4ME_\nu - m_l^2 + 4M^2)^2\Big],\\
c_3 &= \frac{1}{\mathcal{R}_{\rm V}^2}\Big[\mathcal{K}\Big(4ME_\nu - m_l^2 + M_{\rm V}^2 - \frac{\mathcal{K}_\mu \mathcal{R}_{\rm V}}{\mathcal{K}}\Big)^2 - \frac{(\mathcal{K}_\mu \mathcal{R}_{\rm V})^2}{\mathcal{K}}\Big],\\
c_4 &= \frac{\mathcal{K}_\mu}{\mathcal{R}_{\rm V}}\left(4ME_\nu - m_l^2 + M_{\rm V}^2\right)^2.
\end{aligned}$$

For neutrino energy $E_\nu \gg M^{\rm max}$, we conclude that integration over (dq^2) leads to

$$\begin{aligned}
\frac{\mathcal{G}}{(4ME_\nu)^2}\int dq^2\Big(\frac{c_1}{M_{\rm V}^2 - q^2} - \frac{c_1}{4M^2 - q^2}\Big) &\to \frac{\mathcal{G}\mathcal{K}}{\mathcal{R}_{\rm V}^4}\ln\frac{4M^2}{M_{\rm V}^2}\Big(1 + \frac{4M^2 - m_l^2}{2ME_\nu}\Big),\\
\frac{\mathcal{G}}{(4ME_\nu)^2}\int dq^2 \frac{c_2}{(M_{\rm V}^2 - q^2)^2} &\to -\frac{\mathcal{G}\mathcal{K}}{M_{\rm V}^2\mathcal{R}_{\rm V}^3}\Big(1 + \frac{4M^2 - m_l^2}{2ME_\nu}\Big),\\
\frac{\mathcal{G}}{(4ME_\nu)^2}\int dq^2 \frac{c_3}{(M_{\rm V}^2 - q^2)^3} &\to \frac{\mathcal{G}}{2M_{\rm V}^4\mathcal{R}_{\rm V}^2}\Big(\mathcal{K} + \frac{\mathcal{K}(M_{\rm V}^2 - m_l^2) - \mathcal{K}_\mu \mathcal{R}_{\rm V}}{2ME_\nu}\Big),\\
\frac{\mathcal{G}}{(4ME_\nu)^2}\int dq^2 \frac{c_4}{(M_{\rm V}^2 - q^2)^4} &\to \frac{\mathcal{G}\mathcal{K}_\mu}{3M_{\rm V}^6\mathcal{R}_{\rm V}}\Big(1 + \frac{M_{\rm V}^2 - m_l^2}{2ME_\nu}\Big).
\end{aligned}$$

The κ term is the sum of $\kappa_{\rm A}$ and $\kappa_{\rm V}$, therefore

$$\lim_{E_\nu\to\infty}\kappa = \mathcal{G}\,\frac{g_{\rm A}^2 M_{\rm A}^2}{3} + \frac{\mathcal{G}}{3\mathcal{R}_{\rm V}}\left[\frac{3\mathcal{K}}{\mathcal{R}_{\rm V}^3}\ln\frac{4M^2}{M_{\rm V}^2} - \frac{3\mathcal{K}}{M_{\rm V}^2\mathcal{R}_{\rm V}^2} + \frac{3\mathcal{K}}{2M_{\rm V}^4\mathcal{R}_{\rm V}} + \frac{\mathcal{K}_\mu}{M_{\rm V}^6}\right].$$

Recall that $\mathcal{K}_\mu = M_{\rm V}^8(4M^2 - \mu^2 M_{\rm V}^2)$ and $\mathcal{R}_{\rm V} = 4M^2 - M_{\rm V}^2$, hence we obtain

$$\frac{\mathcal{K}_\mu}{M_{\rm V}^6} = M_{\rm V}^2\big(4M^2 - \mu^2 M_{\rm V}^2\big) = M_{\rm V}^2\mathcal{R}_{\rm V} - (\mu^2 - 1)M_{\rm V}^4\,.$$

Next, constant factor $\mathcal{K} = 4M^2 M_{\rm V}^8(\mu^2-1)$, so

$$\frac{\mathcal{K}_\mu}{M_{\rm V}^6} + \frac{3\mathcal{K}}{2M_{\rm V}^4\mathcal{R}_{\rm V}} = M_{\rm V}^2\mathcal{R}_{\rm V} + (\mu^2-1)M_{\rm V}^4\frac{2(M^2-M_{\rm V}^2)+3M_{\rm V}^2}{\mathcal{R}_{\rm V}}\,.$$

It means that the limit of the cross section is equal to

$$\lim_{E_\nu\to\infty}\sigma = \frac{G_{\rm F}^2\cos^2\theta_C}{6\pi}\Bigg[M_{\rm V}^2 + g_{\rm A}^2M_{\rm A}^2 + \frac{2(\mu^2-1)M_{\rm V}^4}{(4M^2-M_{\rm V}^2)^2}(M^2-M_{\rm V}^2) + \frac{3(\mu^2-1)M_{\rm V}^8}{(4M^2-M_{\rm V}^2)^3}\left(\frac{4M^2}{4M^2-M_{\rm V}^2}\ln\frac{4M^2}{M_{\rm V}^2}-1\right)\Bigg].$$

Denoting $\rho = 4M^2/M_{\rm V}^2$ we can write this formula in the following way:

$$\lim_{E_\nu\to\infty}\sigma = \frac{G_{\rm F}^2\cos^2\theta_C}{6\pi}\Bigg[M_{\rm V}^2 + g_{\rm A}^2M_{\rm A}^2 + 2M^2\frac{\mu^2-1}{(\rho-1)^2}\left(1-\frac{4}{\rho}\right) + 3M_{\rm V}^2\frac{\mu^2-1}{(\rho-1)^3}\left(\frac{\rho}{\rho-1}\ln(\rho)-1\right)\Bigg].$$

REFERENCES

[1] H. Budd, A. Bodek, J. Arrington, *Modeling Quasi-elastic Form Factors for Electron and Neutrino Scattering*, presented at NuInt02, Irvine, CA, USA, `hep-ex/0308005`.

[2] C.H. Llewellyn Smith, *Phys. Rep.* **3**, 261 (1972).

[3] W.A. Mann *et al.*, *Phys. Rev. Lett.* **31**, 844 (1973).

[4] S.J. Barish *et al.*, *Phys. Rev.* **D16**, 3103 (1977).

[5] N.J. Baker *et al.*, *Phys. Rev.* **D23**, 2499 (1981).

[6] T. Kitagaki *et al.*, *Phys. Rev.* **D28**, 436 (1983).

[7] D. Allasia *et al.*, *Nucl. Phys.* **B343**, 285 (1990).

[8] G. Sigl, *Phys. Rev.* **D57**, 3786 (1998).