Vol. 37 (2006) *ACTA PHYSICA POLONICA B* No 2

SPECIAL RELATIVITY IN DECAYS OF HYBRIDS

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(Received November 4, 2005; revised version received December 22, 2005)

A decay of a heavy hybrid is expected to produce light mesons flying out with speeds comparable to the speed of light and phenomenological models of the decay must respect symmetries of special relativity. We study consequences of this requirement in a class of simple constituent models with spin. Our models respect boost symmetry because they conform to the rules of a boost-invariant renormalization group procedure for effective particles in light-front QCD. But rotational symmetry of the decay amplitude is not guaranteed and the parameters in the model wave functions must take special values in order to obtain the symmetry. When the effective interaction Hamiltonian responsible for a hybrid decay has the same structure as the gluon–quark–antiquark interaction term obtained by solving the renormalization group equations for Hamiltonians in first order perturbation theory, the non-relativistic image of a hybrid as built from a quark and an antiquark and a heavy gluon that typically resides between the quarks, cannot produce rotationally symmetric amplitude. However, there exists an alternative generic picture in the model that does satisfy the requirements of special relativity. Namely, the distance between the quark and antiquark must be much smaller than the distance between the gluon and the pair of quarks, as if a hybrid were similar to a gluonium in which one gluon is replaced by a quark–antiquark pair.

PACS numbers: 13.25.Jx, 11.80.–m, 13.90.+i

1. Introduction

Special relativity symmetry imposes severe constraints on the constituent picture of decays of hybrids. When one attempts to construct a constituent model of hybrids based on the weak-coupling expansion in QCD [1], treated as a potential candidate to complement the strong-coupling lattice picture [2,3], the parameters of the model cannot be treated as independent. In principle, QCD contains a small number of free parameters: quark masses, a coupling constant, and a scale parameter at which the masses and coupling are specified. In perturbation theory, the coupling depends on the scale through the ratio of that scale to $\Lambda_{\rm QCD}$, the latter being adjusted in the scheme in which the constituent picture is being considered. However, not every scheme can clearly define the concept of constituents. It is particularly hard to describe constituent particles in the region of small virtualities where binding mechanism is at work. In fact, the constituent dynamics is so difficult to derive in QCD and solve precisely, especially for light quarks and gluons, that no clear quantum-mechanical picture in Minkowski space has been derived yet despite extensive studies. Thus, phenomenological images are based on models and such models contain additional parameters due to arbitrary simplifying assumptions that are not under control by a precise theory.

Nevertheless, a phenomenological model constructed within a well-defined theoretical framework may help to finesse the leading approximation through agreement with data. Namely, if QCD is correct and does not require changes to precisely describe data, a model may hint at the structure that one should look for when attempting to solve the theory in a sequence of successive approximations. More importantly, data can provide constraints on models not only through discrete sets of numbers that correspond to the magnitudes of the considered quantities, but also through continuous symmetries. The chief example is the Lorentz symmetry that includes boosts and rotations. The symmetry dictates the shape of functional dependence of a decay amplitude on the coordinates used to describe the outgoing particles. We show how this dictum works in a simple model.

In the standard dynamical approach, rotational symmetry is kinematic (independent of interactions) and boost symmetry is dynamical (a change of frame of reference involves effects caused by interactions, such as a change in the number of constituents). Thus, in the standard approach, a model of a decay of a hybrid (or any other hadron) constructed in the center-of-mass frame of reference (CMF) of the hadron can easily respect the kinematical rotational symmetry in that frame of reference. However, it is not clear in the models that are based on the standard approach how well they respect the dynamical boost symmetry [4]. The boost symmetry would be very useful for checking the validity of all kinds of constituent models since QCD is supposed to yield only fully relativistic answers [5, 6]. Also, when one models a light meson in its own CMF as built from a fixed number of constituents and then uses the same constituent picture when the meson is moving with a speed close to the speed of light, which happens when a light meson is a product in a decay of a hybrid, one has to verify if the model satisfies constraints that the boost symmetry imposes. But the constraints imposed by boost symmetry are hard to satisfy in constituent models in the standard dynamics because boost generators in the standard dynamics depend on interactions and change the number of constituents.

In the light-front (LF) approach discussed here [7,8], boost symmetry is kinematical and it does not involve a change in the number of constituents no matter how fast a meson or a hybrid are moving. Instead, the rotational symmetry is dynamical. Therefore, if one wants to keep a fixed number of constituents, or when one seeks a physical picture in which the contributions from basis states with different than the leading number of constituents are small, it is the rotational symmetry of decay amplitudes (instead of boost symmetry in the standard approach) that begins to impose stringent constraints on the LF wave functions that stand a chance to approximate solutions to QCD, if some constituent picture is actually valid in the theory. Since the group of rotations is compact (the group of boosts is not) and it is already well-understood in non-relativistic quantum mechanics, the LF approach provides an opportunity for studying dynamical constraints of special relativity in an intuitively familiar way. The rotational symmetry is a much more familiar concept of quantum theory than the boost symmetry. This is reflected in the fact that the latter symmetry is rarely discussed in terms of constituents in the context of QCD. But using the LF approach, one can gain some insight concerning structures that may emerge from a relativistic dynamical quantum theory by checking if a set of wave functions chosen in a model can provide a rotationally symmetric decay amplitude of a hybrid. We interpret our findings concerning hybrids in the LF approach from the point of view of models of non-hybrid mesons.

Section 2 discusses theoretical background for the present study. Our model assumptions are described in Section 3. Numerical results obtained from the symmetry constraints on the decay amplitude of a hybrid are described in Section 4 and discussed in Section 5. Section 6 contains our conclusions regarding the constituent structure of hybrids.

2. Theoretical background

A constituent model of a hybrid qualifies as a theoretically reasonable one when it is clear, at least in principle, how the constituents used in the model can be related to quarks and gluons in QCD within a single formulation of the theory. Of course, it is possible that QCD will be eventually solved and precise comparison with data will demonstrate that the theory requires some changes of currently unknown nature and implications [3]. Before this happens, however, models of hybrid decays can be regarded as reasonable from the theoretical point of view if they are designed in agreement with some framework for solving QCD. But since exact solutions are missing and the constituent picture for hadrons continues to pose major conceptual problems (there is no dynamical explanation in QCD of the phenomenological success of the constituent quark model, examples of studies can be found in [9,10]), there exists today a considerable room for varying parameters and adjusting them to data even in reasonable models. This status of theoretically reasonable models of hadrons will continue until QCD is solved precisely enough to derive the model pictures and remove the freedom in choosing their parameters. One should also keep in mind that a model may be suitable for representing a theory for one type of data and not so reasonable for another type. There is an exception to this ambiguity: all theoretically reasonable models of hadrons must respect symmetries of special relativity. The alternative is to explain why the constraints of that symmetry can be ignored.

The requirement of Lorentz symmetry imposes constraints on the wave function of the hybrid as soon as one has a candidate for the dynamical mechanism of the decay. This mechanism should also be derived from QCD. Unfortunately, constituent dynamics is not understood yet in terms of the theory. In the LF QCD, one can apply the renormalization group procedure for effective particles (RGPEP, see *e.g.* [11] which shows the method in a considerably simpler case of heavy quarkonia, rather than the very difficult case of hybrids) and derive interactions of constituent quarks and gluons order by order in perturbation theory assuming an extremely small coupling constant, as if $\Lambda_{\rm QCD}$ were much smaller than it actually is. Certainly, when $\Lambda_{\rm QCD}$ is set to a realistic value, and when the appropriate renormalization group parameter is lowered down to the scale of the binding mechanism, the asymptotically free coupling constant increases to values for which the formal perturbative expansion in powers of the coupling constant is not expected to work well. But numerical tests in asymptotically free models with bound states [12] suggest that perturbation theory in RGPEP may be able to identify parts of the structure of the interaction terms in effective Hamiltonians that dominate in the bound-state dynamics, using as small a coupling constant as one wishes in the process. When a structure is already found, the main effect is the increase of the coupling constant in front of the identified structure [12]. This is true provided that one does not lower the renormalization group parameter in RGPEP too much and the calculation of the effective interaction is not cutting into the mechanism of formation of the bound states of interest.

The first term relevant to hybrid decay that one derives in perturbation theory is the term in which an effective gluon turns into a pair of an effective quark and an effective antiquark. Therefore, we assume here that the interaction that leads to the decay of a hybrid is that of a constituent gluon decaying into a pair of a constituent quark and antiquark. The constituents are identified with effective particles in RGPEP. According to RGPEP, the relevant structure of the interaction is exactly like in the canonical Hamiltonian for LF QCD except for two features. One feature is that the annihilation operator for the effective gluon, the creation operator for the effective quark, and the creation operator for the effective antiquark, all correspond to a small renormalization group scale λ just above the scale where the binding mechanism is active. Another feature is that the interaction vertex contains a form factor f_λ, instead of being local as in the canonical theory that one starts with. The effective interaction term is denoted by $H_{\mathrm{I}\lambda}$.

Once the interaction term $H_{\mathrm{I}\lambda}$ responsible for the decay of a hybrid state, denoted by $|h\rangle$, into two mesons, denoted by $|p\rangle$ and $|b\rangle$ (the letter p is chosen for a light meson, like meson π, and letter b is chosen for a much heavier meson, like meson b_1 of mass 1235 MeV), is specified, the decay amplitude, denoted by $\mathcal{A}$, is evaluated using the formula $\mathcal{A} = \langle p\, b\, | H_{\mathrm{I}\lambda} | h\rangle$. The symmetries of the decay amplitude $\mathcal{A}$ depend on the shapes of the wave functions of the hybrid, meson p, and meson b. Our question is: For what wave functions of the hybrid and two mesons one can obtain a spherically symmetric decay amplitude $\mathcal{A}$ for a 0^{++} hybrid using the interaction term $H_{\mathrm{I}\lambda}$?

We choose the wave functions to correspond to the phenomenological images that underlie constituent models of hadrons. For example, we choose a Gaussian wave function of the relative momentum of quarks to model a meson. We demand that the width of that function is on the order of masses of the involved particles. Similar Gaussian wave functions are introduced for the hybrid state. There are also spin dependent factors for quarks and gluons in the states we consider that were not studied before [5]. Our study includes several choices of these factors. The factors are built from the spinors and Dirac matrices which appear in the current operators that formally can produce meson or hybrid states with the quantum numbers we consider. We check the decay amplitudes of 0^{++} hybrids into two types of mesons: two scalar or two pseudoscalar ones. The interaction term we use here differs from the scalar–gluon term used in the previous study [5] by inclusion of the gluon spin as dictated by QCD. Typically, the resulting decay amplitudes are not spherically symmetric. However, the degree of violation of the spherical symmetry depends on the values of the parameters we introduce in the wave functions and the RGPEP parameter λ. But one can vary the parameters and check if there exist any choices for which an amplitude is spherically symmetric. It turns out that such choices do exist and we find them here by minimizing the deviation of $\mathcal{A}$ from spherical symmetry.

The main assumption that is tested in our study is that the number of constituents can be minimal. We know that the effective particle dynamics derived using RGPEP in LF QCD includes Hamiltonian terms that change the number of effective quarks and gluons. In the standard formulation of particle dynamics that evolves in time $t = x^0$ instead of the LF $x^+ = x^0 + x^3$, special relativity requires that states of hadrons are built from Fock sectors with different numbers of virtual quanta. Even the vacuum state, with no hadrons at all, appears to be a very complex state whose structure eludes efforts of physicists to explain it. But the situation is different in the LF dynamics. When QCD is regulated in transverse (perpendicular to the z-axis) and longitudinal (along the front) direction, there is no creation of quanta from the bare vacuum and hadrons can be considered using an expansion into their Fock components. Moreover, the effective interactions that are obtained from RGPEP contain the vertex form factors f_λ that may have a small width λ in momentum space. These form factors prevent the interaction terms from easily producing additional constituents, even if the coupling constant is not small in comparison to 1. But since we do not know if one can approximate the solution for hadronic states in LF QCD by keeping only the smallest possible number of constituents with some λ, the critical question is if there exists any reasonable choice of the parameters in a model with only a minimal number of constituents for which the model renders a spherically symmetric $\mathcal{A}$. If such choices exist, what do we learn about the allowed model parameters from the symmetry requirement?

It will be shown that all the sets of parameters that we obtain share some features. Some of these features turn out to be independent of all details in our treatment of spin of the effective quarks and the gluon. The conclusion obtained earlier in Ref. [5], using a model with a spinless "gluon", is shown to be also valid when one includes spin. We study several options to do so and we find that the tendency observed in [5] for a spinless gluon is a generic phenomenon, even though some changes do occur. But the general conclusion is that the probability distribution for the constituent quarks and a gluon in a scalar hybrid must resemble a state built from the gluon and an octet diquark. The diquark has a smaller size than the typical distance between the gluon and the diquark. The structure can be imagined as a gluonium with one gluon replaced by a small quark–antiquark pair. This is a result of pure fitting of the model parameters. Our study does not answer the question if or how this picture may arise from the effective LF dynamics in QCD.

Although it is known how the effective dynamics can be derived order by order from QCD, the resulting eigenvalue equations for mesons or hybrids are too complex and the number of basis states too large for solving the equations completely without some guiding rules for simplifying the mathematics. What we observe here is that the constraints of special relativity strongly limit the acceptable wave functions if one assumes that physical states are dominated by the Fock sectors with the smallest possible numbers of effective constituents. The constraints of relativity force the wave function parameters to take values that suggest a dominant role of gluons in the distribution of matter inside hybrids. Gluons seem to dictate to quarks what the latter must do, rather than vice versa, *i.e.*, not as constituent models based on the picture for glueless hadrons suggest. Our main point is not that our model must be correct, but that the constraints of relativity on LF models with a minimal number of constituents are quite restrictive, can be implemented in practice, and point in new directions.

Let us add that the problems with Lorentz symmetry in constituent models of bound states of quarks and gluons occur not only when an outgoing meson is light and has to move fast in the rest frame of a decaying hybrid. They also occur when one considers a decay of a hybrid in fast motion, which happens whenever the decay is a part of a bigger process that includes production and propagation with a high speed of the hybrid itself. Such circumstances may be of interest, for example, in a photoproduction of hybrids in motion.

3. Assumptions

The model we discuss is an extension of the scalar model from Ref. [5] and we adopt notation used there without changes. The new element here is the spin of a constituent gluon. In Ref. [5], gluons were treated as scalar particles. Here, the interaction Hamiltonian $H_{\mathrm{I}\lambda}$ that is responsible for the decay of the constituent gluon is taken directly from LF QCD with the RGPEP width parameter λ near the scale of hadronic masses:

$$\mathcal{H}_{\mathrm{I}\lambda} = g f_\lambda \bar{\psi}_\lambda \gamma^\mu A^a_{\mu,\lambda} t^a \psi_\lambda \,. \tag{1}$$

We display below details of the term that creates a pair of an effective quark and an effective antiquark from an effective gluon. This is the only term that counts in our calculation of the decay amplitude $\mathcal{A}$. The interaction term contains the vertex form factor f_λ. If we denote the invariant mass of the quark–antiquark pair by $\mathcal{M}_{q\bar{q}}$ and the gluon mass by $\mathcal{M}_g$, then [11]

$$f_\lambda = e^{-(\mathcal{M}^2_{q\bar{q}} - \mathcal{M}^2_g)^2/\lambda^4} \,. \tag{2}$$

3.1. $q\bar{q}$ mesons

The $q\bar{q}$ meson wave functions are of the same type as in Ref. [5].

$$|p\rangle = \sum_{12} \int [12]\, p^+ \tilde{\delta}(1 + 2 - p)\, \Psi^p_{J^{\mathrm{PC}}}(1,2)\, b^\dagger_{\lambda 1} d^\dagger_{\lambda 2} |0\rangle \;, \tag{3}$$

where $\Psi^p_{J^{\mathrm{PC}}}(1,2)$ is a product of color, flavor (isospin), spin, and momentum dependent factors:

$$\Psi^p_{J^{\mathrm{PC}}}(1,2) = \chi^\dagger_{c_1} C_p \chi_{c_2}\; \chi^\dagger_{i_1} I_p \chi_{i_2}\; \chi^\dagger_{s_1} S_p(1,2) \chi_{s_2}\; \psi_p(1,2) \;, \tag{4}$$

with $C_p = 1/\sqrt{3}$ (color singlet), $I_p = 1/\sqrt{2}$ (isospin singlet), and $S_p(1,2)$ is a 2×2 spin matrix, sandwiched between two-component spinors. $\tilde{\delta}$ denotes $16\pi^3$ times a three-dimensional δ-function of plus and transverse momenta of the particles indicated in the argument, $\tilde{\delta}(k) = 16\pi^3\delta(k^+)\delta^{(2)}(k^\perp)$, see [5]. The wave function $\psi_p(1,2)$ is chosen to be Gaussian function,

$$\psi_p(1,2) = \mathcal{N}_p\, N_{pm}(\vec{k}_{12})\, N_{ps}(\vec{k}_{12})\ \exp\left[\frac{-\vec{k}_{12}^{\,2}}{2\beta_p^2}\right], \tag{5}$$

where $\vec{k}_{12}$ is the relative three-momentum of the quarks in their center of mass system, see Appendix A. The additional functions N_{pm} and N_{ps} [5] are introduced entirely ad hoc. One option we investigate is that these functions are kept equal 1. In this case, the momentum integrals involve the relativistic momentum-space measure and full complexity of factors resulting from the relativistic spin structure. In particular, the normalization of a meson state is given by an integral of a function that is a product of the square of the Gaussian function, a factor resulting from the relativistic momentum-space measure, and a complex momentum-dependent spin factor. The other options we investigate are that the functions N_{pm} or N_{ps} are chosen to cancel the relativistic momentum-space measure or the spin factor in the normalization integral, respectively. The normalization condition is $\langle p|p'\rangle = p^+\tilde{\delta}(p-p')$. When both the measure and spin factors are canceled by N_{pm} (measure) and N_{ps} (spin), the normalization integral is a plain Gaussian integral as in a non-relativistic quantum mechanics. We investigate these options to find out how strongly the relativity constraints on the amplitude $\mathcal{A}$ depend on different factors. The same type of factors as N_{pm} or N_{ps} in the meson p are introduced in the meson b and denoted by N_{bm} or N_{bs}. All factors N are listed in the Appendix A. The cases we discuss here are described as $N = 1$ (full relativistic complexity of the model wave functions) or $N \neq 1$ (non-relativistic appearance of the normalization integrals of the model wave functions).

For $J^{\mathrm{PC}} = 0^{++}$ mesons, we have the following spin factor

$$\chi_{s_1}^\dagger S_p(1,2)\chi_{s_2} = \bar{u}_1 v_2\,, \tag{6}$$

(the notation is the same as in Ref. [5]), where u_1 and v_2 are Dirac spinors for quarks. For $J^{\mathrm{PC}} = 0^{-+}$ mesons (pseudoscalar mesons) we use

$$\chi_{s_1}^\dagger S_p(1,2)\chi_{s_2} = \bar{u}_1 \gamma^5 v_2\,. \tag{7}$$

The LF spinors we use here are described in the Appendix A.

The above model meson states require comments concerning their parameters, spins, and quark content. First of all, we use the name b-meson here to indicate generically that the meson is relatively heavy, like mesons b_1 or $\eta(1295)$ or others in the same range of masses. The spin of the meson b is assumed to be zero, and we consider only scalars and pseudoscalars to find out the consequences of the special relativity constraints in simplest and most transparent cases. Thus, we do not describe the spin of real b_1 mesons. The parameters of the wave function of a b-meson in our model are not constrained to explain properties of any real meson and they are left free within a considerable range in order to check what, if any, combination of all parameters can produce rotationally symmetric decay amplitudes. The issue here is not if we can fit a model to data when we ignore gluons in a model of ordinary mesons, but if ignoring gluons in a potentially valid effective constituent picture in QCD is allowed by special relativity symmetry even in principle, before a dynamical analysis is attempted.

The same applies in the case of a light meson, called here p-meson, which can have a mass as small as a π-meson. However, in the case of a pion, the assumption that such light meson is dominated by a quark–antiquark component may be considered merely a mock up when one attempts to understand the structure of light mesons in terms of canonical (almost massless) quark degrees of freedom in QCD, where the problem of breakdown of chiral symmetry requires a careful statement, or when one tries to create a strong binding effect in a naive potential model. We are facing a problem that on the one hand hadrons can be classified in terms of constituent quarks and lightest mesons belong to the same scheme as the heavier ones, and on the other the phenomenon of chiral symmetry breaking in canonical QCD is not explained quantitatively. Our model study of symmetry constraints in the effective constituent picture based on RGPEP in QCD is not solving this problem. What matters is that the effective particle picture is not necessarily wrong. Namely, the effective quarks may have large masses and chiral symmetry may be already explicitly broken in the effective Hamiltonian that has the width λ comparable with hadronic masses. At the same time, the binding potentials in H_λ in QCD may differ in the case of π-mesons and in the case of heavier ones. The key question here is not what dynamical mechanism might be responsible for the success of the constituent classification of hadrons, whether it is vacuum condensates in standard dynamics or corresponding special terms in a LF Hamiltonian, but if such minimal constituent picture can satisfy constraints of special relativity, assuming a most plausible Hamiltonian term that can lead to the decay of a hybrid. Therefore, we take the stance at this stage of the development that anything goes that produces relativity with constituents and we ask if this condition can be satisfied in any, even remotely plausible way from the point of view of currently popular models. Our major issue is if the LF QCD constituent picture has a chance to obey rotational symmetry. We do not solve the dynamics of formation of pions here and we will call the light meson p, not π.

3.2. Hybrid meson

Our model for 0^{++} hybrid state is of the form (*cf.* [5])

$$|h\rangle = \sum_{123} \int [123]\, h^+ \tilde{\delta}(1+2+3-h)\, \Psi^h_{J^{\rm PC}}(1,2,3)\, b^\dagger_{\lambda 1} d^\dagger_{\lambda 2} a^\dagger_{\lambda 3} |0\rangle\,. \qquad (8)$$

The creation operator for an effective gluon of width λ, $a^\dagger_{\lambda 3}$, carries spin, and the wave function $\Psi^h_{J^{\rm PC}}(1,2,3)$ depends on this spin. The wave function is a product of color, flavor (isospin), spin, and Gaussian functions of the relative momenta of the three particles:

$$\Psi_{J^{\rm PC}}(1,2,3) = \ \chi^\dagger_{c_1} C^{c_3}_h \chi_{c_2}\ \chi^\dagger_{i_1} I_h \chi_{i_2}\ \chi^\dagger_{s_1} S_h(1,2,3) \chi_{s_2} \psi_h(1,2,3)\ , \qquad (9)$$

with $C^{c_3}_h = t^{c_3}/2$ and $I_h = 1/\sqrt{2}$. The momentum dependent factor (hybrid wave function) is assumed to have the $q\bar{q}$-cluster form [5, 13]

$$\psi_h(1,2,3) = \mathcal{N}_h\ N_{hm}(\vec{k}_q, \vec{k}_g)\ N_{hs}(\vec{k}_q, \vec{k}_g)\ \exp\left[\frac{-\vec{k}_q^{\,2}}{2\beta^2_{hq}}\right]\ \exp\left[\frac{-\vec{k}_g^{\,2}}{2\beta^2_{hg}}\right], \qquad (10)$$

with typical Gaussian functions of the relative momenta. Namely, $\vec{k}_q$ is the three-momentum of the quark in the CMF of the quark–antiquark pair, and $\vec{k}_g$ is the three-momentum of the gluon in the CMF of the quark, antiquark, and gluon (see Appendix A for details). Since we use the LF form of dynamics, the CMFs are well defined and separation of the relative and CMF motion for any state is purely kinematical. The optional additional factors N_{hm} and N_{hs} can be again kept equal 1 or chosen so that the wave function normalization condition has a non-relativistic appearance for three constituents similarly to the wave functions of mesons, see Appendix A. For consistency, we will always either introduce all factors N for both mesons and hybrid equal 1 (the cases labeled $N = 1$), or insert all factors N such that all our states are normalized through the same integrals as in a non-relativistic theory (the cases labeled $N \neq 1$).

The simplest scalar-hybrid spin factor that may be considered in a relativistic theory is

$$\chi^\dagger_1 S_h(1,2,3) \chi_2 = \bar{u}_1 \gamma_\mu \varepsilon^\mu_3 v_2\,, \qquad (11)$$

where ε_3 is the gluon polarization four vector. This case will be referred to as the case $\bar{u}\not{\varepsilon}v$. For the effective gluon in the light-front gauge $A^+ \equiv 0$, the polarization vector has the following components:

$$\varepsilon^\mu_{k\sigma} = \left(\varepsilon^+_{k\sigma} = 0, \varepsilon^-_{k\sigma} = \frac{2k^\perp \varepsilon^\perp_\sigma}{k^+}, \varepsilon^\perp_{k\sigma} = \varepsilon^\perp_\sigma \right) . \tag{12}$$

The sum over gluon polarizations is

$$\sum_\sigma \varepsilon^\mu_{k_3\sigma} \varepsilon^{*\,\nu}_{k_3\sigma} = -g^{\mu\nu} + \frac{k_3^\mu g^{+\nu} + g^{+\mu} k_3^\nu}{k_3^+} . \tag{13}$$

In the above formula, the component k_3^- of the gluon momentum is the same as for massless gluons, $k_3^- = k_3^{\perp\,2}/k_3^+$. But in evaluating kinetic energy of the effective gluons, we introduce the gluon effective mass parameter m_g that can depend on the RGPEP parameter λ. We do not know the value of m_g and we leave it as a free parameter in our Gaussian wave function.

An alternative structure for the spin factor, pertaining to non-Abelian gauge symmetry but not necessarily better than Eq. (11) from the dynamical point of view in the effective theory, is

$$\chi^\dagger_{s_1} \, S_h(1,2,3) \, \chi_{s_2} = \bar{u}_1 \gamma_\mu v_2 \, G^{\mu\nu} P_\nu \, , \tag{14}$$

where $G^{\mu\nu} = k_3^\mu \varepsilon^\nu_{k\sigma} - k_3^\nu \varepsilon^\mu_{k\sigma}$ and $P = k_1 + k_2 + k_3$. The four-vector $\varepsilon^\nu_{k\sigma}$ is the polarization vector for massless gauge bosons. But k_3^- can be calculated as if the gluon mass were 0, or using the parameter m_g, and we do not know which way is more realistic in a dynamical theory. Therefore, we insert the unknown mass m_g in the formula $k_3^- = (k_3^{\perp\,2} + m_g^2)/k_3^+$ and check for what values of m_g the resulting decay amplitude $\mathcal{A}$ is spherically symmetric in the hybrid CMF. This case is referred to as $\bar{u}GPv$.

We also consider alternative versions of the spin factor, where we calculate k_3^- as if the gluon mass should be kept 0 in $G^{\mu\nu}$ and/or in P^ν. These cases are referred to as $\bar{u}\tilde{G}\tilde{P}v$, $\bar{u}G\tilde{P}v$, and $\bar{u}\tilde{G}Pv$, respectively. The tilde means that we put $m_g = 0$ in evaluating k_3^- in the factor that is labeled with the tilde.

The alternative spin factors resemble the one that occurs in the operator structures used in lattice calculations [14,15]. In the hybrid CMF, where $\vec{P} = 0$, this factor reduces to $\bar{u}\gamma^i v G_{i0}$, which is a combination of the components of the quark current and the chromoelectric gluon field.

Note that the gluon momentum component $k_3^- = (k_3^{\perp\,2} + m_g^2)/k_3^+$ does not contribute to the Lorentz product $k_3 \varepsilon_{k\sigma}$ in the chosen gauge and the entire small group of the Poincare transformations that preserve the light-front hyperplane also does not change the gauge condition $A^+ \equiv 0$. Thus, we can safely boost the hybrid state using kinematical relations and our assignment of mass to the effective gluon does not interfere with our choice of gauge. This is important because our model would not be reasonable otherwise. Namely, if the small group and gauge choice would not commute, we would not be able to construct the states of mesons and hybrids in motion without changing the gauge. The latter change would be associated with altering interaction terms in the effective Hamiltonian of width λ. Fortunately, our choice of the LF dynamics, instead of the standard one, offers a possibility of keeping boost invariance using one and the same choice of gauge in all frames of reference that can be reached by the boosts. Note also that our RGPEP procedure respects this commutativity because it is invariant under the small group and our model is reasonable in this respect.

As explained earlier, in contrast to the standard approaches where rotational symmetry is kinematical and boosts are dynamical, in the LF approach the rotational symmetry is dynamical. Therefore, we now have to check to what extent our models can guarantee that the resulting decay amplitude is spherically symmetric in the rest frame of the hybrid. We do this in the next section using both Eqs. (11) and (14) in a number of cases that we have introduced above.

4. Symmetry constraints

The decay amplitude of a scalar hybrid ($J^{\rm PC} = 0^{++}$) into two mesons, either two $J^{\rm PC} = 0^{++}$ mesons or two 0^{-+} mesons, should be spherically symmetric. But a constituent model built in the LF scheme introduces dependence on the angle θ between the z-axis and the direction of flight of the light meson. This effect is the price we pay for boost invariance. The effect was discovered and initially studied in a scalar model in Ref. [5].

Fig. 1 illustrates how badly the rotational symmetry is violated in a decay into two scalar mesons (spin factors $\bar{u}v$) when the light meson mass varies from 664 MeV toward the value of 138 MeV. In this figure the factors N_{hm}, N_{hs}, N_{bm}, N_{bs}, N_{pm}, and N_{ps}, in the wave functions, are kept different from 1 to secure non-relativistic normalization. In the case of 664 MeV, the hybrid mass m_h is just above the threshold of $m_b + m_p$ and the product mesons can barely move. The LF constituent model renders an amplitude that does not depend on the angle θ. In the 138 MeV case, corresponding to the π-mesons, the hybrid mass is far above the threshold and the light meson has a highly relativistic velocity. In this case, the amplitude depends on the angle θ to an unacceptable degree. The effect is caused by the fact that when the outgoing light meson flies against the z-axis with nearly speed of light, it must be built from quarks that have very small momentum k^+ and such quarks are suppressed in the case of wave functions used in the

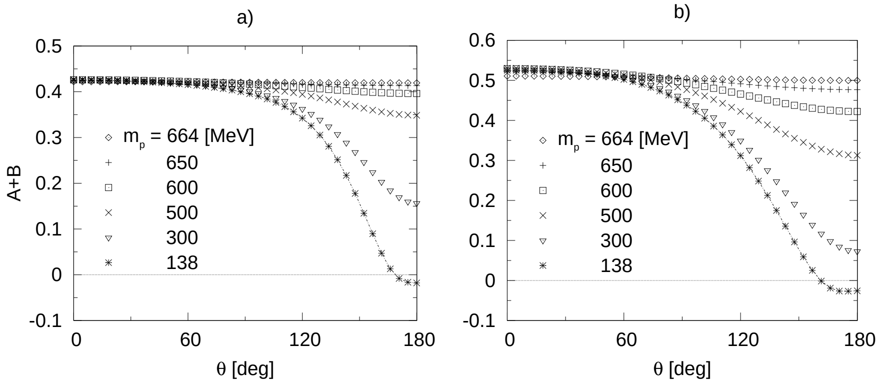

Fig. 1. The angular dependence of the decay amplitude for various masses of the light meson, m_p. The plot (a) is for the hybrid spin factor equal $\bar{u}\gamma^\mu v\varepsilon_\mu$, and (b) is for spin factor $\bar{u}\gamma^\mu v G_{\mu\nu} P^\nu_{123}$. In both cases the hybrid meson decays into two $J^{\rm PC} = 0^{++}$ scalar mesons (spin factors $\bar{u}v$). The model wave functions contain factors N_{ps}, N_{pm}, N_{bs}, N_{bm} and N_{hs}, N_{hm} that secure that the normalization integrals have a non-relativistic appearance of integrals of plain Gaussian functions (case $N \neq 1$). All parameters of the wave functions are given in the first column of Table I.

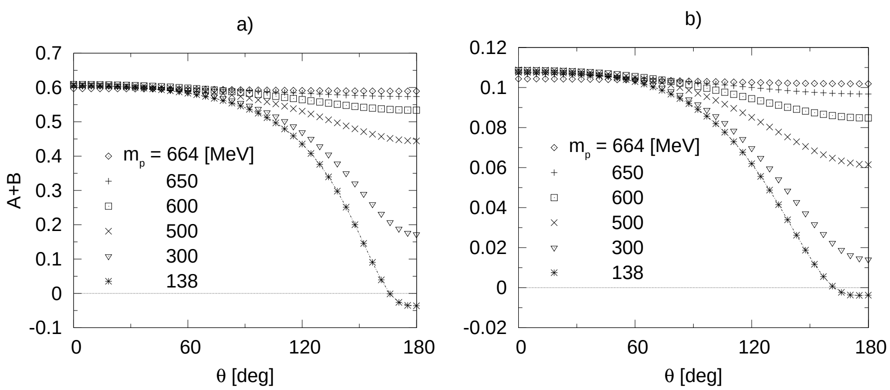

Fig. 2. The same as in Fig. 1, but with the factors $N = 1$.

calculation. The parameters that we use are given in the first column of Table I on page 402 (*cf.* [5]). Figs. 2 to 4 show the same effect, but with all factors N_{hm}, N_{hs}, N_{bm}, N_{bs}, N_{pm}, and N_{ps} equal 1, or in the case of two pseudoscalar ($J^{\rm PC} = 0^{-+}$) mesons instead of the scalar ones, or in the case where the two changes are combined. In order to satisfy constraints of special relativity, the decay amplitude should not depend on the angle θ.

TABLE I

Table 1. The parameters of wave functions (in GeV) that we use in Figs. 1–5 (in the same notation as in Ref. [5]). s means that the parameter is the same as in the left neighboring column. Columns 5**a**–**d** display results of local minimization using Powell's procedure [16], starting from the values given in the last column (labeled "hs"), which is the same as the corresponding column in Table I in Ref. [5].

Fig. #	1–4(a), (b)	5**a**	5**b**	5**c**	5**d**	hs
m_h	1.9	s	s	s	s	s
m_b	1.235	s	s	s	s	s
m_p	varies	0.1375	s	s	s	s
m_q	0.3	0.68	0.75	0.67	0.87	0.365
m_g	0.8	2.6	1.4	3.7	1.3	1.63
β_p	0.4	0.19	0.34	0.14	0.21	0.375
β_b	0.4	0.77	1.2	1.0	0.96	0.719
β_{hg}	1.0	0.28	0.31	0.31	0.52	0.60
β_{hq}	1.0	3.8	4.4	3.8	7.5	4.61
λ	10000	4.8	4.4	4.4	4.4	4.49

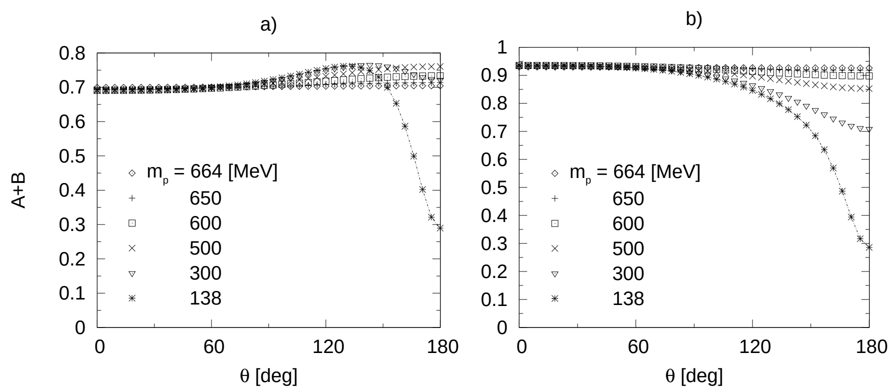

Fig. 3. The same as in Fig. 1, but for hybrid decay into two $J^{\rm PC} = 0^{-+}$ pseudoscalar mesons (spin factors $\bar{u}\gamma^5 v$).

Broadly speaking, the violation of rotational symmetry results from the fact that the model wave functions are not constrained dynamically by any underlying relativistic theory. Given that the model is reasonable, in the sense that (1) the meson states are formed using well-defined degrees of freedom that appear in the LF Hamiltonian H_λ in QCD with a small RGPEP parameter λ, (2) the boost symmetry is preserved exactly, and (3) the decay is driven by an interaction term in the same Hamiltonian, the most ques-

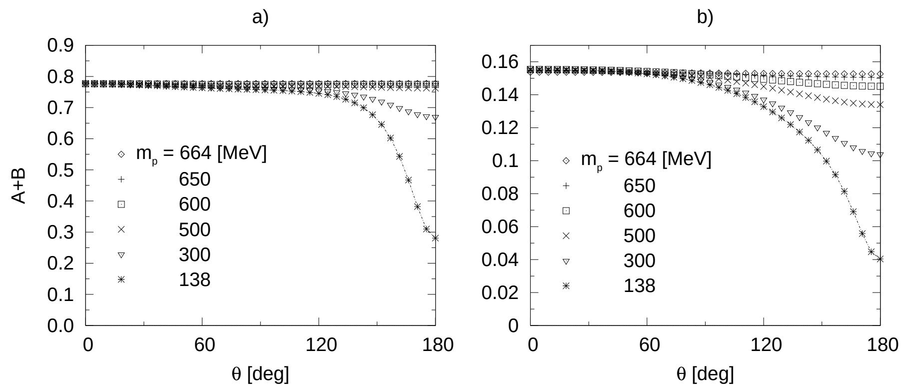

Fig. 4. The same as in Fig. 1, but with the combined effect due to the changes from Fig. 2 and 3: $N = 1$ and the hybrid decays into two pseudo-scalar mesons.

tionable element of the model is the assumption that a small number of constituents is sufficient to build a solution of a relativistic theory. The model assumes that the number of constituents is the smallest possible. It may fail to produce rotational symmetry because the symmetry is dynamical in the LF scheme and the interactions can change the number of constituents [7,8]. We see in Figs. 1 to 4 that all models we test respect rotational symmetry very well in non-relativistic decays. But in the relativistic decays, all the models fail more or less equally badly (on average, decays into pseudoscalars are a bit less wrong than decays into scalars because pseudoscalars are dominated by non-relativistic, momentum-independent components in their wave functions). Does this mean that all models based on the assumption of the smallest number of constituents must be entirely wrong?

We find that the answer to this question is no: a minimal constituent model does not have to be wrong. Since the rotational symmetry is dynamical in the approach we study, it is not known if the parameters listed in Table I in the first column do correspond to a solution of a relativistic theory. Suppose that a different set of parameters should be used in a reasonable model that approximates a solution of a relativistic theory. Can one find a set of parameters in the constituent wave functions for which the required rotational symmetry of the decay amplitude is obtained? This question is found to have a positive answer but the sets of parameters that we find point to a new picture for the hybrids. The picture seems to be generic in the sense that its dominant features are independent of how the spin of the gluon and the spins of quarks are treated. Our numerical studies produce examples of models with a smallest number of constituents in which the rotational symmetry is respected well when one allows the parameters in the wave functions and the RGPEP scale λ in the Hamiltonian to vary. The reader should remember that the number of variable parameters in the class of models we consider is 7 and there exists a great number of possibilities to check, each demanding a multidimensional integration for every value of the angle θ.

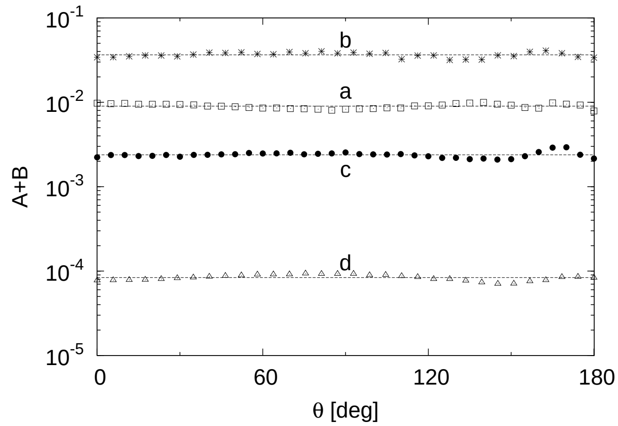

Fig. 5. The hybrid decay amplitude as function of θ in four cases: **a** $\bar{u}\not{\epsilon}v$, $N \neq 1$ and second column of Table I, **b** $\bar{u}\not{\epsilon}v$, $N = 1$ and third column of Table I, **c** $\bar{u}GPv$, $N \neq 1$ and fourth column of Table I, **d** $\bar{u}GPv$, $N = 1$ and fifth column of Table I. Decays into two $J^{\rm PC} = 0^{++}$ mesons.

Fig. 5 shows how well the rotational symmetry can be restored by selecting a different set of parameters in the wave functions (the sets corresponding to Fig. 5 are given in columns 5**a**–**d** in Table I). Curves "**a**" and "**b**" represent the decay amplitudes for a hybrid meson with the spin factor given in Eq. (11) (case $\bar{u}\not{\epsilon}v$) and the wave-function parameters given in the second column of Table I. Curves "**c**" and "**d**" on the same figure show the decay amplitudes for a hybrid with the spin factor given by Eq. (14) (case $\bar{u}GPv$) and with the parameters given in the third column of Table I. Curves "**a**" and "**c**" are obtained with $N \neq 1$, and curves "**b**" and "**d**" are obtained with $N = 1$.

The optimal choice of the parameters that one obtains from the condition of rotational symmetry includes β_p about twice smaller than the typical value of 0.4 GeV in the first column of Table I, which corresponds to the size of a real meson π (we will return to this issue below). We could also find other sets of parameters with even smaller β_p and considerably smaller quark masses, a feature observed already in Ref. [5]. The cases shown here are obtained by starting a Powell local minimization procedure [16, 17], starting from the hs (for "heavy-scalar") set of parameters that was found in a scalar model in Ref. [5]. The nomenclature refers to the relatively heavy scalar particles that played the role of quarks in Ref. [5].

The cases we display here are characterized by quite good spherical symmetry in comparison to other locally optimal choices which we were also able to identify but which displayed more variation with θ. In choosing the minimization we also adopted a criterion that the resulting spherically symmetric amplitude should not be many orders of magnitude smaller than in the case of the parameters in the first column of Table I. Note, however, that the amplitudes in Fig. 5 are, in fact, a whole order of magnitude smaller than in Figs. 1–4. This indicates how important the constraints of relativity can be for analysis of data. We should also add that the size of the coupling constant in the interaction Hamiltonian that drives the decay was fixed as in Ref. [5] and never changed in the fit.

It is clear that one should not consider an unbiased minimization of symmetry violation as a most reasonable approach. The minimization should include additional constraints, including restrictions such as the radius of a meson p, and as much of the dynamical constraints as possible. But in order to impose correlations such as the ones coming from the radius, one has to be very careful about how one calculates the radius and if that calculation does obey requirements of special relativity, which is a problem in itself. Concerning the dynamical constraints, we were not able and not even interested in imposing any such constraints at this stage, because we were only searching for the answer to the question if any choice of the parameters could produce spherical symmetry, and it is interesting that even a minimal model can produce the symmetry of the quality as good as shown in Fig. 5. We remind the reader that the gluon spin introduces functions of momenta that vary rapidly with angles and it was not clear at all that any choice of parameters could lead to a constant amplitude. But once it is established that such result is possible, one can make further observations based on a systematic search through the space of the parameters.

The simplest and least restrictive way of setting bounds on the parameters of the models we test is to limit all of the parameters to fixed intervals around values that are considered reasonable. Such least restrictive parameter bounds adopted in the minimizations described below are given in Table II.

TABLE II

The limits on parameters of the wave functions (in GeV) that we imposed using the Adaptive Simulated Annealing algorithm [18].

param	m_q	m_g	β_p	β_b	β_{hg}	β_{hq}	λ
min	0.1	0.5	0.1	0.1	0.1	0.1	0.1
max	0.6	2.0	0.8	1.6	2.0	8.0	8.0

We performed a global minimization of departures from rotational symmetry using different measures of how much a decay amplitude differs from a constant as a function of the angle θ: a standard deviation from the average value (sum of squares of deviations from the average value, labeled "stddev" in Appendix C), or maximum of the modulus of the deviations from the average value (labeled "maxdev" in the Appendix C), both measured relative to the average. Our global minimization within the assumed bounds is done using Adaptive Simulated Annealing (ASA) [18].

Fig. 6 shows results obtained using Eq. (11) for the spin factor of a hybrid meson: curve "**a**" in case $N \neq 1$ and "**b**" in case $N = 1$, and using Eq. (14): curve "**c**" in case $N \neq 1$ and "**d**" in case $N = 1$, all cases for a decay into two scalar ($\bar{u}v$) mesons. The factors N have considerable impact on the magnitude of the amplitudes and can compensate or dramatically enhance the effect of changing the spin factors. This means that one should probably not trust models of hybrids that are based solely on non-relativistic intuitions.

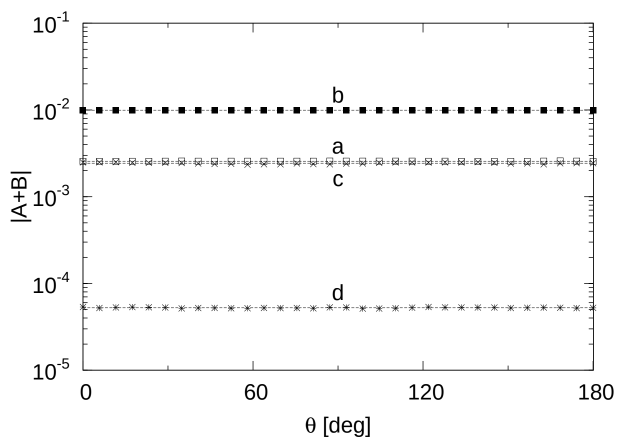

Fig. 6. The hybrid decay amplitude as function of θ for the decay into two $J^{\rm PC} = 0^{++}$ mesons in different cases: **a** $\bar{u}\not{\epsilon}v$, $N \neq 1$, first column of Table III, **b** $\bar{u}\not{\epsilon}v$, $N = 1$, second column of Table III, **c** $\bar{u}GPv$, $N \neq 1$, third column of Table III, **d** $\bar{u}GPv$, $N = 1$, fourth column of Table III.

Results for decays into two $J^{\rm PC} = 0^{-+}$ mesons ($\bar{u}\gamma^5 v$ spin factor) are shown in Fig. 8. The corresponding values of the wave function parameters are given in Table V. Similarly, Figs. 7 and 9 show results for alternate choices of hybrid spin factor: cases referred as $\bar{u}GPv$, $\bar{u}\tilde{G}Pv$ and $\bar{u}\tilde{G}\tilde{P}v$.

In Ref. [5], there were found two locally best sets of values of parameters in each of the two cases: one case with all constituents being scalars, and another one with fermionic quarks and a scalar gluon. These good sets were

TABLE III

The optimal parameters of the wave functions (in GeV) for a decay into two $J^{\mathrm{PC}} = 0^{++}$ mesons. These are results of a global minimization using ASA [18], minimizing standard deviation from the average value of the amplitude, for parameters within limits in Table II. The resulting amplitudes are shown in Fig. 6. The bold face numbers are on the limit of the allowed range.

Fig. #	6**a**	6**b**	6**c**	6**d**
spin term	$\bar{u}\not{\epsilon}v$ $N \neq 1$	$\bar{u}\not{\epsilon}v$ $N = 1$	$\bar{u}GPv$ $N \neq 1$	$\bar{u}GPv$ $N = 1$
m_h	1.9	s	s	s
m_b	1.235	s	s	s
m_p	0.1375	s	s	s
m_q	0.152	0.21	0.17	0.155
m_g	1.28	1.07	1.70	1.90
β_p	0.132	0.219	**0.1**	**0.1**
β_b	0.320	0.536	0.321	0.371
β_{hg}	0.766	1.05	0.267	0.263
β_{hq}	**8.0**	7.83	4.59	5.73
λ	7.68	6.13	2.86	7.98

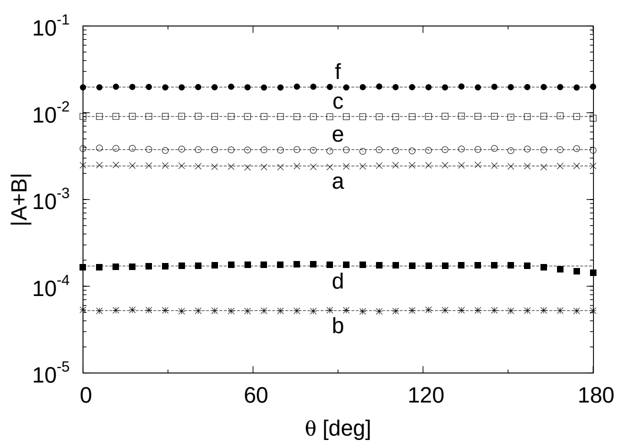

Fig. 7. The hybrid decay amplitude as function of θ for the decay into two $J^{\mathrm{PC}} = 0^{++}$ mesons in different cases: **a** $\bar{u}GPv$ with $N \neq 1$, $\bar{u}GPv$ with $N = 1$, **c** $\bar{u}\tilde{G}Pv$ with $N \neq 1$, **d** $\bar{u}\tilde{G}Pv$ with $N = 1$, **e** $\bar{u}\tilde{G}\tilde{P}v$ with $N \neq 1$, **f** $\bar{u}\tilde{G}\tilde{P}v$ with $N = 1$. Parameters are given in Table IV.

characterized by either light or heavy mass of the quarks (Table I and Fig. 5 in Ref. [5]). In our studies, including the spin of the gluon, the complete ASA

TABLE IV

The optimal parameters of the wave functions (in GeV) for a decay into two $J^{\rm PC} = 0^{++}$ mesons. These are results of global minimization using ASA [18], minimizing standard deviation from the average value of the amplitude, for parameters within limits in Table II. The resulting amplitudes are shown in Fig. 7. The bold face numbers are on the limit of the allowed range.

Fig. #	**7a**	**7b**	**7c**	**7d**	**7e**	**7f**
spin term	$\bar{u}GPv$ $N \neq 1$	$\bar{u}GPv$ $N = 1$	$\bar{u}\tilde{G}Pv$ $N \neq 1$	$\bar{u}\tilde{G}Pv$ $N = 1$	$\bar{u}\tilde{G}\tilde{P}v$ $N \neq 1$	$\bar{u}\tilde{G}\tilde{P}v$ $N = 1$
m_h	1.9	s	s	s	s	s
m_b	1.235	s	s	s	s	s
m_p	0.1375	s	s	s	s	s
m_q	0.17	0.155	0.373	**0.1**	0.214	0.268
m_g	1.70	1.90	1.82	**2.0**	1.82	1.46
β_p	**0.1**	**0.1**	0.286	**0.1**	0.211	0.411
β_b	0.321	0.371	0.209	1.23	0.365	0.470
β_{hg}	0.267	0.263	0.565	1.02	0.244	0.434
β_{hq}	4.59	5.73	4.44	**8.0**	7.60	3.22
λ	2.86	7.98	3.57	**8.0**	4.11	3.86

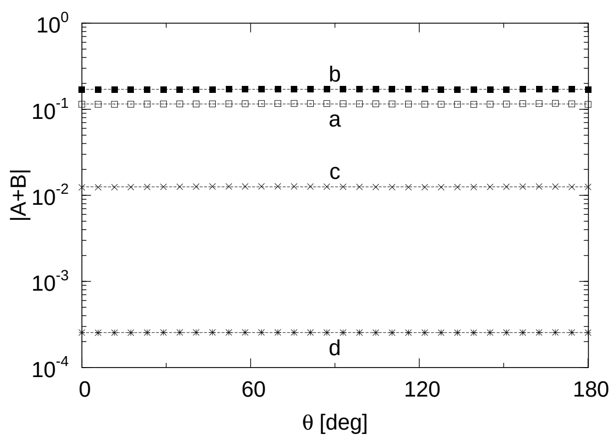

Fig. 8. The hybrid decay amplitude as a function of θ, the same as in Fig. 6, but for a decay into two $J^{\rm PC} = 0^{-+}$ mesons (spin factor $\bar{u}\gamma^5 v$). The optimal wave function parameters are given in the correspondingly marked columns of Table V.

algorithm finds only one best set of the parameters that minimizes deviation from rotational symmetry in every case we consider. These best sets appear with small quark masses and small β_p. Such small β_p implies a too large size of the meson p, apparently corresponding to a too weak binding of too light quarks, as if the number of such light effective constituents could not be only

TABLE V

The optimal parameters of the wave functions (in GeV) for a decay into two $J^{\rm PC} = 0^{-+}$ mesons. These are results of global minimization using ASA [18], minimizing standard deviation from the average value of the amplitude, for parameters within limits in Table IV. The resulting amplitudes are shown in Fig. 8.

Fig. #	8**a**	8**b**	8**c**	8**d**
spin term	$\bar{u}\not{g}v$ $N \neq 1$	$\bar{u}\not{g}v$ $N = 1$	$\bar{u}GPv$ $N \neq 1$	$\bar{u}GPv$ $N = 1$
m_h	1.9	s	s	s
m_b	1.235	s	s	s
m_p	0.1375	s	s	s
m_q	0.15	0.16	0.18	0.19
m_g	1.08	0.88	1.69	1.79
β_p	0.21	0.25	0.16	0.22
β_b	0.59	0.59	0.42	0.30
β_{hg}	0.59	0.72	0.68	0.47
β_{hq}	2.40	2.62	6.96	6.80
λ	3.97	3.71	2.29	2.39

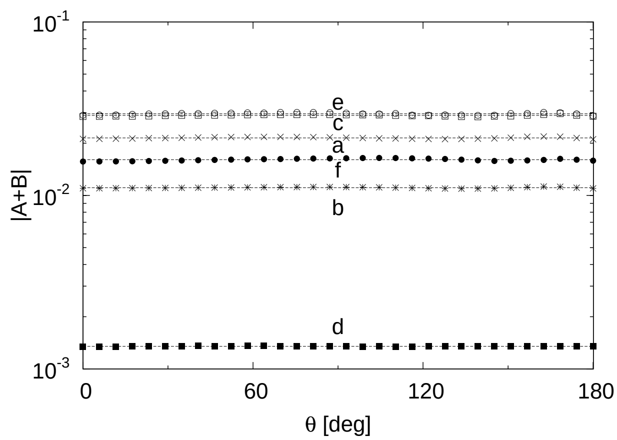

Fig. 9. The hybrid decay amplitude as function of θ for the decay into two pseudoscalar $J^{\rm PC} = 0^{-+}$ mesons in different cases: **a** $\bar{u}GPv$ with $N \neq 1$, **b** $\bar{u}GPv$ with $N = 1$, **c** $\bar{u}\tilde{G}Pv$ with $N \neq 1$, **d** $\bar{u}\tilde{G}Pv$ with $N = 1$, **e** $\bar{u}\tilde{G}\tilde{P}v$ with $N \neq 1$, **f** $\bar{u}\tilde{G}\tilde{P}v$ with $N = 1$. Parameters are given in Table VI.

minimal. But we can find different minima when we impose an additional restriction that the quark mass, m_q, is "heavy", *i.e.* greater than 300 MeV. Other parameters are still limited to the intervals given in Table II.

TABLE VI

The optimal parameters of the wave functions (in GeV) for a decay into two $J^{\rm PC}=0^{-+}$ mesons. These are results of global minimization using ASA [18], minimizing standard deviation from the mean amplitude for parameters within limits in Table II. The resulting amplitudes are shown in Fig. 9.

Fig. #	9**a**	9**b**	9**c**	9**d**	9**e**	9**f**
spin	$\bar{u}GPv$	$\bar{u}GPv$	$\bar{u}\tilde{G}Pv$	$\bar{u}\tilde{G}Pv$	$\bar{u}\tilde{G}\tilde{P}v$	$\bar{u}\tilde{G}\tilde{P}v$
term	$N\neq 1$	$N=1$	$N\neq 1$	$N=1$	$N\neq 1$	$N=1$
m_h	1.9	s	s	s	s	s
m_b	1.235	s	s	s	s	s
m_p	0.1375	s	s	s	s	s
m_q	0.26	0.42	0.29	0.18	0.31	0.31
m_g	1.34	1.85	1.21	1.62	1.22	0.61
β_p	0.21	0.21	0.25	0.19	0.28	0.54
β_b	0.48	0.21	0.59	0.40	0.58	0.56
β_{hg}	0.57	1.40	0.47	0.57	0.43	0.96
β_{hq}	7.53	6.88	7.80	4.75	7.99	4.58
λ	2.26	7.66	2.41	2.71	2.37	2.61

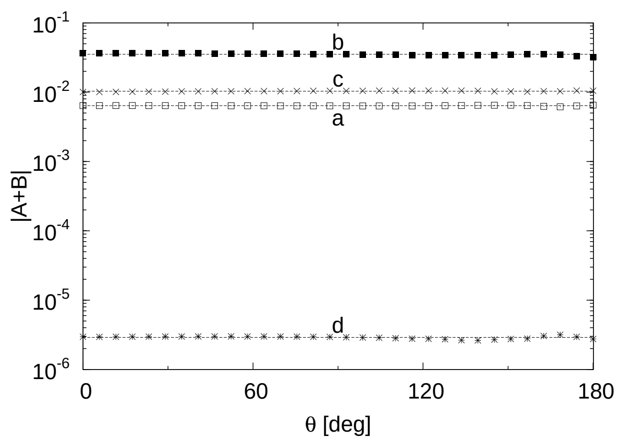

Fig. 10. The hybrid decay amplitude, like in Fig. 6, for the decay into two scalar mesons, as function of θ. Parameters are given in the corresponding columns of Table VII: the lower limit on the quark mass is $m_q\geq 300$ MeV.

An example of such restricted minimization for "heavy" effective constituent quarks is shown in Fig. 10. The corresponding optimal parameters are given in Table VII. In all cases except the case "**d**", the size of the light meson p is now much closer to the size of the real mesons π. This shows that the size (radius) of the light meson may not be as big an issue as one might think on the basis of a search for the best parameters allowing quarks to be much lighter than 300 MeV (this case was discussed earlier).

TABLE VII

The optimal parameters of the wave functions (in GeV) from a global minimization using ASA [18], minimizing standard deviation relative to the mean amplitude with all parameters within limits like in Table II except for the lower limit on the quark mass, $m_q \geq 300$ MeV. The resulting amplitudes are shown in Fig. 10.

Fig. #	10**a**	10**b**	10**c**	10**d**
spin term	$\bar{u}\not{\epsilon}v$ $N \neq 1$	$\bar{u}\not{\epsilon}v$ $N = 1$	$\bar{u}GPv$ $N \neq 1$	$\bar{u}GPv$ $N = 1$
m_h	1.9	s	s	s
m_b	1.235	s	s	s
m_p	0.1375	s	s	s
m_q	**0.3**	0.459	**0.31**	**0.31**
m_g	**1.95**	1.37	1.90	1.89
β_p	0.211	0.353	0.344	**0.1**
β_b	0.295	0.722	0.263	0.169
β_{hg}	0.894	0.754	0.450	1.21
β_{hq}	5.83	7.75	7.69	**8.0**
λ	4.67	7.75	4.44	7.96

Note that the assumption of case "**d**", that the spin factor in the hybrid wave function contains a four-momentum of a massive gluon, leads to a small optimal quark mass and a very small decay amplitude. The constraints of rotational symmetry promise to be very useful in future studies of reasonable models because when they are combined with inspection of observables such as radii one immediately obtains large differences between predictions based on different models.

5. Discussion

First of all, let us note that the inclusion of spin of a constituent gluon does not change the previously obtained result for scalar "gluons" [5] that rotational symmetry is restored when the quark–antiquark pair momentum-space width in the hybrid, β_{hq}, is about the same in size as the width λ in the vertex form factor in the renormalized interaction Hamiltonian H_λ in LF QCD, and both are on the order of 4–6 GeV, much larger than all other parameters. On the basis of our study of many cases with different ways of including the gluon spin and different cases of meson spin factors, we can state that the required relative momentum-space structure of the wave functions appears to be qualitatively independent of the spin of the effective constituents. Quite generally, the parameters β_{hq} and λ appear to have to be about 5 times larger than the other parameters, see Tables I and

III–VII. This result suggests that the quark pair should be thought about as spatially small in comparison to the size of the hybrid. It also suggests that the pair could originate from a gluon that belonged to a gluonium before the interaction changed one gluon into the pair.

One may worry that this result is obtained by minimizing just one observable in a space of seven parameters. Here comes the strength of the continuous symmetry condition on a reasonable model: special relativity provides infinitely many conditions instead of just one — the just one decay amplitude must not be a function of the angle. It was highly questionable that it was possible to even come close to a constant function of the angle before we carried our study including the singular spin factors for gluons. The remarkable fact is that the symmetry cannot be satisfied with a minimal constituent picture unless the hybrid built from a pair and a constituent gluon looks differently than expected assuming that the gluons follow quarks and this picture does not seem to depend on any particular detail but only on the major assumption that a minimal constituent model can approximate the effective LF dynamics.

Another worry concerns the small spatial size of the $q\bar{q}$-pair, about 4 to 5 times smaller than the distance between the gluon and the octet diquark, correlated with λ on the order of 3 or even 5 GeV. The real question is for what values of λ a constituent picture of hybrids may work. In principle, if RGPEP equations were solved exactly, no physical result should depend on λ and no matter what λ is used one should obtain rotationally symmetric decay amplitude provided that the decay is calculated exactly using exact solutions for the participating hadrons. But we know [12] that in order to approximate the full dynamics of an asymptotically free theory by a simple picture one has to lower λ to values that are about twice above the scale of eigenvalues one is seeking to describe. One cannot lower λ to smaller values using perturbation theory in RGPEP because the resulting H_λ would begin to contain too large errors due to cutting into the mechanism of binding. Thus, the scale we obtain from the heuristic fit is quite reasonable. On the other hand, one may worry that no tight diquark clusters are seen in the proton deep inelastic structure. One possible explanation of such special feature of hybrids could be that they contain octet diquarks interacting with constituent gluons, while in the proton we have primarily triplets and diquark antitriplet and no counterpart of the constituent gluon structure. Our symmetry study in hybrids should be seen as groping into the sectors of effective color dynamics that are squeezed out of and cannot be seen in nucleons.

There exist small differences between the case of decays into two scalar and two pseudoscalar mesons, and between various choices for handling the spin of an effective gluon in the hybrid. But the outstanding feature that β_{hq} is about the same as λ and both are larger than the rest of parameters is common to all the cases we studied. Also, the mass of the gluon, m_g, appears to have to be much greater than the masses of the quarks, m_q.

The hybrid spin wave function from Eq. (14), inspired by the lattice operators but with a massive four-momentum for the gluon, gives minima with a much smaller decay amplitude $\mathcal{A}$ than the simplest hybrid spin wave function of Eq. (11). This result shows that one has to be very careful about treatment of spin of gluons in model building.

Let us stress that the absolute size of the amplitude is not under control in our calculation because we did not include the dynamical constraints between the coupling constant g, the RGPEP parameter λ, and the wave function parameters. But there exists a systematic trend in all our results, which requires further study and is not understood here. Namely, when the parameters of the wave functions are varied from the values in the first column of Table I, to the values required by rotational symmetry, like in columns 2–5 of Table I, or the values in Tables III–IV, and VII, the size of the amplitude changes from about 0.9 to 0.1 in Figs. 1–4 to the much smaller values of 10^{-2} or 10^{-3} in Fig. 5, or 10^{-2} or 10^{-4} in Figs. 6 to 10.

These are considerable changes in the order of magnitude. It seems unlikely that the coupling constant can vary by that much and compensate this change. The change is so large only because of the relativistic motion of the light outgoing meson. If the outgoing mesons are slow, rotational symmetry in our model is respected very accurately for typical values of parameters in non-relativistic constituent models, exemplified in the first column of Table I. We are forced to conclude that a relativistic hybrid decay (including fast mesons p) may involve relativistic effects that are not accounted for in the non-relativistic phenomenology, *cf.* [19].

Another feature worth mentioning is that the minima for the wave functions motivated by lattice operator structures, Eq. (14), are narrower than in the case of Eq. (11). If the range of parameters, for which a violation of rotational invariance is small, is very narrow, the symmetry itself becomes a source of detailed information about the necessary values of the parameters even if the corresponding dynamical equations are too difficult to solve with comparable precision. By the same token, one obtains a very strict criterion for judgment of dynamical models that attempt to produce the relevant wave functions.

6. Conclusion

The example of a simple model described here shows that the wave function parameters for hadrons involved in a relativistic decay of a hybrid must be strongly correlated in order that the decay amplitude satisfies requirements of special relativity. Thanks to the use of the LF scheme, boost symmetry is respected exactly and the parameters are constrained by the condition of rotational symmetry. In the example, they have to take values that do not correspond to the picture based on the non-relativistic intuition that the gluons are mainly between two quarks. Instead, the relativistic effective constituent picture almost universally points toward the structure in which a heavy gluon is accompanied by a quark–antiquark pair that resembles a relatively small octet diquark. This is a stunning result because it suggests that the picture with gluons playing a role of a relatively light chain, or a vibrating flux, or string between relatively heavy quarks may be not as realistic as one hopes for on the basis of non-relativistic intuition.

The alternative hybrid structure occurs in a variety of cases that differ in details of the spin factors for gluons and quarks. But the requirement of relativistic symmetry turns out to be very restrictive when one demands that only sectors with the smallest possible number of constituents are important. Therefore, we conclude that the effective constituent dynamics in QCD should be always considered including constraints of special relativity. These constraints appear capable of forcing us to consider hadrons with significant gluon content not as if the gluons were just added to quarks and antiquarks, but as if gluons could actually dominate the dynamics of hybrids and force the quarks to adjust.

Since the hybrid structure we are forced to seriously consider by the results of this analysis contains a spatially tight octet diquark pair, as if the pair emerged from a constituent gluon through a single interaction in an effective LF QCD, one may ask if it is possible that such effective quark–antiquark–gluon states with a tight pair can mediate decays of usual mesons. A decay of a usual meson may proceed by an emission of a gluon from one quark and subsequent decay of the gluon into a new pair of quarks. The emerging two quarks and two antiquarks can form the mesons that are produced in the decay of the usual meson. But if the pair accompanied by the intermediate gluon has to be small in size, as if the three effective particles had to form a structure similar to our finding for a hybrid, the intermediate quark configuration would have to have a small overlap with the initial usual meson configuration. The decay mechanism through an intermediate hybrid meson would have small contribution to the total strong decay width. Would not this width be too small if the hybrid structure were as we obtain? Not necessarily, since in the effective theory there must exist other interactions that are capable of producing four effective quarks from two effective quarks. These interactions do not correspond to the intermediate excitation of a massive effective gluon and they are not characterized by the coupling of such gluons to quark–antiquark pairs. Examples of such interactions are present already in the canonical Hamiltonian in LF QCD. The canonical interactions are not mediated by emission or absorption of gluons, and they must contribute to the mechanism of strong decay of usual mesons in the effective theory characterized by width λ on the order of hadronic masses. In addition, the RGPEP procedure generates more interactions that can turn a quark–antiquark pair into two such pairs without explicit creation and decay of a massive effective gluon corresponding to small λ. Unfortunately, our study is not telling us anything about the dynamical structure of ordinary mesons and interactions that mediate their decays. It is limited to a preliminary study of symmetry constraints in simplest models with a dominant hybrid component.

The wave function parameters that we find to be preferred by the condition of rotational symmetry of the decay amplitude of a model hybrid, may turn out to be invalid when the actual dynamics is included in the analysis. For example, it may turn out that the approximation by the Fock sectors with only the smallest possible number of constituents does not apply. But it is clear that the relativistic constraints cannot be ignored in the search for a leading constituent picture.

Our discussion was limited to 0^{++} hybrids for simplicity, while the most interesting from practical point of view is the structure of exotics [14, 15, 20, 21, 22]. One can change factors in the wave functions and change the quantum numbers of the hybrid states to the exotic values. For example, one can replace the color electric field by a color magnetic field, or introduce p-wave wave functions. Simple introduction of p-wave for gluon (introducing a factor of $\vec{k}_g$ in the hybrid wave function) changes the hybrid states we consider to the $J^{\rm PC} = 1^{-+}$ exotic hybrid mesons, those of most interest experimentally. In such cases, the decay amplitudes calculated in a LF scheme should exhibit the required angular dependence in the CMF of an exotic hybrid. In calculations using the standard form of dynamics, one should make sure that the boost symmetry is respected. However, already on the basis of our analysis of the non-exotic 0^{++} hybrid decays, we suggest that no matter what scheme one uses, a complete set of constraints of special relativity should be seriously taken into account in searches for a suitable constituent picture.

The authors acknowledge several helpful discussions with Adam Szczepaniak and Nikodem Popławski. This work was supported in part by Polish Ministry of Education and Science BST-975/BW-1640.

Appendix A

Basic definitions

Spinors we use are defined as

$$u_{mp\lambda} = B(p,m)\,u_{0\lambda}, \qquad v_{mp\lambda} = B(p,m)\,v_{0\lambda}, \tag{A.1}$$

where the operator $B(p,m)$

$$B(p,m) = \frac{1}{\sqrt{mp^+}}\left[\Lambda_+ p^+ + \Lambda_-(m + \alpha^\perp p^\perp)\right] \tag{A.2}$$

represents a boost that changes the mass m at rest into the four-momentum p. Spinors of fermions at rest $u_{0\lambda}$ are

$$u_{0\uparrow} = \sqrt{2m}\begin{pmatrix}\chi_+\\0\end{pmatrix}, \qquad u_{0\downarrow} = \sqrt{2m}\begin{pmatrix}\chi_-\\0\end{pmatrix}, \tag{A.3}$$

and for anti-fermions at rest $v_{0\lambda}$ are

$$v_{0\uparrow} = \sqrt{2m}\begin{pmatrix}0\\\chi_-\end{pmatrix}, \qquad v_{0\downarrow} = \sqrt{2m}\begin{pmatrix}0\\-\chi_+\end{pmatrix}, \tag{A.4}$$

where $\chi_\pm$ are two-component spinors, $\chi_+ = (1,0)$ and $\chi_- = (0,1)$. We use the convention $\Lambda_\pm = \frac{1}{2}\gamma_0\gamma^\pm$, where $\gamma^\pm = \gamma^0 \pm \gamma^3$.

The integration measure over momenta in a meson, denoted by $[k_1k_2]$ or just $[12]$, is

$$\frac{dk_1^+ d^2k_1^\perp}{2(2\pi)^3 k_1^+}\frac{dk_2^+ d^2k_2^\perp}{2(2\pi)^3 k_2^+} = \frac{dx_{12} d^2k_{12}^\perp}{2(2\pi)^3 x_{12}(1-x_{12})}\frac{dP_{12}^+ d^2P_{12}^\perp}{2(2\pi)^3 P_{12}^+}. \tag{A.5}$$

In terms of the three-vector $\vec{k}_{12}$, the integration measure for two constituents with the same mass m_q, is given by

$$\frac{dx_{12} d^2k_{12}^\perp}{2(2\pi)^3 x_{12}(1-x_{12})} = \frac{4d^3\vec{k}_{12}}{2(2\pi)^3\mathcal{M}_{12}}. \tag{A.6}$$

Therefore,

$$N_{pm}(\vec{k}_{12}) = \sqrt{\frac{\mathcal{M}_{12}}{2m_q}}. \tag{A.7}$$

Similarly,

$$N_{ps}(\vec{k}_{12}) = \sqrt{\frac{1}{\mathrm{Tr}\left[S_p^\dagger(1,2)S_p(1,2)\right]}}. \tag{A.8}$$

In the normalization equation for a hybrid meson, we have a three-particle integration measure

$$\prod_{i=1}^{3} \frac{dk_i^+ d^2 k_i^\perp}{16\pi^3 k_i^+} = \frac{4d^3 k_q}{2(2\pi)^3 \mathcal{M}_q} \frac{d^3 k_g \mathcal{M}_{qg}}{2(2\pi)^3 \sqrt{m_g^2 + k_g^2}\sqrt{\mathcal{M}_q^2 + k_g^2}} \,. \tag{A.9}$$

Therefore,

$$N_{hs}(\vec{k}_q, \vec{k}_g) = \sqrt{\frac{\mathcal{M}_q}{2m_q}} \sqrt{\frac{2m_q + m_g}{\mathcal{M}_{qg}}} \sqrt{\frac{\sqrt{m_g^2 + \vec{k}_g^{\,2}}}{m_g}} \sqrt{\frac{\sqrt{\mathcal{M}_q^2 + \vec{k}_g^{\,2}}}{2m_q}} \,, \tag{A.10}$$

and

$$N_{hs}(\vec{k}_q, \vec{k}_g) = \sqrt{\frac{1}{\sum\limits_{\mathrm{pol}} \mathrm{Tr}\left[S_h^\dagger(1,2,3) S_h(1,2,3)\right]}} \,, \tag{A.11}$$

where $\sum_{\mathrm{pol}}$ means a sum over two transverse polarizations of a gluon.

The decay amplitude of a hybrid into two mesons is

$$\begin{aligned} \mathcal{A}(p,b,h) &= (-1)\frac{2}{3}\frac{1}{\sqrt{2}}\frac{g_\lambda}{(16\pi^3)^2} \int \frac{dx_{14} d^2\kappa_{14}^\perp}{x_{14}(1-x_{14})} \int \frac{dx_{52} d^2\kappa_{52}^\perp}{x_{52}(1-x_{52})} \\ &\quad \times N_p N_b N_h \psi_p^*(1,4)\psi_b^*(5,2)\left[\mathfrak{A}(1,2,3,4,5) + \mathfrak{B}(1,2,3,4,5)\right] \\ &= -\frac{16}{3}\frac{g_\lambda}{(16\pi^3)^2} \int \frac{d^3 k_{52}}{\mathcal{M}_b} \int \frac{d^3 k_{14}}{\mathcal{M}_p} N_p N_b N_h \psi_p^*(\vec{k}_{14})\psi_b^*(\vec{k}_{52}) \\ &\quad \times \left[\mathfrak{A}(1,2,4,5) + \mathfrak{B}(1,2,4,5)\right] \,, \end{aligned}$$

where

$$\mathfrak{A}(1,2,4,5) = \frac{1}{x_3} T_A(1,2,3,4,5) A(1,2,3,4,5)\Big|_{k_3 = k_4 + k_5} \,, \tag{A.12}$$

$$\mathfrak{B}(1,2,4,5) = \frac{1}{x_3} T_B(1,2,3,4,5) B(1,2,3,4,5)\Big|_{k_3 = k_1 + k_2} \,, \tag{A.13}$$

$$A(1,2,3,4,5) = \psi_h(1,2,3) f_\lambda(\mathcal{M}_{45}^2) \,, \tag{A.14}$$

$$B(1,2,3,4,5) = \psi_h(5,4,3) f_\lambda(\mathcal{M}_{12}^2) \,, \tag{A.15}$$

and the spin factors in the decay amplitude are

$$T_A(1,2,3,4,5) = \mathrm{Tr}\left[S_p^\dagger(1,4) S_h(1,2,3) S_b^\dagger(5,2) S_{\mathrm{QCD}}(5,4,3)\right] \,, \tag{A.16}$$

$$T_B(1,2,3,4,5) = \mathrm{Tr}\left[S_p^\dagger(1,4) S_{\mathrm{QCD}}(1,2,3) S_b^\dagger(5,2) S_h(5,4,3)\right] \,. \tag{A.17}$$

Parts A and B refer to the two arrangements of quarks shown in Fig. 11. $S_{\rm QCD}$ is the spin factor coming from the interaction Hamiltonian of QCD:

$$\chi^\dagger_{s_1} S_{\rm QCD}(1,2,3)\chi_{s_2} = \bar{u}_1 \varepsilon^\mu_3 \gamma_\mu v_2\,. \tag{A.18}$$

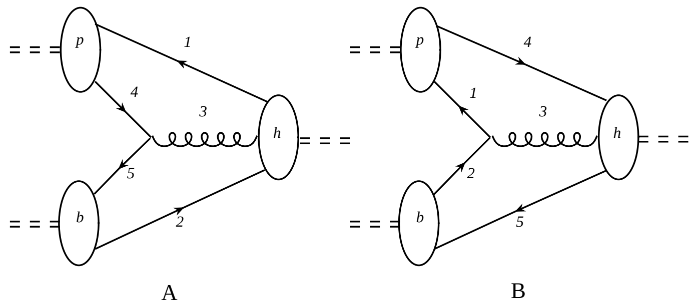

Fig. 11. Hybrid meson decay amplitude into two non-exotic mesons p (the light one) and b (the heavy one).

In both parts of the amplitude, A and B, one has $x_p = p^+/h^+$, $x_b = b^+/h^+ = 1 - x_p$. In meson p, one has $\vec{k}_{14} \equiv \vec{k}_p$, so that $\mathcal{M}_{14} \equiv \mathcal{M}_p = 2\sqrt{m_q^2 + \vec{k}_p^{\,2}}$, and the following relations hold:

$$x_{14} = (\sqrt{m_q^2 + \vec{k}_p^{\,2}} + k_p^3)/\mathcal{M}_p, \tag{A.19}$$

$$\begin{array}{lll} x_1 = x_{14}x_p, & k_1^+ = x_{14}p^+, & k_1^\perp = x_{14}p^\perp + k_p^\perp, \\ x_4 = (1-x_{14})x_p, & k_4^+ = (1-x_{14})p^+, & k_4^\perp = (1-x_{14})p^\perp - k_p^\perp. \end{array} \tag{A.20}$$

In meson b, one has $\vec{k}_{52} \equiv \vec{k}_b$, so that $\mathcal{M}_b \equiv \mathcal{M}_{52} = 2\sqrt{m_q^2 + \vec{k}_b^{\,2}}$, and the analogous relations are:

$$x_{52} = (\sqrt{m_q^2 + \vec{k}_b^{\,2}} + k_b^3)/\mathcal{M}_b, \tag{A.21}$$

$$\begin{array}{lll} k_5^+ = x_{52}b^+, & x_5 = x_{52}x_b, & k_5^\perp = x_{52}b^\perp + k_b^\perp, \\ k_2^+ = (1-x_{52})b^+, & x_2 = (1-x_{52})x_b, & k_2^\perp = (1-x_{52})b^\perp - k_b^\perp. \end{array} \tag{A.22}$$

Evaluating the quarks invariant masses in the hybrid and in the decay vertex, one obtains

$$\mathcal{M}_{12}^2 = (x_1 + x_2)\left[\frac{k_1^{\perp\,2} + m_q^2}{x_1} + \frac{k_2^{\perp\,2} + m_q^2}{x_2}\right] - (k_1^\perp + k_2^\perp)^2\,, \tag{A.23}$$

and

$$\mathcal{M}_{54}^2 = (x_5 + x_4)\left[\frac{k_5^{\perp\,2} + m_q^2}{x_5} + \frac{k_4^{\perp\,2} + m_q^2}{x_4}\right] - (k_5^{\perp} + k_4^{\perp})^2\,. \qquad \text{(A.24)}$$

The three-vectors: $\vec{k}_{12}$ in mesons, and $\vec{k}_{hq}$ and $\vec{k}_{hg}$ in the hybrid, are defined using

$$\left(\sqrt{\vec{k}^2 + m_1^2} + \sqrt{\vec{k}^2 + m_2^2}\right)^2 = \frac{\kappa^2 + m_1^2}{x} + \frac{\kappa^2 + m_2^2}{1-x} = \mathcal{M}^2\,, \qquad \text{(A.25a)}$$

$$\vec{k}^{\perp} = \kappa\,. \qquad \text{(A.25b)}$$

For $m_1 = m_2 = m$, one has

$$4\left(\vec{k}^2 + m^2\right) = \frac{\kappa^2 + m^2}{x(1-x)} = \mathcal{M}^2\,, \qquad \text{(A.26a)}$$

$$\vec{k}^{\perp} = \kappa\,. \qquad \text{(A.26b)}$$

In the part A of the decay amplitude, one has

$$\vec{k}_q^2 = \mathcal{M}_{12}^2/4 - m_q^2\,, \qquad \text{(A.27)}$$

and

$$\vec{k}_g^2 = \frac{\left[\mathcal{M}_{123}^2 - (\mathcal{M}_{12} + m_g)^2\right]\left[\mathcal{M}_{123}^2 - (\mathcal{M}_{12} - m_g)^2\right]}{4\mathcal{M}_{123}^2}\,, \qquad \text{(A.28)}$$

where

$$\mathcal{M}_{123}^2 = \frac{k_1^{\perp\,2} + m_q^2}{x_1} + \frac{k_2^{\perp\,2} + m_q^2}{x_2} + \frac{(k_1 + k_2)^{\perp\,2} + m_g^2}{1 - x_1 - x_2} - (k_1 + k_2 + k_3)^{\perp\,2}\,. \qquad \text{(A.29)}$$

Similarly, in the part B, one has

$$\vec{k}_q^2 = \mathcal{M}_{54}^2/4 - m_q^2\,, \qquad \text{(A.30)}$$

and

$$\vec{k}_g^2 = \frac{\left[\mathcal{M}_{543}^2 - (\mathcal{M}_{54} + m_g)^2\right]\left[\mathcal{M}_{543}^2 - (\mathcal{M}_{54} - m_g)^2\right]}{4\mathcal{M}_{543}^2}\,, \qquad \text{(A.31)}$$

where

$$\mathcal{M}_{543}^2 = \frac{k_5^{\perp\,2} + m_q^2}{x_5} + \frac{k_4^{\perp\,2} + m_q^2}{x_4} + \frac{(k_5 + k_4)^{\perp\,2} + m_g^2}{1 - x_5 - x_4} - (k_5 + k_4 + k_3)^{\perp\,2}\,. \qquad \text{(A.32)}$$

Appendix B

Cross-checks

All our calculations of spin factors were done using two independent methods. One method was to first reduce the spin factors to 2×2 matrices sandwiched between two component spinors χ_σ, using Eq. (12) for the gluon polarization four-vector ε^μ in $A^+ \equiv 0$ gauge. The spin factors were obtained from the trace of the product of 2×2 matrices. The other method made use of the expressions for $\sum u\bar{u}$, $\sum v\bar{v}$ and $\sum \varepsilon^{*\mu}\varepsilon^\nu$. The spin factors were obtained using properties of traces of products of γ matrices. In the evaluation of spin factors, we assume that the gluon is massless and has only two degrees of freedom. But the gluon acquires an effective mass dynamically. Therefore, we used $k_g^2 = m_g^2$ in the momentum-dependent factor of the wave function. In k_3^μ and P_{123}^ν in the lattice-inspired spin factors, we checked what happens in both cases, *i.e.*, when one inserts $k_3^2 = m_g^2$ or $k_3^2 = 0$.

The six-dimensional integrals were carried out using Monte Carlo integration (using the procedure VEGAS [23]). The accuracy of the results of integration (standard deviation output from VEGAS) is shown as error bars in plots, unless the error is smaller than the size of a point on a plot. The VEGAS calculations using C were checked against iterative Gauss quadrature, also in C, and against a separate FORTRAN program performing the same calculations in several representative (but not all regular) cases.

Appendix C

Illustrative examples of minimization

This Appendix provides examples of numerical evidence that we have gathered in all cases (many more than given here) we studied. The figures show how stddev (standard deviation) or maxdev (maximal deviation), both in ratio to the amplitude averaged over the angle θ, change around a global minimum when one changes just one parameter in the wave functions or the effective Hamiltonian width λ. The varied parameter is on the horizontal axis, and the deviation on the vertical axis, stddev marked on the right-hand scale (plotted with black squares) and the maxdev marked on the left-hand scale (plotted with circles).

The examples demonstrate the dominant feature that the parameters β_{hq} and λ are strongly correlated with each other and both much larger than all other parameters. The first example with the hybrid wave function with the spin factor $\bar{u}\not{\varepsilon}v$ and $N \neq 1$. The example illustrates our argument for that the parameters β_{hq} and λ must be both much larger than all other parameters: the minimum resembles one side of a broad valley open toward large values. The remaining four examples concern the cases with various spin factors and always $N = 1$. The examples illustrate that β_{hq} and λ are both strongly correlated and much larger than all other parameters.

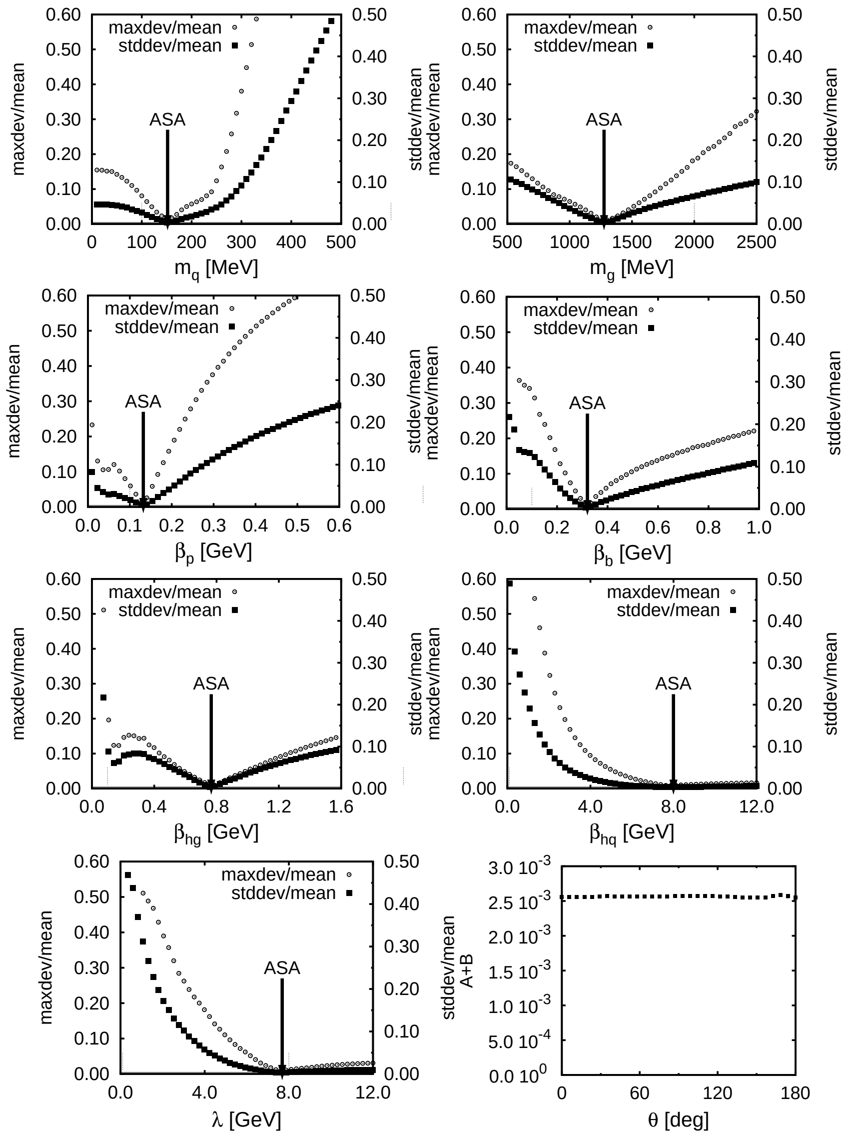

Fig. 12. Variation of rotational symmetry violation *versus* changes of parameters in the wave functions in the case **a** in Fig. 6, *i.e.* $S_h = \bar{u}\not{\epsilon}v$, with extra factors $N \neq 1$, decay into two $J^{\rm PC} = 0^{++}$ mesons. The optimal values of the parameters are given in the first column in Table III. The arrow marked "ASA" points toward the optimal value of a parameter. The last plot shows the amplitude itself for the optimal parameters.

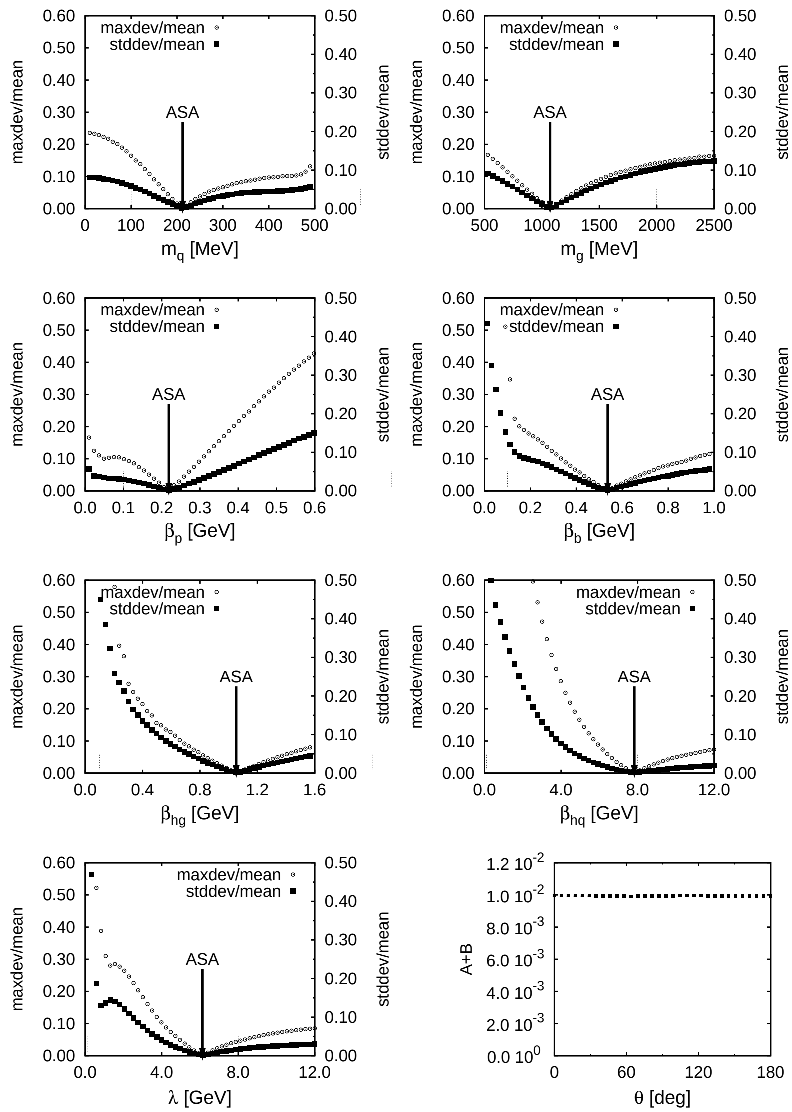

Fig. 13. Variation of rotational symmetry violation *versus* changes of parameters in the wave functions in the case **b** in Fig. 6, *i.e.* $S_h = \bar{u}\not\epsilon v$, with $N = 1$, decay into two $J^{\mathrm{PC}} = 0^{++}$ mesons. The optimal values of the parameters are given in the second column in Table IV. The arrow marked "ASA" points toward the optimal value of a parameter. The last plot shows the amplitude itself for the optimal parameters.

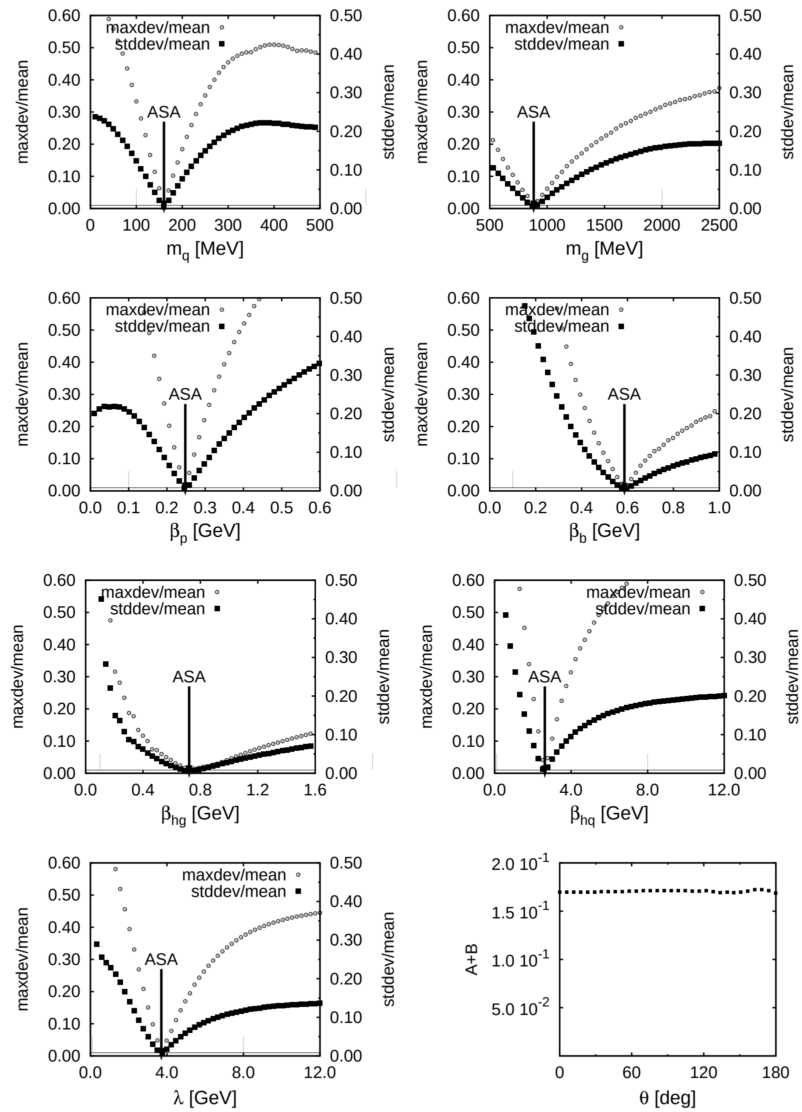

Fig. 14. Variation of rotational symmetry violation *versus* changes of parameters in the wave functions in the case **b** in Fig. 8, *i.e.* $S_h = \bar{u}\not{\epsilon}v$, with $N = 1$, decay into two $J^{\mathrm{PC}} = 0^{-+}$ mesons. The optimal values of the parameters are given in the second column in Table V. The arrow marked "ASA" shows value of parameter found in minimization. The last plot shows the amplitude itself for the optimal parameters.

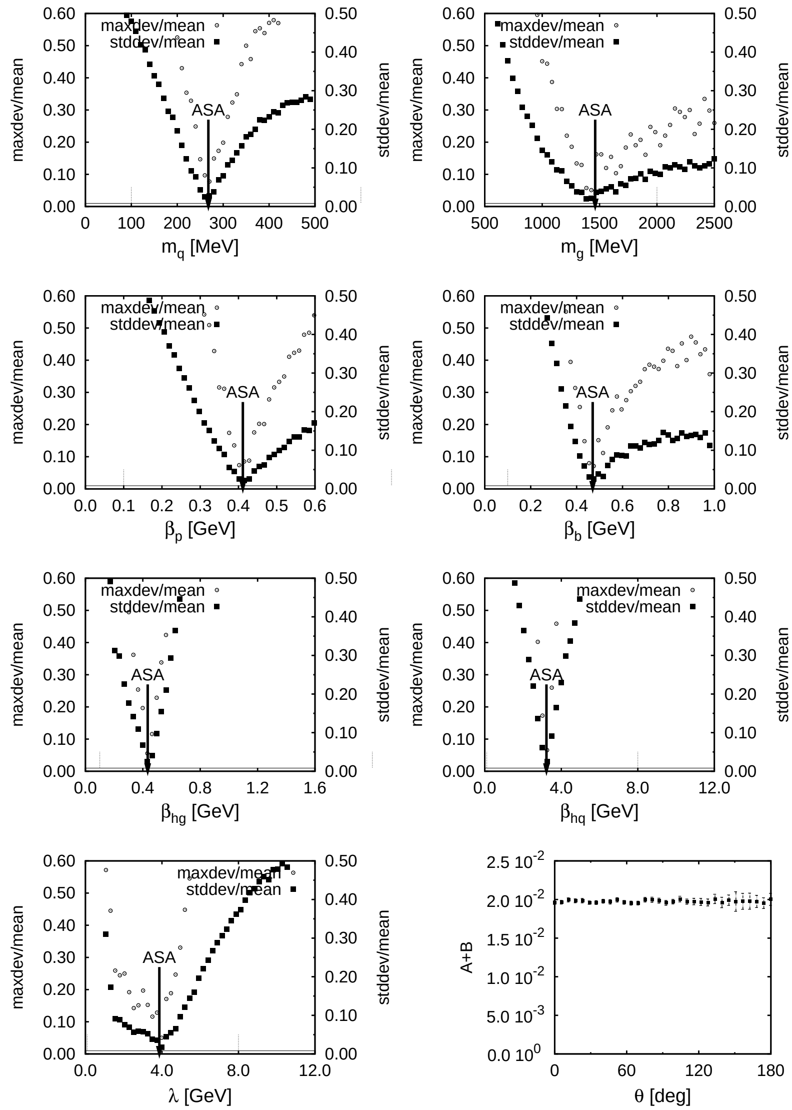

Fig. 15. Variation of rotational symmetry violation *versus* changes of parameters in the wave functions in the case **f** in Fig. 7, *i.e.*, $S_h = \bar{u}\tilde{G}\tilde{P}v$, with $N = 1$, decay into two $J^{\rm PC} = 0^{++}$ mesons. The optimal values of the parameters are given in the sixth column in Table IV. The arrow marked "ASA" points toward the optimal value of a parameter. The last plot shows the amplitude itself for the optimal parameters.

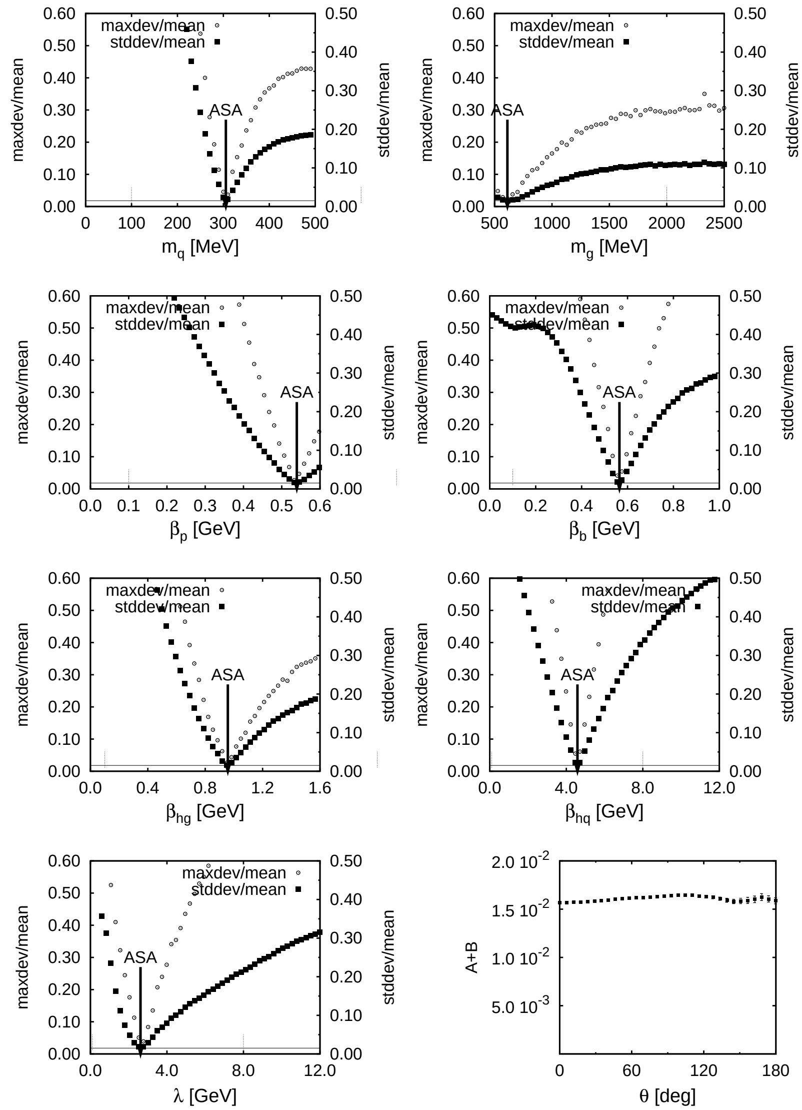

Fig. 16. Variation of rotational symmetry violation *versus* changes of parameters in the wave functions in the case **f** in Fig. 9 $S_h = \bar{u}\tilde{G}\tilde{P}v$, with $N = 1$, decay into two $J^{\mathrm{PC}} = 0^{-+}$ mesons. The optimal values of the parameters are given in the sixth column in Table VI. The arrow marked "ASA" points toward the optimal value of a parameter. The last plot shows the amplitude itself for the optimal parameters.

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