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SIMPLE SOLUTIONS OF FIREBALL HYDRODYNAMICS FOR SELF-SIMILAR ELLIPSOIDAL FLOWS

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A new family of simple, analytic solutions of self-similarly expanding fireballs is found for systems with ellipsoidal symmetry and a direction dependent, generalized Hubble flow. Gaussian, shell like or oscillating density profiles emerge for simple choices of an arbitrary scaling function. New, cylindrically or spherically symmetric as well as approximately one dimensional hydrodynamical solutions are obtained for various special choices of the initial conditions.

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1. Introduction

Hydrodynamics is describing the local conservation of matter, momentum and energy. Due to this nature, hydrodynamical solutions are applied to a tremendous range of physical phenomena ranging from the stellar dynamics to the description of high energy collisions of heavy ions as well as collisions of elementary particles. Some of the most famous hydrodynamical solutions, like the Hubble flow of our Universe or the Bjorken flow in ultra-relativistic heavy ion physics have the properties of self-similarity and scale-invariance. Heavy ion collisions are known to create three dimensionally expanding systems. In case of non-central collisions, cylindrical symmetry is violated, but an *ellipsoidal* symmetry can be well assumed to characterize the final state. The data motivated, spherically or cylindrically symmetric hydrodynamical parameterizations and/or solutions of Refs. [1–10] are generalized here to the case of such an ellipsoidal symmetry, providing new families of exact analytic hydrodynamical solutions.

2. The new family of self-similar ellipsoidal solutions

The non-relativistic (NR) hydrodynamical systems are specified by the continuity, Euler and energy equations:

$$\partial_t n + \nabla(n\boldsymbol{v}) \;=\; 0\,, \tag{1}$$

$$\partial_t \boldsymbol{v} + (\boldsymbol{v}\nabla)\boldsymbol{v} \;=\; -\frac{\nabla p}{mn}\,, \tag{2}$$

$$\partial_t \varepsilon + \nabla(\varepsilon\boldsymbol{v}) \;=\; -p\nabla\boldsymbol{v}\,. \tag{3}$$

Here n denotes the particle number density, $\boldsymbol{v}$ stands for the NR flow velocity field, ε for the energy density, p for the pressure and in the following the temperature field is denoted by T. These fields depend on the time t as well as on the coordinates $\boldsymbol{r} = (r_x, r_y, r_z)$. We assume, for the sake of simplicity, the following equations of state,

$$p \;=\; nT\,, \qquad \varepsilon = \kappa p\,, \tag{4}$$

which close the set of equations for n, $\boldsymbol{v}$ and T. The NR ideal gas corresponds to $\kappa = 3/2$. The new family of exact analytic solutions of the hydrodynamical problem are given for arbitrary values of $\kappa > 0$, $m > 0$. The hydro solutions are determined by the choice of a positive function of a non-negative, real variable $\mathcal{T}(s)$, corresponding to the (dimensionless) scaling function of the temperature. The scaling variable s is defined as

$$s \;=\; \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2}\,. \tag{5}$$

There the scale parameters depend on time, $(X, Y, Z) = (X(t), Y(t), Z(t))$. The ellipsoidal symmetry of the solutions is reflected by the ellipsoidal family of surfaces given by the equation $s = s_0 = \mathrm{const.}$ The temperature and the density field depend on the coordinates (r_x, r_y, r_z) only through the scaling variable s.

The new family of elliptically symmetric solutions of fireball hydrodynamics is given by

$$n(t,\boldsymbol{r}) \;=\; n_0 \frac{V_0}{V}\, \nu(s)\,, \tag{6}$$

$$\boldsymbol{v}(t,\boldsymbol{r}) \;=\; \left(\frac{\dot X}{X}\, r_x, \frac{\dot Y}{Y}\, r_y, \frac{\dot Z}{Z}\, r_z\right)\,, \tag{7}$$

$$T(t,\boldsymbol{r}) \;=\; T_0 \left(\frac{V_0}{V}\right)^{1/\kappa} \mathcal{T}(s)\,, \tag{8}$$

$$\nu(s) \;=\; \frac{1}{\mathcal{T}(s)} \exp\left(-\frac{T_i}{2T_0} \int_0^s \frac{du}{\mathcal{T}(u)}\right)\,, \tag{9}$$

where the constant n_0 is given by $n_0 = n(t_0, \mathbf{0})$, the typical volume of the expanding system is $V = XYZ$, the initial volume being $V_0 = V(t_0)$, the dimensionless scaling function of the density profile is denoted by $\nu(s)$, the constant T_0 is defined by $T_0 = T(t_0, \mathbf{0})$ and a constant of integration is denoted by T_i. The definitions of n_0 and T_0 correspond to the normalization $\nu(s = 0) = 1$ and $\mathcal{T}(s = 0) = 1$. Initially, only one of the temperature and density profiles can be chosen as an arbitrary positive function, the equations of state relates the density and temperature profiles, resulting in the matching condition for the profile functions, as expressed by Eq. (9).

The equations of motion of the scale parameters are

$$X\ddot{X} = Y\ddot{Y} = Z\ddot{Z} = \frac{T_i}{m}\left(\frac{V_0}{V}\right)^{1/\kappa} . \tag{10}$$

This time evolution of the radius parameters X, Y and Z is equivalent to the classical motion of a particle in a non-central potential, governed by the Hamiltonian

$$H = \frac{1}{2m}\left(P_x^2 + P_y^2 + P_z^2\right) + \kappa T_i \left(\frac{X_0 Y_0 Z_0}{XYZ}\right)^{1/\kappa} , \tag{11}$$

where the canonical coordinates are (X, Y, Z) and the canonical momenta are $(P_x, P_y, P_z) = m(\dot{X}, \dot{Y}, \dot{Z})$. This generalizes earlier results [9, 10] from $\kappa = 3/2$ and $\mathcal{T}(s) \equiv 1$ to and to arbitrary $\mathcal{T}(s) > 0$ and $\kappa > 0$.

The conservation of energy by the classical Hamiltonian motion determines the physical meaning of the constant of integration T_i as the initial potential energy corresponding to the initial internal energy of the fireball:

$$E_{\rm tot} \;=\; \frac{m}{2}\left(\dot{X}_0^2 + \dot{Y}_0^2 + \dot{Z}_0^2\right) + \kappa T_i \,, \tag{12}$$

where the total energy is denoted by $E_{\rm tot}$, and the initial velocities are denoted by $(\dot{X}_0, \dot{Y}_0, \dot{Z}_0)$. Due to the repulsive nature of the potential, the coordinates diverge for large values of t. The asymptotic velocities tend to constants [9] of $(\dot{X}_{\rm as}, \dot{Y}_{\rm as}, \dot{Z}_{\rm as})$,

$$E_{\rm tot} \;=\; \frac{m}{2}\left(\dot{X}_{\rm as}^2 + \dot{Y}_{\rm as}^2 + \dot{Z}_{\rm as}^2\right) . \tag{13}$$

This completes the specification of the new family of solutions of fireball hydrodynamics with ellipsoidal symmetry. The form of the dimensionless scaling function $\mathcal{T}(s)$ can be chosen freely from among the positive functions of a non-negative variable. This freedom corresponds to a freedom in the specification of the initial conditions. Thus (uncountably) infinite new solutions of NR hydrodynamics are found.

3. Self-similarity of the elliptic hydro of solutions

Each of these new hydrodynamical solutions is scale invariant:

$$\boldsymbol{r}' = (r_x \frac{X_0}{X}, r_y \frac{Y_0}{Y}, r_z \frac{Z_0}{Z}), \tag{14}$$

$$n(t, \boldsymbol{r}) = n(t_0, \boldsymbol{r}') \left(\frac{X_0 Y_0 Z_0}{XYZ} \right), \tag{15}$$

$$v_x(t, \boldsymbol{r}) = v_x(t_0, \boldsymbol{r}') \frac{\dot{X}}{\dot{X}_0}, \ \ldots \tag{16}$$

$$T(t, \boldsymbol{r}) = T(t_0, \mathbf{r}') \left(\frac{X_0 Y_0 Z_0}{XYZ} \right)^{1/\kappa}. \tag{17}$$

Scale invariance of these solutions is equivalent to their self-similarity. The profile functions depend on time only through the scale parameters (X, Y, Z) and on the coordinates only through the scale parameter s.

4. Limiting cases

The various physically interesting limiting cases fall into two classes. The first class of limiting cases corresponds to various additional symmetry properties imposed on the scaling variable of Eq. (5). The second class of limiting cases corresponds to various choices of $\mathcal{T}(s)$, the scaling function of the temperature. The Lagrangian equations of motion for the scale parameters, Eqs. (10) as well as the shape of the flow velocity field, Eq. (7), are the same for all the choices of $\mathcal{T}(s)$. Trivial prefactors are given in Eqs. (6), (8). Hence these equations will not be repeated in the forthcoming discussion. We provide some physically interesting examples for the matching pair of density and temperature scaling functions $(\nu, \mathcal{T})$ that satisfy Eq. (9).

5. Spherically symmetric family of solutions

By assuming that initially all the scale parameters as well as their time derivatives are equal, $X_0 = Y_0 = Z_0 \equiv R_0$ and $\dot{X}_0 = \dot{Y}_0 = \dot{Z}_0 \equiv \dot{R}_0$, the ellipsoidal family of solutions reduces to the spherical family of solutions of Ref. [7] with a scale parameter $X = Y = Z \equiv R$. The scaling variable and the equations of motion simplify to

$$s = \frac{\boldsymbol{r}^2}{R^2}, \qquad R\ddot{R} = \frac{T_i}{m} \left(\frac{R_0^3}{R^3} \right)^{1/\kappa}. \tag{18}$$

This family generalizes the Zimányi–Bondorf–Garpman (ZGB) solution [1], the spherical Gaussian solution [3], and the Buda–Lund type of hydro solutions [7] to arbitrary scaling functions $\mathcal{T}(s) > 0$ and equation of state parameters $\kappa > 0$. If $\dot{R}_0 = 0$, and the asymptotic velocity of the expansion is $\dot{R}^2_{\rm as} = \langle u \rangle^2 = T_i/m$, and if $\kappa = 3/2$, the equation of motion of the scale parameter is solved in a simple form [7] as $R^2 = R_0^2 + \langle u \rangle^2 (t-t_0)^2$.

6. Cylindrically symmetric family of solutions

By imposing cylindrically symmetric initial conditions, $X_0 = Y_0 = R_{t0}$, and $\dot{X}_0 = \dot{Y}_0 = \dot{R}_{t0}$, one finds that $X = Y = R_t$, as the equations of motion preserve cylindrical symmetry. Introducing $r_t = \sqrt{r_x^2 + r_y^2}$, the scale parameter s and the equations of motion for the longitudinal and the transverse scales read as

$$s \;=\; \frac{r_t^2}{R_t^2} + \frac{r_z^2}{Z^2}\,, \qquad\qquad R_t\ddot{R}_t \;=\; Z\ddot{Z} \;=\; \frac{T_i}{m}\left(\frac{R_{t0}^2 Z_0}{R_t^2 Z_0}\right)^{1/\kappa} . \tag{19}$$

These generalize the equations of motion for scales of the cylindrically symmetric, De–Garpman–Sperber–Bondorf–Zimányi (DGSBZ) solution of Ref. [2] to arbitrary $0 < \kappa \neq 3/2$ and to arbitrary $\mathcal{T}(s) > 0$.

7. One dimensional expansions

The equations of motion of parameters (X, Y, Z) has been studied for the case of $\kappa = 3/2$ in Ref. [9]. Although the expansion is generally 3 dimensional, a big initial compression in one of the directions (r_z) was shown to result in an effectively 1 dimensional expansion in this direction, corresponding to Landau type initial conditions. In this case, an analytic solution for the variable Z is given in Eqs. (23)–(25) of Ref. [9], and the conditions of validity of this approximation were given there by Eqs. (26)–(28). These equations can be further simplified during the late stage of the expansion, when acceleration effects are small. In this limiting case, both Eqs. (10) and the conservation of energy are satisfied by the asymptotic solution:

$$\dot{X}_a \simeq \dot{X}_0\,, \qquad \dot{Y}_a \;\simeq\; \dot{Y}_0\,, \qquad \frac{1}{2}\,m\dot{Z}_a^2 \;\simeq\; \frac{3}{2}\,T_i\,, \tag{20}$$

$$X(t) \;\simeq\; X_0 + \dot{X}_a t\,, \tag{21}$$

$$Y(t) \;\simeq\; Y_0 + \dot{Y}_a t\,, \tag{22}$$

$$Z(t) \;\simeq\; Z_0 + \dot{Z}_a t\,. \tag{23}$$

For simplicity, here we utilized $t_0 = 0$. This asymptotic approximate solution is valid if the conditions of validity of the 1 dimensional expansion given in Ref. [9] are satisfied simultaneously with the following constraints: $Z_0 \ll \dot{Z}_a t$, $X_0 \gg \dot{X}_0 t$ and $Y_0 \gg \dot{Y}_0 t$. Alternatively $\sqrt{m/(3T_i)}Z_0 \ll t \ll$

$\min(X_0/\dot{X}_0, Y_0/\dot{Y}_0)$. Under these conditions all the initial internal energy is converted into kinetic energy in the longitudinal direction, while the kinetic energy in the transverse components is conserved during the time evolution. For $\dot{X}_0 = \dot{Y}_0 = 0$, a one dimensional expansion is obtained.

Various choices for the shape of the temperature scaling function $\mathcal{T}(s)$ generate interesting forms of the hydrodynamical solutions in all the ellipsoidal, cylindrical, spherical or 1 dimensionally expanding classes.

8. Gaussian solutions

The simplest possible choice for the temperature scaling function is $\mathcal{T}(s) = 1$. After a trivial scale transformation, $(X, Y, Z) \to \sqrt{T_i/T_0}(X, Y, Z)$,

$$\nu(s) \; = \; \exp\left(-\frac{s}{2}\right) , \qquad \mathcal{T}(s) = 1 \,. \tag{24}$$

The density profiles are Gaussians and the temperature distribution becomes spatially homogeneous, as follows from Eq. (5), and we recover the elliptic Gaussian solutions described in Refs. [9, 10]. If freeze-out happens at a constant value of the local temperature, $T(t, \boldsymbol{r}) = T_{\mathrm{f}}$, these Gaussian hydrodynamical solutions corresponds to a sudden freeze-out at a constant time, $t = t_{\mathrm{f}}$, in the whole volume of the fireball. Remarkable features of this model are that *(i)* the slope parameters of the transverse momentum spectra increase linearly with mass, with the coefficient of linearity depending on the relative direction to that of the impact parameter, *(ii)* the parameters of the two-particle Bose–Einstein correlation functions oscillate as a function of the angle between the event plane and the transverse momentum of the pair [10].

9. De–Garpman–Sperber–Bondorf–Zimányi solution

One can require that the temperature and the density profiles are described as different powers of the same profile function. Such a similarity is achieved if the temperature and the corresponding density profiles are

$$\mathcal{T}(s) \; = \; (1-s)\, \Theta(1-s) \,, \tag{25}$$

$$\nu(s) \; = \; (1-s)^{\alpha}\, \Theta(1-s) \,, \qquad \alpha = \frac{T_i}{(2T_0)} - 1 \,. \tag{26}$$

These profile functions generalize the spherical ZGB solution [1, 7], and the cylindrically symmetric DGSBZ solution [2] to asymmetric ellipsoids and arbitrary values of κ. The physical meaning of the parameter α is determined here, in terms of the total initial internal energy T_i and the initial central value T_0 of the temperature field.

10. Elliptic Buda–Lund solutions

The Buda–Lund hydro model (BL-H) was developed for the description of single-particle spectra and two-particle Bose–Einstein correlation functions in high energy heavy ion collisions at CERN SPS [4, 5]. The BL-H attempted to characterize the temperature, density and flow fields by their means and variances only, its essential property is that it simultaneously involves a temperature gradient parameter and a Hubble-like flow profile. In its original form, the model was cylindrically symmetric and the (longitudinal) flow profile was relativistic [4, 5]. BL-H type of exact hydro solutions correspond to the scaling functions

$$\mathcal{T}(\int) = \frac{1}{1+bs}, \qquad b = \frac{1}{2}\left\langle \frac{\Delta T}{T} \right\rangle, \tag{27}$$

$$\nu(s) = (1+bs)\exp\left[-\frac{T_i}{2T_0}\left(s + \frac{bs^2}{2}\right)\right]. \tag{28}$$

The dimensionless parameter b is interpreted as a measure of the transverse temperature inhomogeneity [4, 11]. Regardless of the symmetry classes, the BL-H density profile has an approximately Gaussian, conventional shape if $b < T_i/(2T_0)$. On the other hand, the density profile looks like an ellipsoidal ring of fire, with a density minimum at the center and a density pile-up on the surface, if $b > T_i/T_0$. In the spherically symmetric case, similar morphological classes of BL-H solutions were found in Ref [7]. Introducing the notation $T_i = m\langle u\rangle^2$, one finds that

$$\frac{m\langle u\rangle^2}{T_0} > \left\langle \frac{\Delta T}{T} \right\rangle$$

yields ellipsoidal, expanding fireball BL-H solutions, Fig. 1. A detailed analysis of correlations and spectra in Pb + Pb collisions at CERN SPS indicated [11] such a behavior, corresponding to big and expanding fireballs. On the other hand, the analysis of correlations and spectra of $h + p$ reactions at CERN SPS indicated [12–14] small transverse flow and big transverse temperature inhomogeneity,

$$\frac{m\langle u\rangle^2}{T_0} < \left\langle \frac{\Delta T}{T} \right\rangle.$$

This case corresponds to the formation of a shell of fire. Such an example is shown in Fig. 2.

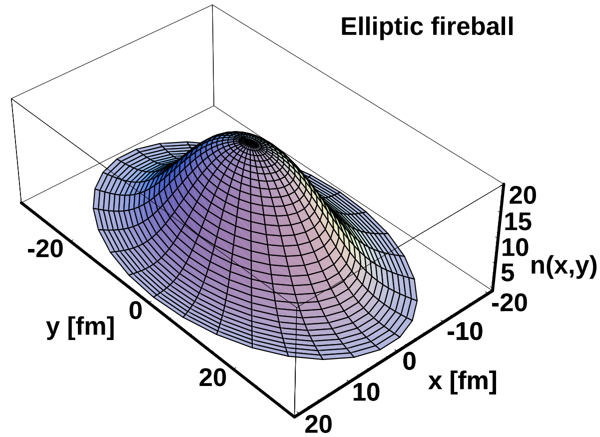

Fig. 1. An approximately Gaussian BL-H fireball profile is shown for $X = 5$ fm, $Y = 8$ fm, $m/T_0 = 1$, $\langle u \rangle = 0.7$, $\langle \Delta T/T \rangle = 0.1$. The vertical scale is arbitrary, the longitudinal coordinate is $r_z = 0$. For the time dependence see Eqs. (10).

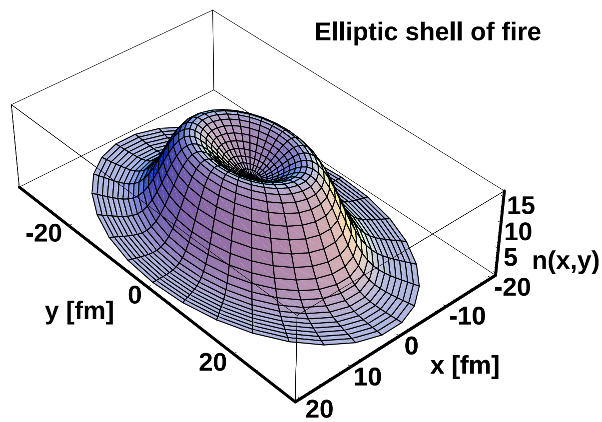

Fig. 2. A BL-H shell profile with $X = 5\,\mathrm{fm}$, $Y = 8\,\mathrm{fm}$, $m/T_0 = 1$, $\langle u \rangle = 0.5$, $\langle \Delta T/T \rangle = 0.71$, otherwise as Fig. 1.

11. Fireballs with density waves

Here we show that a simple choice of the temperature scaling function can lead to periodically oscillating temperature and density waves in exact solutions of NR hydrodynamics. A pair of oscillating scaling functions is, for example

$$\mathcal{T}(s) = (1 + \alpha \cos \beta s)^{-1} , \tag{29}$$

$$\nu(s) = (1 + \alpha \cos \beta s) \exp \left[-\frac{T_i}{2T_0} (s - \frac{\alpha}{\beta} \sin \beta s) \right] . \tag{30}$$

where parameters (α, β) correspond to the amplitude and the period of the oscillations, respectively, as shown in Fig. 3. As time evolves, the shapes of the ellipsoids change in coordinate space: the bigger the initial compression, the faster the expansion in that direction, corresponding to Eqs. (10).

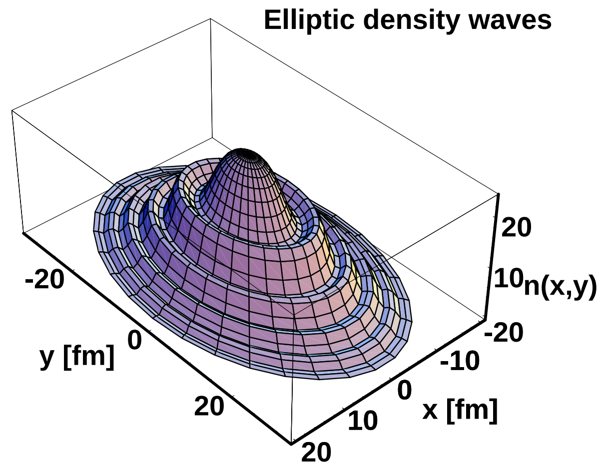

Fig. 3. An oscillating wave-like density profile is obtained, for $X = 5\,\mathrm{fm}$, $Y = 8\,\mathrm{fm}$, $T_i/T_0 = 1$, $\alpha = 0.2$, $\beta = 2.0$.

12. Connection with other exact solutions of fireball hydrodynamics

Before summarizing the results let us also relate the presented family of exact solutions of hydrodynamics to presently known, exact solutions of relativistic hydrodynamics. One of the most well known relativistic hydrodynamical solution is the 1+1 dimensional Landau–Khalatnikov solution, described in Refs. [15–17]. The initial condition here is a uniformly heated piece of matter initially at rest, the equation of state is that of an ultra-relativistic gas with three degrees of freedom, but expanding only 1+1 dimensions. Landau has shown that such expansions lead to an approximately Gaussian rapidity distribution. This Landau–Khalatnikov solution is presently the only known exact solution of relativistic hydrodynamics that describes an exploding fireball with relativistic acceleration. These solutions are, however, not self-similar, and not explicit, one dimensional solutions, hence they are not easily related to the exact, explicit, self-similar and accelerating non-relativistic solutions described in the present manuscript.

However, it is interesting to note, that in the late time limit, the acceleration of the scale parameters vanishes in the non-relativisitic self-similar solutions described here, as given by Eqs. (21)–(23). Hence for very late time, the velocity field becomes spherically symmetric, $\boldsymbol{v} \rightarrow \boldsymbol{r}/t$, which corresponds, within the lightcone, to a well known solution of relativistic hydrodynamics: $u^\mu(x) = x^\mu/\tau$, which is in the 1+1 dimensional case the reknown velocity distribution of the Hwa–Bjorken solution of relativistic hydrodynamics, given in Refs. [18, 19], while in 1+3 dimensions $\boldsymbol{v} = \boldsymbol{r}/t$ or $u^\mu(x) = x^\mu/\tau$ is referred to as the Hubble flow, as it corresponds to the flow velocity field of galaxies in an expanding Friedmann universe. These solutions are boost-invariant and both in the 1+1 dimensional Hwa–Bjorken and in the 1+3 dimensional Hubble case, corresponding to a flat rapidity distribution. The initial boundary conditions are given only within a lightcone, on a $\tau = \tau_0$ boost-invariant hypersurface, the equation of state is characterized by a constant or piecewise constant speed of sound, and the solutions are explicit, accelerationless and self-similar flows.

In Ref. [20] these boost-invariant self-similar solutions were generalized to 1+1 dimensional, non-boost invariant, self-similar, accelerationless solutions, using a broad class of equations of state: $\varepsilon = mn + \kappa p$, $p = nT$. The initial conditions were given, similarity to the case of the Hwa–Bjorken solution, on a boost-invariant hypersurface with $\tau = \tau_0$, but assuming an inhomogeneous initial temperature profile and a corresponding matching initial density profile. In the same paper this solution is extended to axially symmetric, three dimensionally expanding fireballs, that represent the late stages of central heavy ion collisions. However, for non-central collisions, axial symmetry is too restrictive and due to this reason we have generalized these solutions for ellipsoidally expanding relativistic fireballs, and the 1+1 dimensional solution, the 1+3 dimensional axially symmetric solution and the 1+3 dimensional ellipsoidal solutions were written up also in a series of papers for the 3-rd Budapest Winter School on Heavy Ion Physics, published in Refs. [21–23]. These self-similar, relativistic hydrodynamical solutions, in particular, the ellipsoidal expansion described in Ref. [23], correspond exactly to the late time limit of the non-relativistic solutions described here. See Ref. [23] for further details on such a correspondence as well as for a more complete list of citations on early papers on the Hwa–Bjorken solution.

It is worthwhile to mention that there are additional, recently found exact solutions of relativistic hydrodynamics that find accelerationless generalized Hubble type of solutions, with direction dependent Hubble constants. The first of these class of solutions has been found by Bíró in Refs. [24, 25], for cylindrically symmetric expansions at the softest point of the equation of state. Sinyukov and Karpenko recently published a solution which can be considered as the generalization of Refs. [24, 25], as well as that of Ref. [23], for the pre-asymptotic, accelerationless stage of the expansion, when the scales expand already linearly in time, corresponding to Eqs. (21)–(23), but before the time period when the off-sets (X_a, Y_a, Z_a) can be considered negligibly small. The Bíró as well as the Sinyukov–Karpenko solutions are also self-similar and accelerationless relativistic solutions, their shortcoming is that they are valid in the medium only when the pressure is a temperature independent constant, or, when the boundary condition is an expansion to the vacuum, they are obtained only for the special "dust" equation of state, that corresponds to a uniformly vanishing pressure, $p = p_0 = 0$. These solutions can thus be considered as a pre-asymptotic, relativistic solutions corresponding to very late stages of the fireball expansions.

This brief review of known exact solutions of fireball hydrodynamics in the relativistic kinematic domain indicates, that we find a correspondence between the late stages of the presented exact, accelerating, self-similar solutions of non-relativistic, ellipsoidally symmetric fireballs and the known class of relativistic but accelerationless, self-similar, ellipsoidally symmetric solutions of hydrodynamics. The missing link between them is the class of relativistic, accelerating, explicit solutions of hydrodynamics that use a realistic equation of state. The search for this missing class of exact and explicit solutions has been started and the first results will soon be reported elsewhere.

13. Summary

The non-relativistic hydrodynamical problem has been solved for expanding fireballs with ellipsoidal symmetry for the class of self-similar expansions. The flow velocity distribution is a generalized Hubble field in all the cases. An exact solution is assigned to each positive, integrable function of a non-negative variable. The time evolution of the (X, Y, Z) scale parameters corresponds to a Hamiltonian motion of a mass point in a non-central, repulsive potential. The density profiles may be of fireball type, or they may form one or more shells of fire. For initial conditions with higher symmetry, one dimensional, cylindrical and spherical expansions are obtained.

These results provide analytic insight into the time evolution of expanding fireballs with nontrivial, ellipsoidally symmetric morphology, a non-polynomially hard algorithmic problem that is difficult to solve even numerically on von Neumann type computers.

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