# VECTOR MESON DOMINANCE \*

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Historically vector meson physics arose along two different paths to be reviewed in Sections 1 and 2. In Section 3, the phenomenological consequences will be discussed with an emphasis on those aspects of the subject matter relevant in present-day discussions on deep inelastic scattering in the diffraction region of low values of the Bjorken variable.

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### 1. The gauge principle applied to properties of hadrons

In the 1960ies, among particle theorists, there reigned the fairly widespread opinion that the theory of strongly interacting particles was to be formulated in terms of the unitarity and analyticity properties of the S-matrix by themselves, rather than relying on local quantum field theory. All hadronic states being considered as equally elementary and at the same footing, their dynamics was conjectured to be determined by the "bootstrap conditions" intensively put forward by Chew [1].

In his 1960 paper entitled "Theory of strong interactions", published in Annals of Physics [2], J.J. Sakurai advocated an entirely different point of view. Starting from the success of the principles of quantum field theory in quantum electrodynamics (QED), he emphasized that one should expect these general principles to also hold for the physics of strong interactions. In particular, the notion of conserved currents, the gauge principle and the universality of couplings should be applied to strong-interaction physics. Taking advantage of the 1954 Yang and Mills generalization [3] of U(1) gauge invariance from QED to local SU(2) gauge transformations, Sakurai predicted

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the existence of vector mesons coupled to the hadronic isospin and hypercharge currents. The predicted vector mesons were indeed experimentally established in the years between 1961 and 1963, compare Table I.

#### TABLE I

The vector mesons.

Ι	Y	$J^P$	
1	0		$\rho$ (770)
0	0	$1^{-}$	$\omega$ (780), $\phi$ (1020)
1/2	$\pm 1$		$K^{*}$ (890)

Vector mesons played an important role in the generalization of  $SU(2)_{isospin}$  to  $SU(3)_{flavor}$ . In "The Eightfold Way: A Theory of Strong Interactions" Gell-Mann said [4]:

"The most attractive feature of the scheme is that it permits the description of eight vector mesons by a unified theory of the Yang Mills type (with a mass term). Like Sakurai, we have a triplet of vector mesons coupled to the isospin current, ...".

The non invariance of the mass terms of the vector mesons was ignored for the time being, local gauge transformations being considered as a means to generate interactions with universal couplings among the nucleons and the vector mesons themselves. For the  $\rho$  meson triplet, for example [2],

$$f_{\rho} \equiv f_{\rho NN} = f_{\rho \pi \pi} = f_{\rho \rho \rho}.$$
 (1)

In connection with the concept of mass, it may be appropriate to remind ourselves of the situation at present. The mass problem, shifted to the masses of the leptons and quarks, even to-day still awaits the discovery of the Higgs particle to be considered as being (partially?) solved.

In 1964, in "A schematic model of baryons and mesons" the three-dimensional representation of (flavour) SU(3) became "physical" by introducing "quarks" [5]:

"It is fun to speculate about the way the quarks would behave, if they were physical particles of finite mass ...". (M. Gell-Mann).

Subsequently the quarks were endowed with an additional degree of freedom [6] beyond electromagnetic charge. They became "colored", and the application of the local gauge principle to the color degree of freedom of the quarks in 1972 led to  $SU(3)_{Color}$  and Quantum Chromodynamics (QCD) [7].

The vector mesons, now recognized as  $(q\bar{q})^{J=1}$  bound states, nevertheless find their place as "Dynamical gauge bosons of hidden local symmetry" (Bando *et al.* 1989 [8]) in the framework of a low-energy effective Lagrangian of massless two-flavored QCD, thus arriving at Sakurai's massive Yang–Mills Lagrangian from a novel point of view.

### 2. Electromagnetic interactions of hadrons

In the 1950ies the first measurements of the electromagnetic form factors of the nucleons in electron-scattering experiments were performed. The interpretation of the form factor measurements was the second path that led to the existence of vector mesons.

Based on a picture of the nucleon as a core surrounded by a pion cloud, the form factor measurements were interpreted as empirical evidence for an isoscalar vector meson,  $\omega \to 3\pi$ , by Nambu [9] in 1957, and for an isovector meson,  $\rho^0 \to 2\pi$ , by Frazer and Fulco [10] in 1959.

Subsequently, this interpretation of the nucleon form factors was generalized to hold for the totality of all photon-hadron interactions, formulated in terms of an operator identity [11–13], known as current-field identity (CFI). The electromagnetic current, the source of the Maxwell field,  $j_{\mu}^{\text{elm}} = J_{\mu}^{(3)} + \frac{1}{2}J_{\mu}^{(Y)}$ , was identified with a linear combination of isovector and isoscalar vector meson fields. For *e.g.* the isovector part (with  $m_{\rho}$  denoting the  $\rho^0$ -meson mass,  $\rho_{\mu}(x)$  denoting the  $\rho^0$ -meson field and  $f_{\rho} \equiv 2\gamma_{\rho}$ the coupling), the CFI reads

$$j_{\mu}^{(3)} = -\frac{m_{\rho}^2}{2\gamma_{\rho}}\rho_{\mu}(x) \equiv -\frac{m_{\rho}^2}{f_{\rho}}\rho_{\mu}(x).$$
(2)

Consistency of (2) with electromagnetic current conservation requires the vector mesons to be coupled to conserved hadronic currents.

The CFI immediately implies a proportionality between the amplitudes of interactions induced by real or virtual photons ( $\gamma^*$ ) and the corresponding vector meson-induced processes (*e.g.* [14]),

$$[\gamma^* A \to B] = -e \frac{m_\rho^2}{2\gamma_\rho} \frac{1}{q^2 - m_\rho^2} [\rho^0 A \to B] + (\omega) + (\phi).$$
(3)

According to (3), the photon virtually dissociates, or "fluctuates" in modern jargon, into an on-shell vector meson that subsequently interacts with hadron A to yield the hadron state B. It is important to note that the photon fluctuates into an on-shell vector meson, since the dependence on the photon virtuality  $q^2$  is (by an implicit assumption) solely determined by the propagator in (3).

Specializing (3) to A = B, we deduce universality of *e.g.* the  $\rho^0$ -meson coupling,  $(f_{\rho AA} = f_{\rho BB} = ... = f_{\rho})$  from universality of the electromagnetic coupling

$$\frac{e}{2\gamma_{\rho}}f_{\rho AA} = \frac{e}{2\gamma_{\rho}}f_{\rho BB} = \dots = e \tag{4}$$

connecting the approach of the present section with the one of Section 1, where universality arose as a consequence of the gauge principle.

The CFI, when applied to e.g. the  $\gamma^* \to \rho^0 \to 2\pi$  transition, measurable in  $e^+e^-$  annihilation, allows one to express the coupling constant  $\gamma_{\rho}$  as integral over the  $\rho^0$  meson peak,

$$\frac{e^2}{4\gamma_{\rho}^2} = \frac{\alpha\pi}{\gamma_{\rho}^2} = \frac{1}{4\pi^2 \alpha} \int_{4m_{\pi}^2} \sigma_{e^+e^- \to \rho^0 \to 2\pi}(m^2) dm^2.$$
(5)

Based on (5), the on-shell vector meson couplings to the photon were accurately measured [15] as soon as the  $e^+e^-$  storage rings at Novosibirsk and Orsay started to produce data around 1966/1967.

The CFI says that the vector mesons are the (only) source of the Maxwell field, e.g. for the isovector part of the photon interaction, we have

$$\partial^{\mu}F^{(3)}_{\mu\nu} = \frac{em_{\rho}^2}{2\gamma_{\rho}}\rho_{\nu}.$$
(6)

What is the underlying Lagrangian? A mixing term proportional to  $\rho_{\mu}A^{\mu}$  is suggestive, but violates [16] electromagnetic gauge invariance and yields an imaginary (!) photon mass when summing up to photon–vector meson transition to all orders in the photon propagator. The correct form of the Lagrangian, consistent with gauge invariance, was given by Kroll, Lee, Zumino in 1967 [12]. It may be written in the "current-mixing" form (where  $J_{\mu}^{(\rho)}$  is the source of the  $\rho^0$  field),

$$L_{\rm mix} = -\frac{1}{2} \frac{e}{f_{\rho}} \rho_{\mu\nu} F^{\mu\nu} + \frac{e}{f_{\rho}} A_{\mu} J^{(\rho)\mu}, \qquad (7)$$

or, equivalently, in the "mass-mixing" form,

$$L'_{\rm mix} = \frac{e'm_{\rho}^2}{f_{\rho}}\rho'_{\mu}A'^{\mu} - \frac{1}{2}\left(\frac{e'}{f_{\rho}}\right)^2 m_{\rho}^2 A'^2_{\mu} \tag{8}$$

with  $e^2 = e'^2 / (1 + e'^2 / f_{\rho}^2)$  and appropriate linear relations between the fields in (7) and (8). Recalculating the photon propagator to all orders in the mixing according to (8) now yields a vanishing photon mass. For most phenomenological applications of vector meson dominance, the first term in (8) is sufficient.

The CFI provides a powerful means to describe photon-hadron interactions. It is without exaggeration to say that, throughout the 1960ies, it dominated our understanding of the electromagnetic interaction of the hadrons [14, 17].

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### 3. Phenomenological consequences

I will concentrate on those phenomenological consequences from vector meson dominance that are most relevant for the present-day discussions, in particular on deep-inelastic scattering (DIS) in the "diffraction region" of small  $x \cong Q^2/W^2 \ll 1$ .

## 3.1. Vector meson photoproduction, the total photoproduction cross section, shadowing in photo- and electroproduction from complex nuclei

According to the CFI, the amplitudes for photoproduction of vector mesons and for vector meson scattering are proportional to each other [14]. For example, for the  $\rho^0$  vector meson

$$A_{\gamma p \to \rho^0 p} = \frac{e}{2\gamma_{\rho}} A_{\rho^0 p \to \rho^0 p}.$$
(9)

The  $\rho^0$ -scattering amplitude on the right-hand side in (9) can be predicted from pion scattering by applying [18] the additive quark model,  $\sigma_{\rho^0 p} =$  $(1/2) (\sigma_{\pi^+ p} + \sigma_{\pi^- p})$ . From (9), vector meson photoproduction must be weakly dependent on energy and develop a diffraction peak in the forward direction, as observed in hadron-hadron interactions. These features, known as "hadronlike behavior of the photon" [14,19] were established in the 1960ies by the experiments at DESY and SLAC.

With vector meson dominance applied to the forward Compton scattering amplitude, as pointed out by Stodolsky [20] in 1967, at sufficiently high energy, W, we have the important sum rule

$$\sigma_{\gamma p}(W^2) = \sum_{\rho^0, \omega, \phi} \sqrt{16\pi} \sqrt{\frac{\alpha \pi}{\gamma_V^2}} \sqrt{\frac{d\sigma_{\gamma p \to V p}^0}{dt}} (W^2).$$
(10)

In (10), for simplicity we have ignored the correction due to the (small) real part of the vector meson-production amplitude on the right-hand side. By the time of the 1971 Conference on Electron Photon Interactions at Cornell University, it had become clear that (10) was of approximate validity. The fact that the right-hand side of (10) yields 78 % [21] of the total photoproduction cross section,  $\sigma_{\gamma p}(W^2)$ , became the starting point of Generalized Vector Dominance (GVD) [22,23] in 1972<sup>1</sup>. The remaining 22 % were attributed to a continuum of more massive vector state contributions, becoming dominant as soon as the virtuality of the photon becomes large,  $Q^2 \gg m_{\rho}^2$ . I will come back to that.

<sup>&</sup>lt;sup>1</sup> Compare also "Extended Vector Dominance" [24].

When hadrons, *e.g.* pions, are scattered from large complex nuclei, the nucleons inside the nucleus find themselves in the shadow created by the ones at the surface, since the mean free path of hadrons in nuclear matter is smaller than the radius of the nucleus,  $l_h \ll R$ . For very small  $l_h$ , one expects hadron–nucleus-interaction cross sections to be proportional to the surface of the nucleus (*e.g.* [25]) of mass number A, *i.e.*  $\sigma_h$  nucleus  $\sim A^{2/3}$ . The actually measured power is slightly larger than 2/3, since  $l_h$  is not sufficiently small compared with R.

What is the dependence on A for photon-nucleus interactions? Is it that  $\sigma_{\gamma \text{ nucleus}} \sim A$  or rather  $\sigma_{\gamma \text{ nucleus}} \sim A^{2/3}$ , since photons behave hadronlike? The question was posed and answered by Stodolsky [20] in 1967<sup>2</sup>. He realized that the process of  $\gamma$ -nucleus scattering must be treated as a two-channel problem, since the photon in the forward Compton amplitude on nuclei may convert to a vector meson and reconvert to a photon on a single nucleon as well as on two different nucleons in the nucleus. The scale that determines the A dependence may be identified with the lifetime [27] of a vector meson fluctuation. For sufficiently large lifetime, or fluctuation length, d, namely for 2u

$$d = \frac{2\nu}{M_V^2} \gg R,\tag{11}$$

the A dependence becomes hadronlike. Shadowing occurs, provided the photon energy  $\nu$  is sufficiently large. The generalization to virtual photons reads (e.g. Ref. [29])

$$d(x, Q^2, m_V^2) = \frac{Q^2}{Mx(Q^2 + M_V^2)},$$
(12)

where  $x \cong Q^2/2M\nu$  is the Bjorken scaling variable [28].

Shadowing in photoproduction  $(Q^2 = 0)$  in the years 1969 to 1973 was experimentally found in experiments performed at DESY and SLAC. Compare Fig. 1 [30]. After many years of confusion, in 1989, the EMC-NMC collaboration established [31] shadowing in electroproduction  $(Q^2 \cong 10 \text{ GeV}^2)$ , compare Fig. 2 [29]<sup>3</sup>.

The EMC-NMC result is of great importance with respect to the presentday discussions on deep-inelastic scattering (DIS) at low values of  $x \cong Q^2/W^2$ . Since diffractive production and propagation of  $(q\bar{q})^{J=1}$  (vector) states is essential for the two-step process causing shadowing, and the contributions of the low-lying vector mesons  $\rho^0, \omega, \phi$  become negligible at large  $Q^2$ , the 1989 EMC-NMC result provided unambiguous evidence for diffractive production of high-mass  $(q\bar{q})^{J=1}$  (vector) states in electroproduction prior to the HERA experiments. Moreover, the shadowing results require those high-mass states to interact hadronlike.

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<sup>&</sup>lt;sup> $^{2}$ </sup> Compare also Refs. [25, 26].

<sup>&</sup>lt;sup>3</sup> See [31] also for further references on the theoretical analysis of the EMC-NMC effect.



Fig. 1. Shadowing in photoproduction (from [30]).



Fig. 2. Shadowing in DIS (from [29]).

## 3.2. $e^+e^-$ annihilation into hadrons, quark-hadron duality

In the 1960ies, expectations on how the cross section for  $e^+e^- \rightarrow$  hadrons would behave at asymptotic energies ranged [32] from  $\sigma(e^+e^- \rightarrow \text{hadrons}) \sim$  $1/s^3$  as expected from the CFI with a finite number of vector mesons, to  $\sigma(e^+e^- \rightarrow \text{hadrons}) \sim 1/s$ . In his 1966 paper [33] Bjorken said:

"A speculative argument is presented that the rate of  $e^+e^- \rightarrow$  hadrons is comparable to the rate of  $e^+e^- \rightarrow \mu^+\mu^-$  in the limit of large energies."

As a consequence of the 1969 SLAC experiment on DIS, to be summarized below, a scaling behavior of  $\sigma(e^+e^- \rightarrow \text{hadrons}) \sim 1/s$  became stronger acceptance and led to the concept of quark-hadron duality: the low-lying vector meson peaks are smoothly interpolated by  $e^+e^-$  annihilation into quark-antiquark pairs,  $e^+e^- \rightarrow q\bar{q}$  of the appropriate flavor [34,35],

$$\frac{\alpha\pi}{\gamma_V^2} = \frac{1}{4\pi^2\alpha} \int_{\text{Peak}V} ds \sigma_{e^+e^- \to \text{hadrons}}(s) = \frac{\alpha R_{e^+e^-}^{(V)}}{3\pi} \frac{\Delta M_V^2}{M_V^2}, \quad (13)$$

where  $R_{e^+e^-}^{(V)}$  contains the squares of the relevant quark charges. Compare Fig. 3. Quark–hadron duality became subsequently refined in terms of QCD sum rules [36].



Fig. 3. Quark-hadron duality (from [35]).

### 3.3. Generalized Vector Dominance and modern picture of DIS at low x

The measurements on DIS carried out by the SLAC-MIT collaboration in 1969 [37] revealed that the transverse part of the photoabsorption cross section  $\sigma_{\gamma_T^*}(W^2, Q^2)$  was decreasing as  $\sigma_{\gamma_T^*}(W^2, Q^2) \sim 1/Q^2$  in strong disagreement with the  $\rho^0, \omega, \phi$  dominance prediction, where  $\sigma_{\gamma_T^*}(W^2, Q^2) \sim 1/Q^4$ . The observed (approximate) scaling behaviour [28] of the structure function  $F_2(x, Q^2) \simeq F_2(x)$  led Feynman to the parton-model interpretation [38] of the data.



Fig. 4. Generalized vector dominance (from [22]).

In the generalized vector dominance approach (GVD) [22] from 1972, the coupling of the photon to  $\rho^0, \omega, \phi$  mesons was supplemented by the coupling to a continuum of more massive vector states<sup>4</sup>. A successful representation of the experimental data in the region of low  $x \simeq Q^2/W^2 \lesssim 0.1$  (*i.e.* large  $\omega'$ ) was obtained, compare Fig. 4. As a verification of the proposed picture, it was concluded [22] that

"Comparisons of higher-mass vector states diffractively produced in photoand electroproduction with the final states in  $e^+e^-$  collisions would be of enormous importance."

In the SLAC-MIT experiment from 1969 the region of  $x \leq 0.05$  was not accessible. The exploration of DIS and diffractive production at low x only started with HERA in 1993. Consistency requirements between DIS and the suggested scaling in  $e^+e^-$  annihilation led to the refinement of GVD to off-diagonal GVD [41] that anticipated a general structure close to the one contained in the modern approach based on two-gluon exchange from QCD.

<sup>&</sup>lt;sup>4</sup> Compare also the review [39]. For GVD applied to  $\gamma\gamma \rightarrow$  hadrons, compare [40].

The modern picture of DIS at low x [42] has much in common [43, 44] with (off-diagonal) GVD. The novel element is the dependence of the virtual forward Compton amplitude on the transverse momentum of the exchanged gluon arising from the two-gluon-exchange [45] structure. The effective value of the transverse momentum of the gluon,  $\langle \vec{l}_{\perp}^2 \rangle$ , introduces a novel scale, characteristic for DIS at low x. The scale is known as "saturation scale",  $A_{\rm sat}^2(W^2)$ , since it governs the transition of the total photoabsorption cross section at fixed  $Q^2$  from a strong to a weak energy dependence, frequently interpreted as an indication for parton saturation [46]. Since the photon fluctuates to a vector state of two on-shell quarks<sup>5</sup>, the saturation scale that determines the energy dependence of the  $q\bar{q}$  color-dipole scattering on the proton depends on the energy<sup>6</sup> [47, 48]. From the fit with

$$\Lambda_{\rm sat}^2(W^2) = \frac{1}{6} \left\langle \vec{l}_{\perp}^2 \right\rangle = B' \left( \frac{W^2}{1 \,\,{\rm GeV}^2} \right)^{C_2} \tag{14}$$

we found  $C_2^{\exp} = 0.27 \pm 0.01$ ,  $B' = 0.340 \pm 0.063 \text{ GeV}^2$ . The experimental data for the total photoabsorption cross section,  $\sigma_{\gamma^* p}(W^2, Q^2)$  lie on a single curve [48] against the novel scaling variable  $\eta = (Q^2 + m_0^2)/\Lambda_{\text{sat}}^2(W^2)$ . Compare Fig. 5 [47, 48, 51].



Fig. 5. Scaling of  $\sigma_{\rho^* p} = \sigma_{\rho^* p}(\eta)$ , (from [51]).

<sup>&</sup>lt;sup>5</sup> The mass, the photon fluctuates into, from the analysis of the two-gluon exchange amplitude is obtained from the photon four momentum,  $q^2 = \frac{k_q^2 + \vec{k}_\perp^2}{z} + \frac{k_q^2 + \vec{k}_\perp^2}{1-z}$ , by putting  $k_q^2 = k_{\bar{q}}^2 = m_q^2$ .

<sup>&</sup>lt;sup>6</sup> In this respect, among other things, we differ from Ref. [49], where the saturation scale depends on x. The energy is also used as basic variable in Ref. [50].

By making use of the duality between the above-mentioned description in terms of  $q\bar{q}$  scattering and a formulation based on  $\gamma^*$ -quark–gluon scattering in terms of parton distributions, we were recently able to derive [51] a theoretical value of  $C_2^{\text{theor.}} = 0.276$  in surprisingly good agreement with the experimental result. More about that in the session on DIS and diffraction at this conference.

## 4. Conclusions

In this brief review only a minor part of the relevant photon-hadron phenomenology could be addressed.

Vector meson physics introduced local gauge transformations to hadron interactions. With respect to photons interacting with hadrons, it led to useful concepts: the fluctuating photon, hadronlike behavior, quark-hadron duality, among others, which stood the test of time.

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