NON-FORWARD BFKL KERNEL AT NLO*

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The kernel of the BFKL equation at next-to-leading order is presented and discussed in general case of non-zero momentum transfer t and any possible t-channel colour state.

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1. Introduction

In the BFKL approach [1] amplitudes of processes $A+B \longrightarrow A'+B'$ at large center of mass energy $\sqrt{s}$ and fixed momentum transfer $\sqrt{-t}$, $s \gg |t|$, can be written as

$$\mathcal{A}_{AB}^{A'B'} = \left\langle \Phi_{A'A} \left| e^{\hat{\mathcal{K}} \ln(s/s_0)} \right| \Phi_{B'B} \right\rangle . \tag{1}$$

Here the impact factors $\Phi_{A'A}$ and $\Phi_{B'B}$ describe the transitions $A \to A'$ and $B \to B'$ due to scattering on Reggeized gluons, the BFKL kernel $\hat{\mathcal{K}}$ describes interaction of these gluons and s_0 is some energy scale.

For the case of the forward scattering, *i.e.* $t=0$ and vacuum quantum numbers in the t-channel, the kernel $\hat{\mathcal{K}}$ was found at next-to-leading order (NLO) several years ago [2]. However, the BFKL approach is not limited to this particular case and, what is more, from the beginning it was developed for arbitrary t and for all possible t-channel colour states. Evidently, the forward kernel carries only restrictive information about the BFKL dynamics. Moreover, the non-forward case has an advantage of smaller sensitivity to large-distance contributions, since the diffusion in the infrared region is limited by $\sqrt{|t|}$. Unfortunately, the NLO calculation of the non-forward kernel was not completed till last year. The reason is a complexity of the two-gluon contribution.

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The kernel $\hat{\mathcal{K}}$ is given by the sum of the "virtual" $\hat{\Omega}$ and "real" $\hat{\mathcal{K}}_{\rm r}$ contributions. Due to colour conservation its colour structure is determined by representations R of the colour group in the t-channel. At that the "virtual" part does not depend on R and is expressed through the gluon Regge trajectory, which has been known at NLO for a long time [3]. The "real" part is related to particle production in Reggeon–Reggeon collisions and consists of pieces coming from one-gluon, two-gluon and quark–antiquark pair production:

$$\hat{\mathcal{K}}_{\rm r} = \hat{\mathcal{K}}_G + \hat{\mathcal{K}}_{Q\bar{Q}} + \hat{\mathcal{K}}_{GG}. \tag{2}$$

The first piece was found in [4]. It depends on R only through a common coefficient. Each of $\hat{\mathcal{K}}_{Q\bar{Q}}$ and $\hat{\mathcal{K}}_{GG}$ is written as a sum of two terms with coefficients depending on R. For the $Q\bar{Q}$ case both these terms were found [5]. Instead, for the GG case till recently only the term related to the octet representation was calculated. Since the quark contribution to the kernel is known, we shall discuss in the following the case of pure gluodynamics.

2. The two-gluon contribution

For the representation R of the colour group this contribution can be written [6] as

$$\hat{\mathcal{K}}_{GG}^{(R)} = 2c_R\mathcal{K}_{GG}^{(8)} + b_R\mathcal{K}_{GG}^{(s)}\,, \tag{3}$$

where $\mathcal{K}_{GG}^{(8)}$ is the two-gluon contribution for the adjoint representation (gluon channel), $\mathcal{K}_{GG}^{(s)}$ is so called "symmetric" two-gluon contribution, c_R and b_R are the group coefficients. Remarkably, only planar diagrams contribute to $\mathcal{K}_{GG}^{(8)}$ due to the colour structure. In contrast, $\mathcal{K}_{GG}^{(s)}$ is defined in such a way that all diagrams have equal colour weights. A marvelous feature of $\mathcal{K}_{GG}^{(s)}$ is the absence of infrared singularities, *i.e.* divergences in the limit $\varepsilon \to 0$. The disappearance of the singularities is rather tricky: it takes place due to independence of infrared singular terms in the $\mathcal{K}_{GG}^{(s)}$-integrand from relative rapidity of produced gluons. Because of this the singularities vanish after the subtraction, necessary in order to avoid double counting of the region of large invariant mass of the gluons.

For the colour group SU$(N_{\rm c})$ with $N_{\rm c}=3$ the possible representations $\mathcal{R}$ are $\underline{1}$, $\underline{8_a}$, $\underline{8_s}$, $\underline{10}$, $\underline{\overline{10}}$, $\underline{27}$. Corresponding coefficients are

$$c_1 = 1\,, \quad c_{8_a} = c_{8_s} = \frac{1}{2}\,, \quad c_{10} = c_{\overline{10}} = 0\,, \quad c_{27} = -\frac{1}{4N_{\rm c}} \tag{4}$$

and $b_R = c_R(c_R - 1/2)$.

The colour octet contribution $\mathcal{K}^{(8)}_{GG}$ was found in [4]. It is concerned with the gluon Reggeization and looks rather simple. The "symmetric" contribution is much more complicated because of the non-planar diagrams, which do not contribute to $\mathcal{K}^{(8)}_{GG}$. The complexity of contributions of non-planar diagrams is known since the calculation of the non-forward kernel for the QED Pomeron, which was found only in the form of two-dimensional integral [7]. In QCD the situation is considerably worse because of existence of the cross-pentagon and cross-hexagon diagrams in addition to the QED-type cross-box diagrams. It requires the use of additional Feynman parameters. At arbitrary space-time dimension, $D = 4 + 2\varepsilon$, no integration over these parameters can be done in elementary functions. It occurs, however, that in the limit $\varepsilon \to 0$ the integration over additional Feynman parameters can be performed, so that the result can be written as two-dimensional integral, as well as in QED. Unfortunately, the representation of $\hat{\mathcal{K}}^{(s)}_{GG}$ in this form is rather complicated [6]. Here we present another form of this contribution in the transverse momentum space:

$$\mathcal{K}^{(s)}_{GG}(\vec{q}_1,\vec{q}_2;\vec{q}) = \frac{\alpha_{\rm s}^2 N_{\rm c}^2}{4\pi^3}\Big([J^s(\vec{q}_1,\vec{q}_2;\vec{q}) + J^s(-\vec{q}_2,-\vec{q}_1;-\vec{q})] + [\vec{q}_n \leftrightarrow \vec{q}-\vec{q}_n]\Big)\,, \quad (5)$$

where $\vec{q}_n$ and $\vec{q}-\vec{q}_n \equiv -\vec{q}^{\,\prime}_n$ ($n = 1,2$) are the t-channel Reggeized gluon momenta

$$\begin{aligned} J^s(\vec{q}_1,\vec{q}_2;\vec{q}) \;=\; & \frac{\vec{k}^{\,2}}{2} + \frac{5}{2}(\vec{q}_1\vec{q}_2) + \frac{\vec{q}^{\,2}}{2}\left(\frac{13}{18} - \zeta(2)\right) - \frac{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})(\vec{q}_1^{\,\prime\,2}-\vec{q}_2^{\,\prime\,2})}{2\vec{k}^{\,2}} \\ & + 2\left(\vec{q}_1\vec{q}_2 - \vec{q}_1^{\,2}\frac{\vec{k}\vec{q}_1^{\,\prime}}{\vec{k}^{\,2}}\right)\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{k}^{\,2}}\right) - \vec{q}^{\,2}\left(\frac{11}{12}\ln\left(\frac{\vec{q}^{\,2}}{\vec{k}^{\,2}}\right) + \frac{5}{6}\ln 2\right) \\ & - \frac{\vec{k}^{\,2}}{2}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{k}^{\,2}}\right)\ln\left(\frac{\vec{q}_2^{\,2}}{\vec{k}^{\,2}}\right) + \frac{\vec{q}^{\,2}}{4}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}^{\,2}}\right)\ln\left(\frac{\vec{q}_1^{\,\prime\,2}}{\vec{q}^{\,2}}\right) + \left(\vec{q}_1\vec{q} + \frac{\vec{q}_1^{\,2}(\vec{k}\vec{q}_2^{\,\prime}) - \vec{q}_1^{\,\prime\,2}(\vec{k}\vec{q}_2)}{\vec{k}^{\,2}}\right) \\ & \times\left(\frac{1}{2}\ln\left(\frac{\vec{q}_2^{\,2}}{\vec{k}^{\,2}}\right)\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right) + (\vec{q}_2\vec{k})I\left(\vec{q}_1^{\,2},\vec{q}_2^{\,2},\vec{k}^{\,2}\right)\right) - \vec{q}_2^{\,2}\,(\vec{q}_1^{\,\prime}\vec{q})\,I\left(\vec{q}_1^{\,2},\vec{q}_2^{\,2},\vec{k}^{\,2}\right) \\ & + \int\frac{d^2k_1}{\pi}\Bigg[\left(-\frac{\vec{k}_1^{\,2}\vec{k}_2^{\,2}}{2} + (\vec{k}_1\vec{k}_2)^2 + (Q_1^i\Omega_1^{ij}Q_2^{\prime j})(Q_1^i\Omega_2^{ij}Q_2^{\prime j}) - \vec{Q}_2^{\,\prime 2}(Q_1^i\Omega_1^{ij}\Omega_2^{jl}Q_1^l)\right) \\ & \times\frac{1}{\vec{Q}_1^{\,2}\vec{Q}_2^{\,\prime\,2} - \vec{k}_1^{\,2}\vec{k}_2^{\,2}}\ln\left(\frac{\vec{Q}_1^{\,2}\vec{Q}_2^{\,\prime\,2}}{\vec{k}_1^{\,2}\vec{k}_2^{\,2}}\right) - \frac{1}{2} + \frac{5}{6}\frac{\vec{q}^{\,2}}{\vec{k}_1^{\,2}+\vec{k}_2^{\,2}}\Bigg]. \end{aligned} \quad (6)$$

Here $\vec{k} = \vec{q}_1 - \vec{q}_2 = \vec{q}_1^{\,\prime} - \vec{q}_2^{\,\prime}$, $\vec{k}_2 = \vec{k} - \vec{k}_1$, $\vec{Q}_n = \vec{q}_1 - \vec{k}_n$, $\vec{Q}_n^{\,\prime} = \vec{q}_1^{\,\prime} - \vec{k}_n$, $\Omega_n^{ij} = \delta^{ij} - 2k_n^i k_n^i/\vec{k}_n^{\,2}$,

$$I(a,b,c) = \int_0^1 \frac{dx}{a(1-x) + bx - cx(1-x)}\ln\left(\frac{a(1-x)+bx}{cx(1-x)}\right). \quad (7)$$

3. Non-forward BFKL kernel at NLO

In the pure gluodynamics a complete expression for the kernel in the transverse momentum space is

$$\begin{aligned}\mathcal{K}^{(R)}\left(\vec{q}_1,\vec{q}_2;\vec{q}\right) &= \left[\omega\left(-\vec{q}_1^{\,2}\right)+\omega\left(-\vec{q}_1^{\,\prime\,2}\right)\right]\vec{q}_1^{\,2}\vec{q}_1^{\,\prime\,2}\delta^{(D-2)}\left(\vec{q}_1-\vec{q}_2\right)\\ &\quad +2c_R\mathcal{K}_{\rm r}^{(8)}\left(\vec{q}_1,\vec{q}_2;\vec{q}\right)+2b_R\,\mathcal{K}_{GG}^{(s)}\left(\vec{q}_1,\vec{q}_2;\vec{q}\right)\,, \end{aligned}\tag{8}$$

where

$$\begin{aligned}\omega(t) &= -2\bar{g}_\mu^2\left(\frac{1}{\varepsilon}+\ln\left(\frac{-t}{\mu^2}\right)\right)-\bar{g}_\mu^4\left[\frac{11}{3}\left(\frac{1}{\varepsilon^2}-\ln^2\left(\frac{-t}{\mu^2}\right)\right)+\left(\frac{67}{9}-2\zeta(2)\right)\right.\\ &\quad\left.\times\left(\frac{1}{\varepsilon}+2\ln\left(\frac{-t}{\mu^2}\right)\right)-\frac{404}{27}+2\zeta(3)\right]\end{aligned}\tag{9}$$

is the gluon Regge trajectory, $\zeta(n)$ is the Riemann zeta function ($\zeta(2) = \pi^2/6$),

$$\bar{g}_\mu^2=\frac{g_\mu^2N\Gamma(1-\varepsilon)}{(4\pi)^{2+\varepsilon}}\,,\tag{10}$$

$\Gamma(x)$ is the Euler function, g_μ is the renormalized coupling in the $\overline{MS}$ scheme, concerned with the bare coupling constant g through the relation

$$g=g_\mu\mu^{-\varepsilon}\left[1+\frac{11}{3}\frac{\bar{g}_\mu^2}{2\varepsilon}\right]\,,$$

and

$$\begin{aligned}\mathcal{K}_{\rm r}^{(8)}(\vec{q}_1,\vec{q}_2;\vec{q}) &= \frac{\bar{g}_\mu^2\mu^{-2\varepsilon}}{\Gamma(1-\varepsilon)\pi^{1+\varepsilon}}\left\{\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}+\vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,2}}{\vec{k}^{\,2}}-\vec{q}^{\,2}\right)\left(\frac{1}{2}+\bar{g}_\mu^2\left(\frac{\vec{k}^{\,2}}{\mu^2}\right)^{\varepsilon}\right.\right.\\ &\quad\left.\left.\times\left(\frac{67}{18}-\zeta(2)+\varepsilon\left(-\frac{202}{27}+7\zeta(3)+\frac{11}{6}\zeta(2)\right)\right)\right)+\bar{g}_\mu^2\right.\\ &\quad\times\left[\frac{\vec{q}^{\,2}}{4}\left(\frac{22}{3}\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{q}^{\,2}\vec{k}^{\,2}}\right)+\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}^{\,2}}\right)\ln\left(\frac{\vec{q}_1^{\,\prime 2}}{\vec{q}^{\,2}}\right)+\ln\left(\frac{\vec{q}_2^{\,2}}{\vec{q}^{\,2}}\right)\ln\left(\frac{\vec{q}_2^{\,\prime 2}}{\vec{q}^{\,2}}\right)\right.\right.\\ &\quad\left.+\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right)-\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}+\vec{q}_2^{\,2}\vec{q}_1^{\,\prime\,2}}{2\vec{k}^{\,2}}\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\\ &\quad+\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}-\vec{q}_2^{\,2}\vec{q}_1^{\,\prime\,2}}{\vec{k}^{\,2}}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\left(\frac{11}{6}-\frac{1}{4}\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{k}^{\,4}}\right)\right)\\ &\quad+\frac{1}{2}\left[\vec{q}^{\,2}(\vec{k}^{\,2}-\vec{q}_1^{\,2}-\vec{q}_2^{\,2})+2\vec{q}_1^{\,2}\vec{q}_2^{\,2}-\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}-\vec{q}_2^{\,2}\vec{q}_1^{\,\prime\,2}+\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}-\vec{q}_2^{\,2}\vec{q}_1^{\,\prime\,2}}{\vec{k}^{\,2}}\right.\\ &\quad\left.\left.\left.\times(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})\right]I(\vec{q}_1^{\,2},\vec{q}_2^{\,2},\vec{k}^{\,2})\right]\right\}+\frac{\bar{g}_\mu^2\mu^{-2\varepsilon}}{\Gamma(1-\varepsilon)\pi^{1+\varepsilon}}\left\{\vec{q}_n\leftrightarrow\vec{q}-\vec{q}_n\right\}.\end{aligned}\tag{11}$$

The kernel (11) is singular at $\vec{k}^{\,2}=0$ so that the region of small $\vec{k}^{\,2}$ values such that $\varepsilon|\ln(\vec{k}^{\,2}/\mu^2)|\sim 1$ is important at integration of the kernel. Therefore, the expansion of $(\vec{k}^{\,2}/\mu^2)^\varepsilon$ is not done in Eq. (11). Moreover, the terms $\sim\varepsilon$ are taken into account in the coefficient of the expression divergent at $\vec{k}^{\,2}=0$ in order to save all contributions non-vanishing in the limit $\varepsilon\to 0$ after the integration. Note that apart from the coefficient the infrared singularities are the same for all representations in the "real" contribution. Moreover, they do not depend on t, *i.e.* they are the same as in the forward kernel.

4. Summary

Equations (5)–(11) determine the NLO kernel for arbitrary representation R. The most important are the colour octet and the colour singlet representations. The first of them is required for the proof of the multi-Regge form of QCD amplitudes with gluon exchanges (see for instance [8] and references therein). This form is the basis of the BFKL approach. The colour singlet representation is related to scattering of physical (colorless) particles. The total kernel (8) is infrared safe in this case due to cancellation of infrared singularities in "virtual" and "real" parts of the kernel. In [6] the infrared safety is demonstrated and formulations free from the singularities are suggested.

The non-forward kernel has a wide field for phenomenological applications. Apart from this there are extremely intriguing problems of conformal invariance and dipole picture of high energy scattering related to the kernel.

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