# THE ENERGY DEPENDENCE OF THE SATURATION SCALE *

Masaaki Kuroda

Institute of Physics, Meiji-Gakuin University, Yokohama 244, Japan

Dieter Schildknecht

Fakultät für Physik, Universität Bielefeld
Universitätsstrasse 25, 33615 Bielefeld, Germany
Dieter.Schildknecht@physik.uni-bielefeld.de

(Received January 2, 2006)

At low $x \cong Q^2/W^2 \ll 1$, in deep inelastic scattering, the photon fluctuates into a $q\bar{q}$ vector state that interacts via two gluons with the proton. The energy dependence is determined by the saturation scale, in our approach given by $\Lambda^2_{\rm sat}(W^2) \sim (W^2)^{C_2}$. Imposing DGLAP evolution, we find $C_2^{\rm theory} = 0.27$ in agreement with the model-independent analysis of the HERA data. Different values of the exponent $C_2$ are correlated with different ratios of the longitudinal to transverse structure function. This stresses the need for experiments to separate longitudinal and transverse contributions in deep inelastic scattering.

PACS numbers: 13.60.Hb, 12.38.Qk

In deep inelastic scattering (DIS) at low $x \cong Q^2/W^2 < 0.1$ the photon fluctuates into on-shell quark–antiquark vector states (*e.g.* [1]) which interact with the proton via coupling of two gluons [2] to the $q\bar{q}$ color–dipole [3]. The cross section depends on the effective transverse three momentum of the gluon, $\vec{l}_\perp$, absorbed by the $q\bar{q}$ pair in the imaginary part of the virtual-photon forward Compton amplitude. The effective transverse momentum of the absorbed gluon gives rise to a novel scale, the "saturation scale", characteristic for DIS in the $x \to 0$ limit. Since the photon fluctuates into an on-shell $q\bar{q}$ pair, the $\gamma^* p$ energy, $W$, besides $Q^2$, is the appropriate variable

* Presented at the PHOTON2005 Conference, 31 August–4 September 2005, Warsaw, Poland.

for the virtual photon-absorption cross section to depend on. The color–dipole cross section, $\sigma_{q\bar{q}p}(\vec{r}, W^2)$, and the saturation scale, $\Lambda^2_{\rm sat}(W^2)$, become functions of $W^2$.

In the high-energy limit, with

$$\Lambda^2_{\rm sat}(W^2) = \frac{1}{6}\left\langle \vec{l}_\perp^{\,2} \right\rangle = \frac{1}{6} B' \left( \frac{W^2}{1\ {\rm GeV}^2} \right)^{C_2}, \tag{1}$$

the fit to the DIS data gave [4]

$$\begin{aligned} C_2^{\rm exp} &= 0.27 \pm 0.01, \\ B' &= 0.340 \pm 0.063\ {\rm GeV}^2. \end{aligned} \tag{2}$$

In terms of $\Lambda^2_{\rm sat}(W^2)$, the cross section of this QCD-based generalized vector dominance — color–dipole picture (GVD-CDP) in the limit of $Q^2 \to 0$ and $Q^2 \to \infty$ reads

$$\sigma_{\gamma^* p}(W^2, Q^2) = \frac{\alpha}{3\pi} R_{e^+e^-} \sigma^{(\infty)} \begin{cases} \ln \frac{\Lambda^2_{\rm sat}(W^2)}{Q^2 + m_0^2}, & (Q^2 \ll \Lambda^2_{\rm sat}(W^2)), \\ \frac{\Lambda^2_{\rm sat}(W^2)}{2Q^2}, & (Q^2 \gg \Lambda^2_{\rm sat}(W^2)). \end{cases} \tag{3}$$

The cross section depends on the single scaling variable [4, 5]

$$\eta \equiv \frac{Q^2 + m_0^2}{\Lambda^2_{\rm sat}(W^2)}, \tag{4}$$

and at HERA, $\Lambda^2_{\rm sat}(W^2)$ varies in the range of $2\ {\rm GeV}^2 \lesssim \Lambda^2_{\rm sat}(W^2) \lesssim 7\ {\rm GeV}^2$. In (3), $\sigma^{(\infty)} = 27.5$ mb and $R_{e^+e^-} = 3\sum_f Q_f^2 = 10/3$. The cross section (3), at any fixed $Q^2 \gg \Lambda^2_{\rm sat}(W^2)$ contains the spectacular strong rise of the structure function

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha} \sigma_{\gamma^* p}(\eta(W^2, Q^2)) \tag{5}$$

with energy at HERA [6]. For $\Lambda^2(W^2) \gg Q^2$, the saturation limit of weak energy dependence, as seen in photoproduction, will be reached. Except for exceedingly small $Q^2$, this only happens at energies far beyond the HERA energy range.

A fundamental question concerns the magnitude of the exponent $C_2$ in (1) that determines the rise of the cross section at large $Q^2$. It turned out [7] that an examination of the expression for the proton structure function (5) in the large-$Q^2$ limit in (3) in terms of the dual language of sea-quark and gluon distributions, upon applying DGLAP evolution [8], provides an answer to this very important question on the magnitude of $C_2$.

The structure function in the large-$Q^2$ limit of (3) takes the form [7]

$$F_2(x,Q^2) = \frac{R_{e^+e^-}}{36\pi^2}\left(T(W^2) + \frac{1}{2}L(W^2)\right), \tag{6}$$

where $T(W^2)$ and $L(W^2)$ are integrals over the first moments of the gluon transverse momentum formed with the dipole cross sections in momentum space. The longitudinal part of $F_2(x,Q^2)$ in (6) is expressible in terms of the saturation scale,

$$L(W^2) = \frac{\sigma^{(\infty)}}{\pi}\Lambda^2_{\rm sat}(W^2). \tag{7}$$

The (successful) representation of the experimental results according to (3) was based on the assumption of $T(W^2) = L(W^2)$ in (6), *i.e.* equal magnitude of transverse and longitudinal $(q\bar{q})^{J=1}$ scattering cross sections.

We turn to the sea-quark and gluon distributions corresponding to the structure function (6) and the saturation scale in (7). The structure function measures the sea-quark distribution,

$$(qq^-)_{\rm sea} \equiv x\Sigma(x,Q^2) = \frac{1}{3\pi^2}\left(T(W^2) + \frac{1}{2}L(W^2)\right), \tag{8}$$

and the gluon distribution is related to the longitudinal part of $F_2$, *i.e.* with (7)

$$\alpha_{\rm s}(Q^2)xg(x,Q^2) = \frac{1}{8\pi}L\left(W^2 = \frac{Q^2}{x}\right) = \frac{1}{8\pi}\frac{\sigma^{(\infty)}}{\pi}\Lambda^2_{\rm sat}(W^2). \tag{9}$$

So far, our results have been based on a general analysis of the generic structure of the two-gluon exchange from QCD. We adopt the assumption that the sea-quark and gluon distributions in good approximation have identical dependence on the kinematic variables, in our case $W^2 \cong Q^2/x$. They are assumed to be proportional to each other,

$$x\Sigma(x,Q^2) = \frac{8}{3\pi}\left(r + \frac{1}{2}\right)\alpha_{\rm s}(Q^2)xg(x,Q^2), \tag{10}$$

with $r = {\rm const} \geq 0$. Substitution of the sea-quark and the gluon distribution from (8) and (9) into (10) yields

$$T(W^2) = rL(W^2) \tag{11}$$

and $F_2$ in (6) becomes (with (7)),

$$F_2(x,Q^2) = \frac{R_{e^+e^-}}{36\pi^2}T(W^2)\left(1 + \frac{1}{2r}\right) \cong \begin{cases} T, & (r \gg 1), \\ \frac{3}{2}T, & (r = 1), \\ L, & (r \to 0). \end{cases} \tag{12}$$

The above-mentioned (successful) representation of the experimental data corresponds to $r = 1$.

In the range of sufficiently large $Q^2$, we are concerned with, the evolution of $F_2(x, Q^2)$ is in very good approximation determined by the gluon structure function alone, [8]

$$\frac{\partial F_2(\frac{x}{2}, Q^2)}{\partial \ln Q^2} = \frac{R_{e^+e^-}}{9\pi} \alpha_s(Q^2) x g(x, Q^2). \tag{13}$$

Upon substitution of (12) and (9), with (11), the evolution in $Q^2$ is converted into a derivative with respect to $W^2$ that determines the $W^2$ dependence

$$(2r+1) \frac{\partial}{\partial \ln W^2} \Lambda^2_{\rm sat}(2W^2) = \Lambda^2_{\rm sat}(W^2). \tag{14}$$

Substituting the power law (1), into (14), we obtain

$$(2r+1) 2^{C_2} C_2 = 1, \tag{15}$$

or

$$C_2^{\rm theor} = \begin{cases} 0, & (r \gg 1), \\ 0.276, & (r = 1), \\ 0.65, & (r = 0). \end{cases} \tag{16}$$

According to (16), the exponent $C_2$ determining the rise of $\sigma_{\gamma^* p}(W^2, Q^2)$ at fixed $Q^2 \gg \Lambda^2_{\rm sat}(W^2)$, or, equivalently, the rise of $F_2(x, Q^2)$ as a function of $x = Q^2/W^2$ at fixed $Q^2$, is uniquely connected with the relative magnitude of sea *versus* gluon distributions. For a given sea distribution, a very small gluon distribution ($r \gg 1$ according to (10)) and a purely transverse structure function (according to (12)) implies a very weak energy dependence. In contrast, in the limit of $r \to 0$, where the relative magnitude of gluon-to sea distribution is maximal, we must have an extraordinarily strong energy dependence, $C_2 = 0.65$. The previous fit to the experimental data was based on $r = 1$. The result of the fit given in (2) is in excellent agreement with the theoretical result (16).

The foregoing intimate connection between a strong rise of $F_2(x, Q^2)$ with decreasing $x$ at fixed $Q^2$ and a large longitudinal photoabsorption cross section, and a weak rise for a small longitudinal contribution, can be further illuminated by measurements allowing to separate longitudinal and transverse cross sections. Such measurements are in fact urgently needed.

This work was supported by Deutsche Forschungsgemeinschaft under grant No. Schi 189/6-1.

## REFERENCES


[1] D. Schildknecht, in the session "Photon: Properties and Interactions — the Historical Evolution", at this conference, PHOTON2005, Warsaw, 31 August–04 September, 2005.

[2] F.E. Low, *Phys. Rev.* **D12**, 163 (1975); S. Nussinov, *Phys. Rev. Lett.* **34**, 1286 (1975); S. Nussinov, *Phys. Rev.* **D14**, 246 (1976); J. Gunion, D. Soper, *Phys. Rev.* **D15**, 2617 (1977).

[3] N.N. Nikolaev, B.G. Zhakarov, *Z. Phys.* **C49**, 607 (1991).

[4] D. Schildknecht, B. Surrow, M. Tentyukov, *Phys. Lett.* **B499**, 116 (2001); G. Cvetic, D. Schildknecht, B. Surrow, M. Tentyukov, *Eur. Phys. J.* **C20**, 77 (2001); D. Schildknecht, B. Surrow, M. Tentyukov, *Mod. Phys. Lett.* **A16**, 1829 (2001).

[5] D. Schildknecht, Diffraction 2000, *Nucl. Phys. B* (Proc. Suppl.) **99**, 121 (2001); D. Schildknecht, in: G. Bruni *et al.* (Eds.), The 9th International Workshop on Deep Inelastic Scattering, DIS 2001, Bologna, Italy, World Scientific, Singapore 2002, p. 798.

[6] ZEUS Collaboration, S. Chekanov *et al.*, *Eur. Phys. J.* **C21**, 443 (2001); H1 Collaboration, C. Adloff *et al.*, *Eur. Phys. J.* **C21**, 33 (2001).

[7] M. Kuroda, D. Schildknecht, *Phys. Lett.* **B618**, 84 (2005).

[8] K. Prytz, *Phys. Lett.* **B311**, 286 (1993).