

SUM RULES FOR PARITY VIOLATING
COMPTON AMPLITUDES *

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After discussion of legitimacy of the dispersive approach in the Standard Model sum rules (s.r.) for parity violating (p.v.) amplitudes are presented. These are s.r. for polarizabilities and p.v. analogue of the Gerasimov–Drell–Hearn sum rule. Phenomenological implications are reviewed.

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1. Introduction

In this talk results reflecting the p.v. structure of the electromagnetic current will be considered. More precisely, the sum rules for Compton amplitudes derived in [1] and their possible phenomenological consequences [2] will be discussed. In this context real photon initiated processes will be of interest. Of course, both real photon and electron-initiated collisions are of interest when one looks for p.v. effects. However, convenient feature of (real) photon case is the absence of direct Z exchange between projectile and target, so it is a unique situation where the p.v. structure of the electromagnetic current itself is singled out without further elaboration. On the other hand, disentangling virtual photon p.v. contributions from electroproduction seems to be difficult. Already at $Q^2 > 0.1 \text{ GeV}^2$ its contribution to the measured asymmetries is estimated to be a few percent of that coming from the neutral current [3].

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2. Sum rules

As we want to discuss dispersion formulae for collision amplitudes, it is a suitable place to ask to what extent the usual properties of these amplitudes (existence of asymptotic states and of interpolating local fields) are exhibited in the Standard Model. The asymptotic states have to correspond to Fock space of stable particles, so we are left with photons, electrons, neutrinos (at least the lightest one), protons and stable atomic ions. Let us mention here that the existence of unstable fields is a source of concern in Quantum Field Theory [4, 5]. Next, each stable particle should correspond to irreducible Poincaré (unitary) representation. Here there is a trouble with charged particles [6, 7], connected with QED infrared radiation. In fact a well defined treatment of the infrared region exists in perturbative calculus only. This is the reason that our considerations concerning Compton amplitudes will be limited to the order α in p.c. part and to order α^2 in p.v. part (they are infrared safe and at low energies are αG_F order contributions). Still we are left with the problem of asymptotic states and interpolating fields in QCD part of SM — we shall rely on the results of Oehme: “the analytic properties of physical amplitudes are the same as those obtained on the basis of an effective theory involving only the composite, physical fields” [8] (in other words confinement does not spoil old axiomatic proofs for hadronic interactions [9]).

Therefore, it is legitimate to consider Compton amplitudes for any stable targets and the results derived should be valid in any order of strong interactions but to the order α and α^2 for p.c. and p.v. parts of amplitude, respectively. In what follows we shall be interested in forward Compton amplitudes without spin-flip, $f_{s,h}(\omega)$; ω is photon’s lab energy, s and h denote target’s z -component of spin and photon’s helicity, respectively. (z -axis is taken in the direction of photon’s momentum.) We shall use normalization

$$\text{Im} f_{s,h}(\omega) = \omega \sigma_{s,h}^T(\omega). \quad (1)$$

It is convenient to consider p.v. amplitudes averaged over the spin of the target

$$f_h^{(-)\gamma} = \frac{1}{2S+1} \sum_{s_i} f_{s_i,h}^- \quad (2)$$

and p.v. amplitudes averaged over the photon’s helicity

$$f_s^{(-)\text{tg}} = \frac{1}{2}(f_{s,+1}^- + f_{s,-1}^-), \quad (3)$$

where

$$f_{s,h}^- = \frac{1}{2}(f_{s,h} - f_{-s,-h}) \quad (4)$$

is p.v. part of amplitude $f_{s,h}$. Low energy theorems are known for targets of any spin [10–13]. As a result one knows that for $\omega \rightarrow 0$

$$\text{Re} f_s^{(-)\text{tg}} = -4\pi\omega^2 a_s^{(-)\text{tg}} \quad (5)$$

and

$$\text{Re} f_h^{(-)\gamma} = -4\pi\omega^3 a_h^{(-)\gamma}, \quad (6)$$

where $a^{(-)}$ are finite p.v. forward spin polarizabilities defined in analogy with p.c. forward spin polarizabilities [14]. Using twice subtracted dispersion relations, sum rules for p.v. polarizabilities have been obtained [1]:

$$a_s^{(-)\text{tg}}(0) = \frac{1}{4\pi^2} \int_{\omega_{\text{th}}}^{\infty} \frac{\sigma_{-s}^T - \sigma_s^T}{\omega'^2} d\omega', \quad (7)$$

$$a_h^{(-)\gamma}(0) = \frac{1}{4\pi^2} \int_{\omega_{\text{th}}}^{\infty} \frac{\sigma_{-h}^T - \sigma_h^T}{\omega'^3} d\omega. \quad (8)$$

Assuming superconvergence of the type $f(z)/z \rightarrow 0$ for (crossing-odd) amplitude $f_h^{(-)\gamma}$ then the p.v. analogue of DGH [15, 16] sum rule has been obtained [1]:

$$\int_{\omega_{\text{th}}}^{\infty} \frac{\sigma_h^T - \sigma_{-h}^T}{\omega'} d\omega' = 0. \quad (9)$$

3. Phenomenological implications

Experimental examination of p.v. sum rules would lead to additional constraints on weak hadronic effective couplings. These couplings cannot be uniquely determined from existing decay data and their values are model dependent [17]. For example p.v. πNN coupling h_π^1 can be in the range $(0.2\text{--}11.4) \cdot 10^{-7}$. If h_π^1 were large (as in so called “best fit” [17]) then the exchange of π mesons would be dominant in description of the low energy p.v. induced processes. It was pointed out in [1], chapter 4, that in such a case the sum rule (7) exhibits order of magnitude inconsistency between HB χ PT [18, 19] value of $a_s^{(-)\text{tg}}$ and r.h.s. of Eq. (7) calculated from the photoproduction contributions obtained in analysis [3] where model including elaborated Born type exchanges (with resonances and form factors taken into account) was used. It was also shown — from inspection of sum rule (9) — that in the case of “best fit” one should expect quite large p.v. photoproduction cross sections in the energy region 0.5–1 GeV. (Compare discussion and Eq. 4.14

in [1] — on the l.h.s. of this equation factor $\frac{1}{2}$ is lacking.) More detailed analysis of the sum rule (9) has been given in [2] where eight different models describing hitherto known data [17] are discussed. Two of these models predict large h_π^1 and [2] confirm that then s.r. (9) demand large values of p.v. photoproduction above 0.5 GeV. The remaining models can be accommodated with quick saturation hypothesis (comp. chapter 2, [2]). It is also shown that by measuring experimental asymmetries at energies below 0.5 GeV one can in principle select one or two models which would be compatible with the data. This can happen due to different behaviour of asymmetries as a function of energy (see Fig. 3 in [2]). Verification of p.v. sum rules in the near future depends, of course, on experimental possibilities. It is plausible that at least a part of low energy contributions can be experimentally checked in existing experimental facilities (B. Wojtsekhowski, private communication).

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