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DARK MATTER AND THE ILC*

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We discuss the solution to the Dark Matter problem provided by the lightest neutralino of the Minimal Supersymmetric Standard Model (MSSM) and highlight the role of the International Linear Collider (ILC) in determining its cosmological relic density.

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1. Introduction

As deduced from the WMAP satellite measurement of the temperature anisotropies in the Cosmic Microwave Background, in combination with data on the Hubble expansion and the density fluctuations in the Universe, cold Dark Matter (DM) makes up $\approx 25\%$ of the energy of the Universe [1]. The DM cosmological relic density is precisely measured to be

$$\Omega_{\rm DM}\, h^2 = 0.113 \pm 0.009 \tag{1}$$

which leads to $0.087 \leq \Omega_{\rm DM}\, h^2 \leq 0.138$ at the 99% confidence level. In these equations, $\Omega \equiv \rho/\rho_{\rm c}$, where $\rho_{\rm c} \simeq 2 \times 10^{-29} h^2 {\rm g/cm}^3$ is the "critical" mass density that yields a flat universe, as favored by inflationary cosmology and as verified by the WMAP satellite itself; $\rho < \rho_{\rm c}$ and $\rho > \rho_{\rm c}$ correspond, respectively, to an open and closed universe, *i.e.* a metric with negative or positive curvature. The dimensionless parameter h is the scaled Hubble constant describing the expansion of the Universe.

In the Minimal Supersymmetric Standard Model (MSSM), there is an ideal candidate for the weakly interacting massive particle (WIMP) which is expected to form this cold DM: the lightest neutralino χ_1^0 which is a mixture of the supersymmetric partners of the neutral gauge and Higgs bosons and is in general the Lightest Supersymmetric Particle (LSP). This electrically

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neutral particle is absolutely stable when the symmetry called R-parity is conserved, is massive and thus non-relativistic or cold. Furthermore, it has only weak interactions and for a wide range of the MSSM parameter space, its annihilation rate into SM particles fulfills the requirement that the resulting cosmological relic density is within the range measured by WMAP. This is particularly the case in the widely studied minimal Supergravity (mSUGRA) scenario [3] and in some of its variants; see Ref. [2].

In this brief note, we discuss the contribution of the LSP neutralino to the overall matter density of the Universe and highlight the role of the International Linear Collider (ILC) in determining this relic density.

2. The Dark Matter relic density

To derive the cosmological relic density, the standard treatment is based on the assumption [besides that the LSP should be effectively stable, *i.e.* its lifetime should be long compared to the age of the Universe, which holds in the MSSM with conserved R-parity which is discussed here] that the temperature of the Universe after the last period of entropy production must exceed $\sim 10\%$ of $m_{\chi_1^0}$, an assumption which is quite natural in the framework of inflationary models [2]. In the early Universe all particles were abundantly produced and were in thermal equilibrium through annihilation and production processes. The time evolution of the number density of the particles is governed by the Boltzmann equation

$$\frac{dn_{\chi_1^0}}{dt} + 3Hn_{\chi_1^0} = -\langle v\,\sigma_{\rm ann}\rangle \left(n^2_{\chi_1^0} - n^{{\rm eq}\,2}_{\chi_1^0}\right) , \tag{2}$$

where v is the relative LSP velocity in their center-of-mass frame, $\sigma_{\rm ann}$ is the LSP annihilation cross section into SM particles and $\langle \ldots \rangle$ denotes thermal averaging; $n_{\chi_1^0}$ is the actual number density, while $n^{\rm eq}_{\chi_1^0}$ is the thermal equilibrium number density. The Hubble term takes care of the decrease in number density due to the expansion, while the first and second terms on the right-hand side represent, respectively, the decrease due to annihilation and the increase through creation by the inverse reactions. If the assumptions mentioned above hold, χ_1^0 decouples from the thermal bath of SM particles at an inverse scaled temperature $x_{\rm F} \equiv m_{\chi_1^0}/T_{\rm F}$ which is given by [2]

$$x_{\rm F} = 0.38 M_{\rm P} \langle v\sigma_{\rm ann}\rangle c(c+2) m_{\chi_1^0}\,(g_* x_{\rm F})^{-1/2} \,, \tag{3}$$

where $M_{\rm P} = 2.4\times 10^{18}$ GeV is the (reduced) Planck mass, g_* the number of relativistic degrees of freedom which is typically $g_* \simeq 80$ at $T_{\rm F}$, and c a numerical constant which is taken to be $1/2$; one typically finds

$x_{\rm F} \simeq 20$ to 25. Today's LSP density in units of the critical density is then given by [2]

$$\Omega_\chi h^2 = \frac{2.13 \times 10^8/\mathrm{GeV}}{\sqrt{g_*} M_{\rm P} J(x_{\rm F})}\,, \quad \text{with} \quad J(x_{\rm F}) = \int_{x_{\rm F}}^{\infty} \frac{\langle v\sigma_{\rm ann}\rangle(x)}{x^2} dx\,. \tag{4}$$

Eqs. (3), (4) provide an approximate solution of the Boltzmann equation which has been shown to describe the exact numerical solution very accurately for all known scenarios. Since χ_1^0 decouples at a temperature $T_{\rm F} \ll m_\chi$, in most cases it is sufficient to use an expansion of the LSP annihilation rate in powers of the relative velocity between the LSPs

$$v\,\sigma_{\rm ann} \equiv v\,\sigma(\chi_1^0\chi_1^0 \to \mathrm{SM\,particles}) = a + bv^2 + \mathcal{O}(v^4)\,. \tag{5}$$

The entire dependence on the model parameters is then contained in the coefficients a and b, which essentially describe the LSP annihilation cross section from an initial S- and P-wave, since the expansion of the annihilation rate of Eq. (5) is only up to $\mathcal{O}(v^2)$. S-wave contributions start at $\mathcal{O}(1)$ and contain $\mathcal{O}(v^2)$ terms that contribute to Eq. (5) via interference with the $\mathcal{O}(1)$ terms. In contrast, P-wave matrix elements start at $\mathcal{O}(v)$, so that only the leading term in the expansion is needed. There is no interference between S- and P-wave contributions, and hence no $\mathcal{O}(v)$ terms.

In generic scenarios the expansion Eq. (5) reproduces exact results quite well. However, it fails in some exceptional cases [2] all of which can be realized in some part of the MSSM parameter space, and even in mSUGRA:

(i) The expansion breaks down near the threshold for the production of heavy particles, where the cross section depends very sensitively on the c.m. energy $\sqrt{s}$. In particular, due to the non-vanishing kinetic energy of the neutralinos, annihilation into final states with mass exceeding twice the LSP mass ("sub-threshold annihilation") is possible. This is particularly important in the case of neutralino annihilation into W^+W^- and hh pairs, for relatively light higgsino-like and mixed LSPs, respectively.

(ii) The expansion Eq. (5) also fails near s-channel poles, where the cross section again varies rapidly with $\sqrt{s}$. In the MSSM, this happens if twice the LSP mass is near M_Z, or near the mass of one of the neutral Higgs bosons. In models with universal gaugino masses, the Z-pole region is now excluded by chargino searches at LEP2 and we are left only with the Higgs pole regions which are important as will be seen later.

(iii) If the mass splitting between the LSP and the next-to-lightest superparticle NLSP is less than a few times $T_{\rm F}$, co-annihilation processes involving one LSP and one NLSP, or two NLSPs, can be important. Co-annihilation is important in three cases: higgsino or SU(2) gaugino like LSPs and when

the LSP is degenerate in mass with $\tilde{\tau}_1$ or with the lightest top squark (the latter case hardly occurs in mSUGRA scenarios).

3. The relic density in the mSUGRA scenario

The mSUGRA model [3] is the most widely studied implementation of the MSSM and it manages to describe phenomenologically acceptable spectra with only four parameters plus a sign:

$$m_0,\ m_{1/2},\ A_0,\ \tan\beta,\ \mathrm{sign}\mu\,, \tag{6}$$

where $m_0, m_{1/2}$ and A_0 are the common soft SUSY-breaking terms of all scalar masses, gaugino masses and trilinear scalar interactions, defined at the Grand Unification scale. $\tan\beta$ is the ratio of the vacuum expectation values (vev's) of the two Higgs doublets at the weak scale and μ is the supersymmetric higgs(ino) mass parameter.

We use the Fortran code SUSPECT [4] to solve the RGE and to calculate the spectrum of physical sparticles and Higgs bosons, following the procedure outlined in Ref. [5]. In addition to leading to consistent electroweak symmetry breaking (EWSB), a given set of input parameters has to satisfy experimental constraints [6]. The ones relevant for this study are:

— The total cross section for the production of any pair of sparticles at the highest LEP energy (209 GeV) must be less than 20 fb.

— Searches for neutral Higgs bosons at LEP impose a lower bound on m_h; allowing for a theoretical uncertainty, one requires $m_h > 111$ GeV.

— Recent measurements of the muon magnetic moment lead to the constraint on the SUSY contribution: $-5.7\times10^{-10} \leq a_{\mu,\mathrm{SUSY}} \leq 4.7\times10^{-9}$.

— Allowing for experimental and theoretical errors, the branching ratio for radiative b decays should be $2.65\times10^{-4} \leq B(b\to s\gamma) \leq 4.45\times10^{-4}$.

— Finally, the calculated $\tilde{\chi}_1^0$ relic density has to be in the range (1).

The output is shown and partly commented in Fig. 1 The black regions are those satisfying the DM constraint, Eq. (1). In general, there are four familiar regions where this constraint is satisfied. *(i)* Scenarios where both m_0 and $m_{1/2}$ are rather small (the "bulk region") are most natural from the point of view of EWSB but are severely squeezed by lower bounds from searches for sparticles and Higgs bosons. *(ii)* In the "co-annihilation" region one has $m_{\chi_1^0} \simeq m_{\tilde{\tau}_1}$, leading to enhanced destruction of sparticles since the $\tilde{\tau}_1$ annihilation cross section is about ten times larger than that of the LSP; this requires $m_{1/2} \gg m_0$. *(iii)* The "focus point" or "hyperbolical branch"

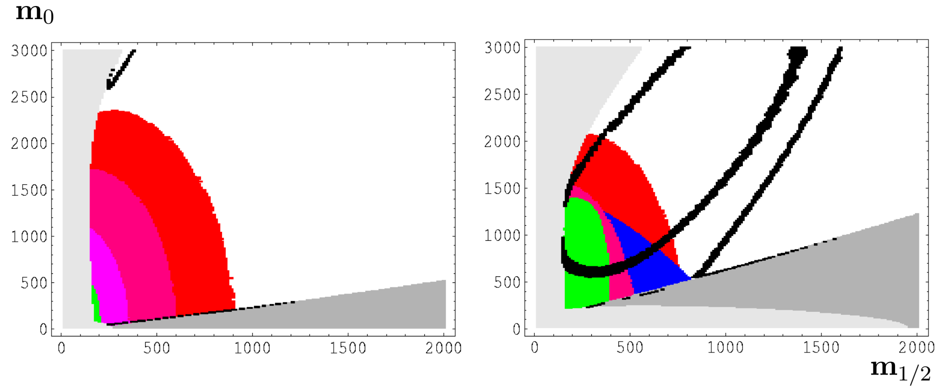

Fig. 1. The mSUGRA $(m_{1/2}, m_0)$ parameter space with all constraints imposed for $A_0 = 0, \mu > 0$ and $\tan\beta = 10$ (left) and 50 (right). The top quark mass is fixed to the new central value, $m_t = 172.7$ GeV. The light grey regions are excluded by the requirement of correct electroweak symmetry breaking, or by sparticle search limits. In the dark grey regions $\tilde{\tau}_1$ would be the LSP. The light pink regions are excluded by searches for neutral Higgs bosons at LEP, whereas the green regions are excluded by the $b \to s\gamma$ constraint. In the blue region, the SUSY contribution to $g_\mu - 2$ falls in the correct range whereas the red regions are compatible with having an SM-like Higgs boson near 115 GeV. Finally, the black regions satisfy the DM constraint.

region occurs at $m_0 \gg m_{1/2}$, and allows $\tilde{\chi}_1^0$ to have a significant higgsino component, enhancing its annihilation cross sections into final states containing gauge and/or Higgs bosons; however, if m_t is much higher than its current central value of 173 GeV, this solution requires multi-TeV scalar masses. *(iv)* Finally, if $\tan\beta$ is large, the s-channel exchange of the CP-odd Higgs boson A can become nearly resonant, again leading to an acceptable relic density (the "A-pole" region).

Recently, a fifth cosmologically acceptable region of mSUGRA parameter space has been revived. In a significant region of parameter space one has $2m_{\chi_1^0} \lesssim m_h$, so that s-channel h exchange is nearly resonant. This "h-pole" region featured prominently in early discussions of the DM density in mSUGRA but seemed to be all but excluded by the combination of rising lower bounds on m_h and $m_{\chi_1^0}$ from searches at LEP [6]. However, in recent years improved calculations [7] of the mass of the light CP-even h boson have resurrected this possibility for top mass values close to or larger than 178 GeV (thus, this region does not appear in Fig. 1).

4. SUSY particle masses and the ILC

Bounds on physical masses might be a more meaningful way to show the possibilities of mSUGRA than the ubiquitous plots of allowed regions in the space of basic input parameters, Fig. 1, in which one always fixes the values of some other free parameters (*e.g.*, A_0, $\tan\beta$, m_t). One obtains the least biased view of the allowed ranges of masses by simply scanning over the entire parameter set that is consistent with a given set of constraints.

Table I lists lower bounds on the masses of some new (s)particles in mSUGRA, first without (Set I) and then with (Set II) the DM constraint. The lower bounds on many new (s)particles simply coincide with the bounds established by collider experiments. This is true for the lighter chargino, stau and scalar Higgs states, and essentially also holds true for the lighter stop. The bounds on the masses of the gluino and third neutralino are essentially the same as that in a more general MSSM, as long as gaugino mass unification is maintained. Clearly the DM constraint still allows some

TABLE I

Sparticle mass bounds in mSUGRA obtained by scanning over the entire allowed parameter space, defined by $m_t \in [171\,\text{GeV}, 185\,\text{GeV}]$, $(m_{\tilde{t}_1} + m_{\tilde{t}_2})/2 \leq 2$ TeV, the lower bounds on sparticle and Higgs masses from collider experiments, the constraint on $b \to s\gamma$, simple 'CCB' constraints and a conservative interpretation of the constraint from $g_\mu - 2$ (essentially the overlap of the 2σ regions using τ decay and e^+e^- collider data). Set II adds the DM constraint to the above set of constraints. Set III is like Set II, except that the scanned region has been artificially limited to the h-pole region, where $m_{\tilde{\chi}_1^0} \leq m_h/2$. Only *lower* bounds are listed for Sets I and II, while for Set III the allowed range is given; a dash (–) means that the upper bound is directly set by the upper bound on the average stop mass.

(s)particle	mass bounds [GeV]		
	Set I	Set II	Set III
$\tilde{\chi}_1^0$	50	53	[53, 61]
$\tilde{\chi}_1^\pm$	105	105	[105, 122]
$\tilde{\chi}_3^0$	136	137	[280, –]
$\tilde{\tau}_1$	99	99	[630, –]
h	114	114	[114, 122]
$H^\pm$	128	128	[246, –]
$\tilde{g}$	374	383	[383, 482]
$\tilde{d}_R$	444	444	[774, –]
$\tilde{t}_1$	102	110	[110, –]

new (s)particles to be quite light. One should emphasize, however, that usually the lower bounds in the table cannot be saturated simultaneously. Nevertheless, the possibility of light sparticles even in this simplest of all potentially realistic SUSY models that allow WIMP Dark Matter should be quite encouraging to experiments.

Set III shows these bounds (including the DM constraint) when one confines oneself to the h-pole region discussed at the end of the previous subsection. In this case there are significant upper bounds on the masses of all gauginos. The reason is that one needs $2m_{\tilde{\chi}_1^0} \simeq m_h \leq 120$ GeV here, leading in mSUGRA with the assumed universality of gaugino masses at the GUT scale, to relatively light charginos and neutralinos as well as gluinos.

5. The determination of the relic density at the ILC

Thus, in many scenarios SUSY particles can be produced abundantly at the next generation of high-energy colliders, in particular at the LHC and the ILC. However, to determine the predicted WIMP relic density (see the flowchart shown in the upper panel of Fig. 2), one must experimentally constrain all processes contributing to the LSP pair annihilation cross section; this requires detailed knowledge not only of the LSP properties, but also of all other particles contributing to their annihilation. This is not a simple task and all unknown parameters entering the determination of $\Omega_\chi h^2$ need to be experimentally measured or shown to have marginal effects.

Many high precision measurements are possible at the LHC, but many of them can be vastly improved at the ILC. Because of the clean environment and the knowledge of the c.m. energy of the initial beams, sparticle masses can be determined with high accuracy through kinematic endpoints and threshold scans. The results of one study in a given mSUGRA scenario are summarized in the lower panel of Fig. 2, where the achievable precision at collider experiments are compared with the satellite determination of $\Omega_\chi h^2$. The figure shows that the ILC will provide a part per mille determination of $\Omega_\chi h^2$ in the case under study, matching WMAP and even the huge accuracy expected from Planck. The many possible implications of such measurements are outlined in the flowchart in the lower part of the figure.

Thus, if DM is composed of the lightest neutralinos, the LHC and particularly the ILC will be able to determine the WIMP's properties and pin down its relic density. If these determinations match cosmological observations to high precision, then (and only then) we will be able to claim to have determined what dark matter is. Such an achievement would be a great success of the particle physics/cosmology connection and would give us confidence in our understanding of the Universe.

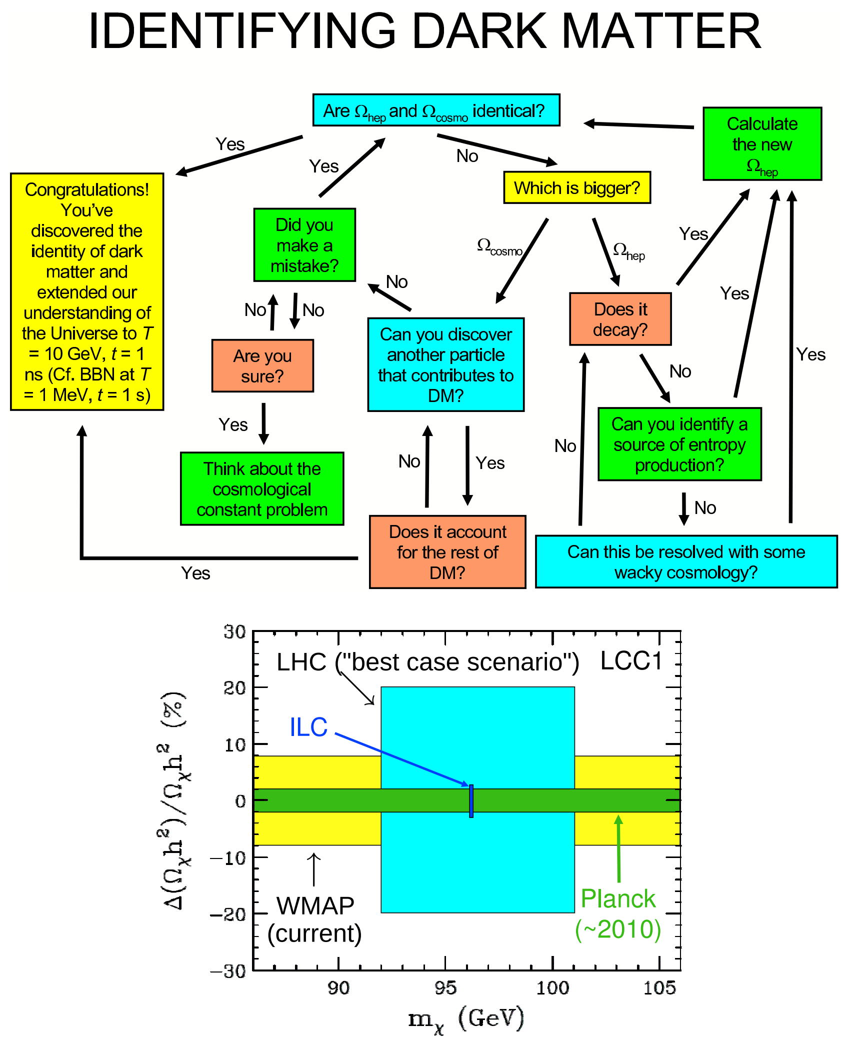

Fig. 2. Upper: Flowchart illustrating the possible implications of comparing $\Omega_{\rm hep}$, the predicted DM density determined from high energy physics, and $\Omega_{\rm cosmo}$, the actual DM density determined by WMAP and Planck. Lower: Constraints in the $(m_{\chi_1^0}, \Delta(\Omega_\chi h^2)/\Omega_\chi h^2)$ plane from the ILC and LHC. Constraints on $\Delta(\Omega_\chi h^2)/\Omega_\chi h^2$ from WMAP and Planck (which provide no constraints on $m_{\chi_1^0}$) are also shown; from Ref. [8].

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