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SOME NEW DEVELOPMENTS IN NONLINEAR OPTICS*

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We review some new developments in modern nonlinear optics. They are related to optics with both photons and atoms. The goals of the presentation were to give a cursory review on contemporary challenges in nonlinear optics (focusing on problems that are of interest to the Warsaw nonlinear optics group) and to show how fruitful it can be to transfer ideas across the border between different fields.

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1. Short introduction to nonlinear optics

Nonlinear optics is the study of the interaction of intense laser light with matter. It is a young field of research, which takes its beginning from the observation of second harmonic generation in 1961 by Franken's group [1] (this milestone is flawed, as the second harmonic signal actually does not appear on the experimental plate presented there as Fig. 1). Almost immediately a series of experiments followed, where other wave mixing phenomena were obtained (third harmonic generation, sum and difference frequency generation, *etc.*). The field now ranges from fundamental studies of interaction of laser light with matter to applications such as laser frequency conversion, optical switching and quantum gates. Here we will give a few examples of new developments, concentrating on new sources of coherent radiation and solitons. A second goal of this presentation is to show how fruitful it can be to transfer ideas across the border between different fields, in this case nonlinear and atom optics.

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Tunable sources of coherent radiation are very desirable. The last decade has seen spectacular developments in ultrafast laser technology, due to the introduction of solid state active materials and of new mode-locking and amplification techniques. These advances, together with the discovery of new nonlinear optical crystals, have fostered the introduction of ultrafast optical parametric amplifiers and oscillators as practical sources of femtosecond pulses tunable across the visible and infrared spectral ranges. Spontaneous parametric down-conversion is an important process in nonlinear optics. A nonlinear crystal splits incoming individual photons into pairs of lower energy whose combined energy and momentum are equal to the energy and momentum of the original photon. "Parametric" refers to the fact that the state of the crystal is left unchanged in the process, which is why energy and momentum are conserved (this is related to phase matching in nonlinear optics; phase matching in this particular case is illustrated in Fig. 1). The process is spontaneous in the same sense as is spontaneous emission. It is initiated by random vacuum fluctuations. Consequently, the photon pairs are created at random times. Nevertheless, if one of the pairs (the "signal") is detected at any time, then we know its partner (the "idler") is also present. This then allows for the creation of optical fields containing a single photon. As of 2005, this is the predominant mechanism for experimentalists to create single photons (also known as Fock states). The single photons as well as photon pairs are often used in quantum information experiments and applications such as quantum cryptography and Bell tests [2].

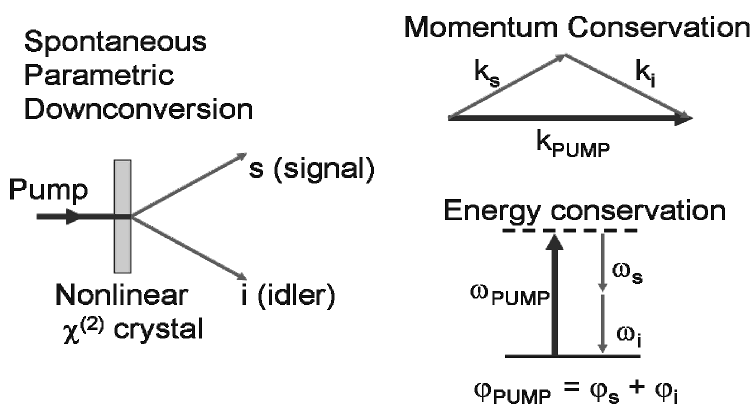

Fig. 1. Schematic presentation of the features of parametric down conversion, including phase matching and energy conservation (from Wikipedia).

2. Atom optics

It is well known that photons do not interact in free space. To create polarization (linear or nonlinear) responsible for wave mixing phenomena it is necessary for light to propagate within a medium, be it an atomic vapor, a crystal, a plasma, *etc.* Far from resonances it becomes possible to eliminate the material dynamics and obtain effective nonlinear equations for the optical fields only. The information of the medium remains in the nonlinear susceptibilities, as mentioned in the previous section.

The situation appears to be fundamentally different for atoms: after all, it is known that for high enough fluxes or densities, atoms undergo collisions. The presence of other atoms modifies the evolution of a given atom, hence atom optics is inherently nonlinear. Most of the collisions are incoherent and we had to wait until 1995, when for the first time neutral atom gasses were Bose–Einstein condensed. This created a condition analogous to that familiar from nonlinear optics. The miracle is in the peculiar behavior of ultra cold Bose atoms, which all like to occupy the same quantum state. In this case elastic collisions merely result in a coherent phase shift between atoms, an effect analogous to the creation of coherent "nonlinear polarization". Moreover, the macroscopic population of this quantum state allows us to introduce a control parameter, a wavefunction which satisfies a nonlinear Gross–Pitaevskii equation:

$$i\hbar\,\frac{\partial\psi(\boldsymbol{r},t)}{\partial t} = \left(-\frac{1}{2m}\triangle + V(\boldsymbol{r},t) + N_0U_0|\psi|^2\right)\psi\,, \tag{1}$$

where $V(\boldsymbol{r},t)$ is an external potential, $U_0 = (4\pi a_0\hbar^2)/m$ is the atom–atom interaction strength that is proportional to the s-wave scattering length a_0, and N_0 is the total number of atoms.

The Gross–Pitaevskii equation is formally reminiscent of the nonlinear wave equation describing the paraxial propagation of light in a nonlinear medium characterized by an instantaneous cubic nonlinearity. In view of this analogy it is not surprising that many nonlinear effects first predicted and demonstrated in optics could be repeated with matter waves. For example, four-wave mixing of atom waves (4WM), a process in which three matter waves combine to produce a fourth while conserving energy and momentum, has been demonstrated by researchers at NIST [3]. This experiment provided the first example in atoms of "nonlinear optics" effects which are known for laser beams. In lasers, these effects arise when the light is so intense that it changes the index of refraction of the material which it traverses. The behavior of the material thereby depends on the intensity of light, and this non-linearity can lead to the self-focusing of light and the creation of new colors from a single one. The NIST researchers created three overlapping

Bose–Einstein condensates of sodium atoms moving at different velocities relative to one another. The three BECs interfered to create a fourth condensate moving at a different velocity. This four-wave mixing phenomenon, which also occurs in light waves, can be used to explore the uniquely quantum mechanical properties of matter waves (see Fig. 2). For methods to strengthen the process see Ref. [4].

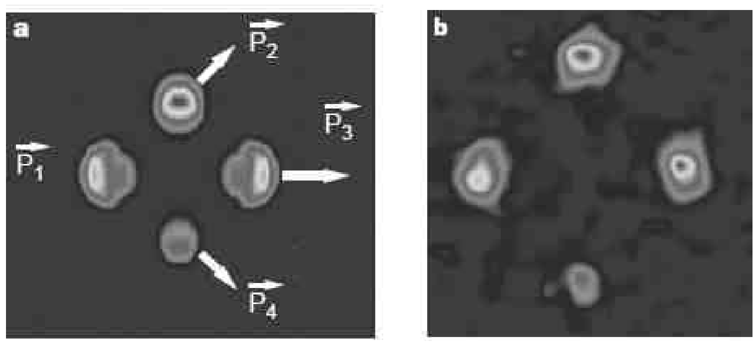

Fig. 2. Numerical simulation and experimental results for 4WM. Left: Calculated two dimensional atomic distribution after 1.8 ms, showing the 4WM. We note that atoms are removed primarily from the inner ends of the wave-packets because these regions overlap for the longest time. Right: An image of the experimental atomic distribution showing the fourth (small) wave-packet generated by the 4WM process. We have verified that if we make initial wave-packets such that energy and momentum conservation cannot be simultaneously satisfied, no 4WM signal is observed, as expected. From Deng *et al.,* 1999.

A four wave process, analogous to that described above, is also possible by mixing atoms and photons. For instance, the MIT group investigated Rayleigh scattering off a Bose–Einstein condensate [5]. Observation of super-radiant Rayleigh scattering was conducted in the following way. An elongated condensate was illuminated by a single off-resonant laser beam. Collective scattering leads to photons scattered predominantly along the long axial direction and atoms emerging at 45 degrees. Notice that this is a four wave mixing process in which two particles are annihilated: one photon from the laser beam and one atom from the initial stationary condensate, and two particles are created: one photon at the direction of the long axis and one atom at 45 degrees. This is illustrated in Fig. 3.

In the future, practical matter-wave devices based on phenomena described above are likely to be built on chips, very much like electronic sensors. It has recently been possible to generate a Bose–Einstein condensate directly on a chip and to couple atomic condensates into waveguides. These spectacular and rapid developments augur well for the future.

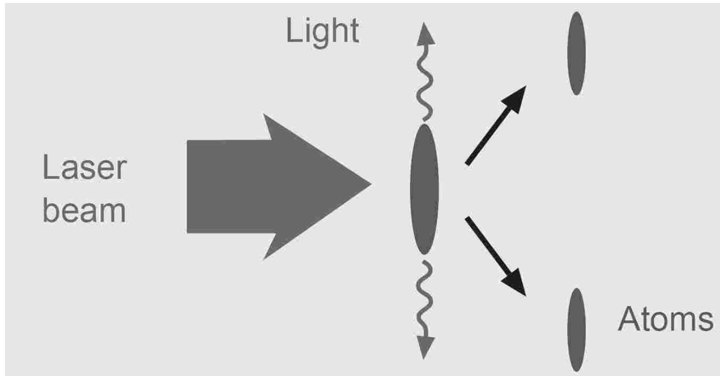

Fig. 3. Exposing an elongated condensate to a single off-resonant laser beam resulted in the observation of highly directional scattering of light and atoms. This collective light scattering is caused by the coherent center-of-mass motion of the atoms in the condensate. A directional beam of recoiling atoms was built up by matter wave amplification.

3. Solitons

In August 1834 Scott Russell, an important engineer who never managed to succeed in obtaining a university chair, was investigating the Union Canal for its possible use for steamships. Indeed, the good captain was best known for his work on ship design. On this day he was mounted on a horse and luckily observed a boat drawn by a pair of horses that suddenly stopped. A bow wave so created detached itself from the boat and continued down the canal in the form of a single hump, easily followed by the horseman. The wave kept both its shape and its velocity for a long time (one foot high and thirty feet long propagating at 8 to 9 mph) [6]. This discovery is one of the finest examples of serendipity in XIX Century physics. A similar soliton was recently created at the same spot (1995, see `http://www.ma.hw.ac.uk/solitons/press.html`).

In the work published in 1965 Zabusky and Kruskal numerically investigated an equation describing the strong interaction of phonons [7]. They found that an initially cosine profile broke up into a sequence of solitary waves, which they called solitons due to their apparent quantal nature (see Fig. 4)

The soliton equation involved (Korteveg de Vries) was nonlinear, but two years later became the first in a long sequence to be solved by an "inverse scattering method" [8]. The method involves linearization in some space. This fortunate fact and the plethora of applications subsequently found across the board of physics catapulted solitons into the very forefront of modern research.

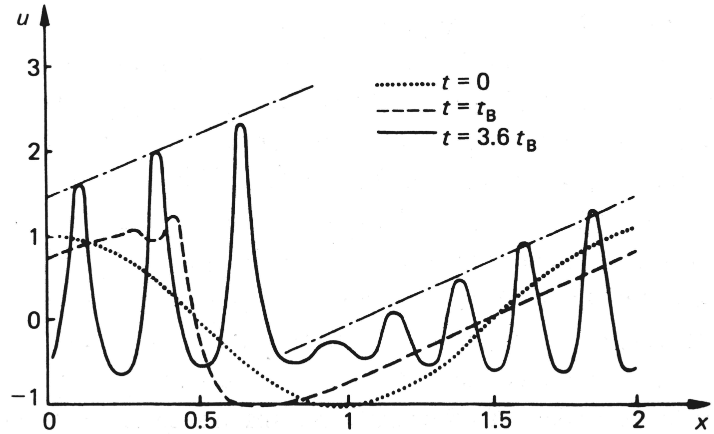

Fig. 4. Evolution of an initially periodic profile, $\cos(\pi x)$, as given by the KdV equation. The breaking time for the wave profile, when the 3rd term is neglected, is $t_{\rm B}$. After a while patterns roughly repeat themselves. From Zabusky and Kruskal [7].

Solitons have been observed in plasma physics experiments and crystal lattices [9]. Solitons are used in models of high temperature superconductors systems and of energy transport in DNA. They are even used as models in nuclear theory. The fact that solitons are so robust makes them ideal for fiber optical communication systems [10]. Nowadays, research on soliton telecommunications is basically focused on solutions provided by dispersion management or compensation. In this regime, optical pulse propagation in the presence of fiber nonlinearity turns out to be extremely stable.

Solitons can be essentially one dimensional, as the water wave ones mentioned above, two dimensional *e.g.* cylindrical, known as lumps, or three dimensional, often referred to as bullets. In the case of solitons, nonlinear effects balance the diffractive spreading of a pulse. In isotropic optical media the most common type of nonlinearity is the so called Kerr nonlinearity, which is cubic. Unfortunately the balance between dispersion and Kerr nonlinearity is only possible in 1D. For the stability of higher dimensional pulses a saturable nonlinearity and/or modulation of parameters is necessary [11, 12]. As an example we suggest considering two and three dimensional bright solitons in a BEC.

Bose–Einstein condensates are media in which two of the above kinds of soliton can be created (2D and 3D). Suppose the condensate is confined by a one dimensional optical lattice. If we combine the temporal modulation of the nonlinearity with confinement by the one dimensional lattice, both two and three dimensional optical solitons can be created. Strong confinement will lead to two dimensional solitons, moderate confinement to three dimensional ones. Weak confinement further allows the individual solitons to interact [13]. Modulation of the nonlinearity is achieved by variation of the external magnetic field, which causes the so called Feshbach resonance [14]. If the magnetic field oscillates around an appropriate value we have a periodic modulation of the scattering length, including a change of sign. These oscillations, when fast, can stabilize the soliton. Fully three dimensional BEC solitons can be created by a combination of Feshbach resonance management and a one dimensional optical lattice. Such a lattice can be created by illuminating the BEC by a pair of counter-propagating laser beams such that they form a periodic interface pattern.

Somewhat unexpectedly we found one region of stable 2D solitons in parameter space and *two* for 3D soliton formation. This situation is illustrated in Fig. 5. The two regions where stable 3D solitons exist are distinct. Only one of them has a counterpart for 2D solitons. The other region appears when the frequency of the modulation exceeds the lowest excitation frequency of the confining potential which is fixed. The results of Fig. 5 come from direct simulations, a simplified variational analysis and some simple theoretical considerations. At first glance it might seem strange that adding

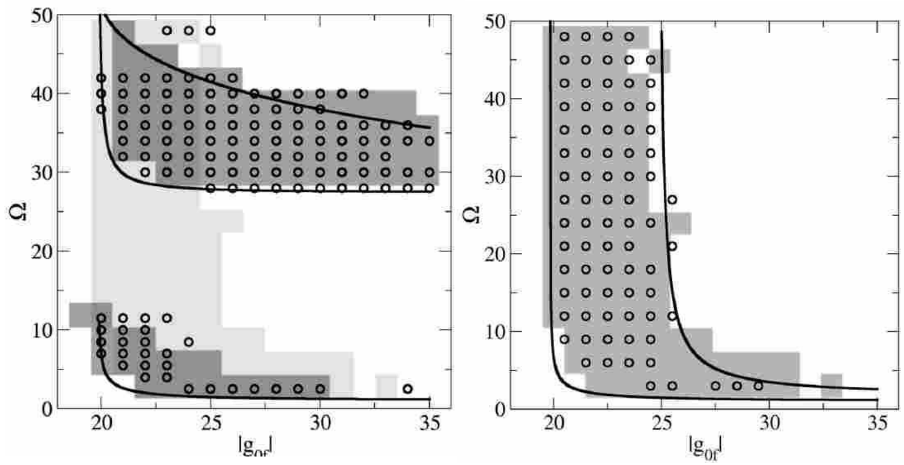

Fig. 5. Stability regions for solitons in the $(|g_{0f}|, \Omega)$ plane, as predicted by the variational approximation (shaded area), and by direct simulations of the Gross–Pitaevskii equation (circles). Here $|g_{0f}|$ is the average value of the nonlinear interaction, Ω is the frequency of the modulation of the scattering length. The frame on the left was obtained from a 3D analysis and that on the right is the result of a 2D treatment. The lower region on the left corresponds to 2D solitons, and solitons in the upper region are fully 3D. As the confinement increases, the lower region obtained from the 3D analysis becomes more and more like that following from 2D. For more details see Ref. [13].

a degree of freedom stabilizes a soliton situation. The key to this dichotomy seems to be the presence of a periodic modulation. This can be illustrated by a simple case involving oscillators. Take as the one dimensional version a forced oscillator problem:

$$\ddot{x} + \omega_0^2 x = y \cos(\omega_0 t)\,. \tag{2}$$

If y is fixed, the solution has a secular component $x = yt/(2\omega_0)\sin(\omega_0 t) + F(t)$, where $F(t)$ is a periodic function, and so the amplitude will grow as t. If, however, we allow a second degree of freedom, such that y also oscillates (for instance $\ddot{y} + \varepsilon^2 y = 0$) the solution stabilizes, unless $\varepsilon = \pm 2\omega_0$. In general this can be the case when there are periodic modulations.

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