MUON PAIR PRODUCTION IN RELATIVISTIC NUCLEAR COLLISIONS*

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The exclusive production of one $\mu^+\mu^-$ pair in collisions of two ultra-relativistic nuclei is considered. We present the simple method for calculation of the Born cross section for this process. Then we found that the Coulomb corrections to this cross section (which correspond to multi-photon exchange of the produced $\mu^\pm$ with nuclei) are small while the unitarity corrections are large. This is in sharp contrast to the exclusive e^+e^- pair production where the Coulomb corrections to the Born cross section are large while the unitarity corrections are small. We calculated also the cross section for the production of one $\mu^+\mu^-$ pair and several e^+e^- pairs in the leading logarithmic approximation. Using this cross section we found that the inclusive production of $\mu^+\mu^-$ pair coincides in this approximation with its Born value.

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1. Introduction

The lepton pair production in ultra-relativistic nuclear collisions were discussed in numerous papers (see review [1] and references therein). For the RHIC and LHC colliders the charge numbers of nuclei $Z_1 = Z_2 \equiv Z$ and their Lorentz factors $\gamma_1 = \gamma_2 \equiv \gamma$ are given in Table I.

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TABLE I

Colliders and the Born cross sections for lepton pair production.

Collider	Z	γ	$\sigma_{\rm Born}^{e^+e^-}$ [kb]	$\sigma_{\rm Born}^{\mu^+\mu^-}$ [b]
RHIC, Au–Au	79	108	36.0	0.23
LHC, Pb–Pb	82	3000	227	2.6

The cross section of one e^+e^- pair production in the Born approximation, described by Feynman diagram of Fig. 1, was obtained many years ago [2]. Since the Born cross section $\sigma_{\rm Born}^{e^+e^-}$ is huge (see Table I), the e^+e^- pair production can be a serious background for many experiments.

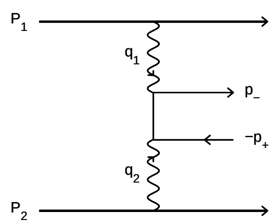

Fig. 1. The Feynman diagram for the lepton pair production in the Born approximation.

It is also important for the problem of beam lifetime and luminosity of colliders. It means that the various corrections to the Born cross section are of great importance. At present, there are a lot of controversial and incorrect statements in papers devoted to this subject. The corresponding references and critical remarks can be found in [1, 3, 4]. Since the parameter $Z\alpha$ may be not small ($Z\alpha \approx 0.6$ for Au–Au and Pb–Pb collisions), the whole series in $Z\alpha$ has to be summed to obtain the cross section with sufficient accuracy. The exact cross section for one pair production σ_1 can be written in the form

$$\sigma_1 = \sigma_{\rm Born} + \sigma_{\rm Coul} + \sigma_{\rm unit}\,, \tag{1}$$

where two different types of corrections have to be distinguished. The Coulomb correction $\sigma_{\rm Coul}$ corresponds to multi-photon exchange of the produced $e^\pm$ with nuclei (Fig. 2); it was calculated in [3]. The unitarity correction $\sigma_{\rm unit}$ corresponds to the exchange of light-by-light blocks between nuclei

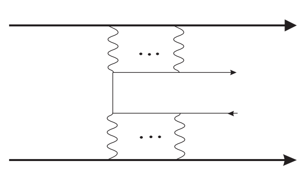

Fig. 2. The Feynman diagram for the Coulomb correction.

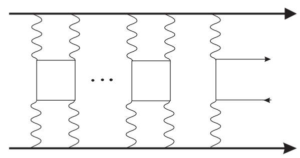

Fig. 3. The Feynman diargam for the unitarity correction.

(Fig. 3); it was calculated in [4]. The results of [4] recently were confirmed in [5] by direct summation of the Feynman diagrams. It was found that the Coulomb corrections are large while the unitarity corrections are small (see Table II).

TABLE II

Coulomb and unitarity corrections to the e^+e^- pair production.

Collider	$\sigma_{\rm Coul}/\sigma_{\rm Born}$	$\sigma_{\rm unit}/\sigma_{\rm Born}$
RHIC, Au–Au	-25%	-4.1%
LHC, Pb–Pb	-14%	-3.3%

Muon pair production may be easier for an experimental observation. The calculation scheme for the $\mu^+\mu^-$ pair production is quite different from that for the e^+e^- pair production.

2. Born cross section for one $\mu^+\mu^-$ pair production

The production of one $\mu^+\mu^-$ pair

$$Z_1 + Z_2 \to Z_1 + Z_2 + \mu^+\mu^- \,, \tag{2}$$

in the Born approximation is described by the Feynman diagram of Fig. 1. When two nuclei with charges Z_1e and Z_2e collide with each other, they emit equivalent (virtual) photons with the 4-momenta q_1, q_2, energies ω_1, ω_2 and the virtualities $Q_1^2 = -q_1^2$, $Q_2^2 = -q_2^2$. Upon fusion, these photons produce a $\mu^+\mu^-$ pair with the total four-momentum $q_1 + q_2$ and the invariant mass squared $W^2 = (q_1 + q_2)^2$, besides we denote $(P_1 + P_2)^2 = 4E^2 = 4M^2\gamma^2$, $L = \ln(\gamma^2)$, $\alpha \approx 1/137$ and use the system of units in which $c = 1$ and $\hbar = 1$.

The Born cross section of the process (2) can be calculated with a good accuracy using the equivalent photon approximation (EPA) — see, for example, Ref. [6]. Let the numbers of equivalent photons be dn_1 and dn_2. The most important contribution to the production cross section stems from photons with very small virtualities $Q_i^2 \ll \mu^2$ where μ is the muon mass. In this very region the Born differential cross section $d\sigma_{\rm B}$ for the considered process is related to the cross section $\sigma_{\gamma\gamma}$ for the real $\gamma\gamma \to \mu^+\mu^-$ process by the equation

$$d\sigma_{\rm B} = dn_1 dn_2\, d\sigma_{\gamma\gamma}(W^2)\,, \qquad W^2 \approx 4\,\omega_1\,\omega_2\,. \tag{3}$$

The number of equivalent photons are (see Eq. (D.4) in Ref. [6])

$$dn_i(\omega_i, Q_i^2) = \frac{Z_i^2\alpha}{\pi}\left(1-\frac{\omega_i}{E_i}\right)\frac{d\omega_i}{\omega_i}\left(1-\frac{Q^2_{i\,\rm min}}{Q_i^2}\right)F^2(Q_i^2)\,\frac{dQ_i^2}{Q_i^2}\,, \tag{4}$$

where

$$Q_i^2 \ge Q^2_{i\,\rm min} = \frac{\omega_i^2}{\gamma^2}\,, \tag{5}$$

and $F(Q^2)$ is the nucleus electromagnetic form factor. It is important that the integral over Q^2 converges fast at $Q^2 > 1/R^2$, were $R = 1.2\,A^{1/3}$ fm is the nucleus radius and $A \approx M/m_p$ ($R{=}7\,$fm, $1/R{=}28\,$MeV for Au and Pb). Since $Q^2_{\rm min} \lesssim 1/R^2$, the main contribution to the cross section is given by virtual photons with energies $\omega_i \lesssim \gamma/R$.

In calculation below we use the form factor in a simple approximate form

$$F(Q^2) = \frac{1}{1+Q^2/\Lambda^2}\,, \tag{6}$$

which leads to

$$dn_i(\omega_i) = \frac{Z_i^2\alpha}{\pi}\, f\left(\frac{\omega_i}{\Lambda\gamma}\right)\frac{d\omega_i}{\omega_i}\,, \tag{7}$$

with

$$f(x) = \left(1+2\,x^2\right)\,\ln\left(\frac{1}{x^2}+1\right) - 2\,. \tag{8}$$

Finally we obtain the Born cross section as a simple two dimension integral:

$$\sigma_{\rm B} = \frac{Z_1^2 Z_2^2\alpha^2}{\pi^2}\int\limits_{4\mu^2}^{\infty}\frac{dW^2}{W^2}\,G(W^2)\,\sigma_{\gamma\gamma}(W^2) = \frac{(Z_1\alpha Z_2\alpha)^2}{\pi\mu^2}\,J\left(\frac{\gamma\Lambda}{\mu}\right)\,, \tag{9}$$

where

$$G(W^2) = \int\limits_{\omega_{\rm min}}^{\omega_{\rm max}}\frac{d\omega}{\omega}\, f\left(\frac{\omega}{\Lambda\gamma}\right)\, f\left(\frac{W^2}{4\Lambda\gamma\omega}\right)\,. \tag{10}$$

It is easy to show that an accuracy of this calculation is determined by the omitted terms of the order of $\eta_1 = \Lambda^2/(W^2\,L)$, *i.e.* $\eta_1 \sim 5\%$ for the collisions considered.

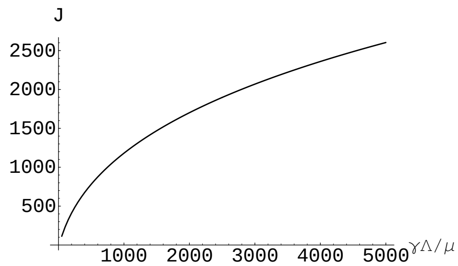

Fig. 4. The function $J(\gamma\Lambda/\mu)$ from Eq. (9).

A numerical evaluation of the integrals in Eqs. (9), (10) yields the function $J(\gamma\Lambda/\mu)$ presented in Fig. 4.

Let us now consider the probability of muon pair production $P_{\rm B}(\rho)$ in collision of two nuclei at a fixed impact parameter ρ. The Born cross section $\sigma_{\rm B}$ can be obtained by the integration of $P_{\rm B}(\rho)$ over the impact parameters:

$$\sigma_{\rm B} = \int P_{\rm B}(\rho)\, d^2\rho\,. \tag{11}$$

We calculate this probability in the LLA:

$$P_{\rm B}(\rho) = \int dn_1 dn_2\, \delta(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2 - \boldsymbol{\rho})\, \sigma_{\gamma\gamma}(W^2) = \frac{28}{9\pi^2}\, \frac{(Z_1\alpha Z_2\alpha)^2}{(\mu\rho)^2}\, \Phi(\rho)\,. \tag{12}$$

There are two scales in dependence of function $\Phi(\rho)$ on ρ:

$$\Phi(\rho) = \left(4\ln\frac{\gamma}{\mu\rho} + \ln\frac{\rho}{R}\right)\ln\frac{\rho}{R} \qquad \text{at} \quad R \ll \rho \le \frac{\gamma}{\mu}\,, \tag{13}$$

$$\Phi(\rho) = \left(\ln\frac{\gamma^2}{\mu^2\rho R}\right)^2 \qquad \text{at} \quad \frac{\gamma}{\mu} \le \rho \ll \frac{\gamma^2}{\mu^2 R}\,. \tag{14}$$

We compare equations for $\Phi(\rho)$ with the numerical calculations based on the exact matrix element calculated with the approach outlined in [7]. There is a good agreement for the Pb–Pb collisions: the discrepancy is less then 10% at $\mu\rho > 10$ and it is less then 15% at $\mu\rho > 2\mu R = 7.55$.

3. Coulomb and unitarity corrections

The Coulomb correction corresponds to Feynman diagram of Fig. 2. Due to restriction of transverse momenta of additional exchange photons on the level of $1/R$, the effective parameter of the perturbation series is not $(Z\alpha)^2$

but $(Z\alpha)^2/(R\mu)^2$. Besides, the contribution of the additional photons is suppressed by logarithmic factor. Indeed, the cross section for two-photon production mechanism is proportional to L^3, while the cross section for multiple-photon production mechanism is proportional to L^2. Therefore, the real suppression parameter is of the order of $\eta_2 = (Z\alpha)^2/[(R\mu)^2 L]$, which corresponds to the Coulomb correction less then 1 %.

The unitarity correction $\sigma_{\rm unit}$ to one muon pair production corresponds to the exchange of light-by-light blocks between nuclei (Fig. 3). We start with more general process — the production of one $\mu^+\mu^-$ pair and n electron–positron pairs ($n \geq 0$) in collision of two ultra-relativistic nuclei

$$Z_1 + Z_2 \to Z_1 + Z_2 + \mu^+\mu^- + n\,(e^+e^-)\,, \tag{15}$$

taking into account the unitarity corrections which correspond to the exchange of the blocks of light-by-light scattering via the virtual lepton loops. The corresponding cross section $d\sigma_{1+n}$ can be calculated by a simple generalization of the results obtained in paper [5] for the process without muon pair production: $Z_1 + Z_2 \to Z_1 + Z_2 + n\,(e^+e^-)$. Our result is the following

$$\frac{d\sigma_{1+n}}{d^2\rho} = P_{\rm B}(\rho)\,\frac{[\bar{n}_e(\rho)]^n}{n!}\,{\rm e}^{-\bar{n}_e(\rho)}\,, \tag{16}$$

where $\bar{n}_e(\rho)$ is the average number of the e^+e^- pairs produced in collisions of two nuclei at a given impact parameter ρ.

In particular, the cross section for the one $\mu^+\mu^-$ pair production including the unitarity correction is

$$\sigma_{1+0} = \int\limits_{2R}^{\infty} P_{\rm B}(\rho)\,{\rm e}^{-\bar{n}_e(\rho)}\,d^2\rho\,. \tag{17}$$

This expression can be rewritten in the form $\sigma_{1+0} = \sigma_{\rm B} + \sigma_{\rm unit}$, where

$$\sigma_{\rm B} = \int\limits_{2R}^{\infty} P_{\rm B}(\rho)\;d^2\rho \tag{18}$$

is the Born cross section discussed in Sec. 2 and

$$\sigma_{\rm unit} = -\int\limits_{2R}^{\infty} \left[1 - {\rm e}^{-\bar{n}_e(\rho)}\right]\,P_{\rm B}(\rho)\,d^2\rho\,, \tag{19}$$

corresponds to the unitarity correction for one muon pair production.

In LLA we find $\sigma_{\rm unit} \sim -1.2$ barn for the Pb–Pb collisions at LHC, which correspond approximately to -50% of the Born cross section. It is seen that unitarity corrections are large, in other words, the exclusive production of one muon pair differs considerably from its Born value.

4. Inclusive production of one $\mu^+\mu^-$ pairs

The experimental study of the exclusive muon pair production seems to be a very difficult task, because this process requires that the muon pair should be registered without any electron–positron pair production including $e^\pm$ emitted at very small angles. Otherwise, the corresponding cross section will be close to the Born cross section.

Indeed, it is clearly seen from the expression for σ_{1+n} that after summing up over all possible electron pairs we obtain the Born cross section $\sum_{n=0}^{\infty} \sigma_{1+n} = \sigma_{\rm B}$. Therefore, there is a very definite prediction: the inclusive muon pair production coincides with the Born limit. This direct consequence of calculations which take into account strong field effects, may be easier to test experimentally then the prediction for cross sections of several e^+e^- pair production.

5. Conclusion

The exclusive production of one $\mu^+\mu^-$ pair in collisions of two ultra-relativistic nuclei is considered. We present the simple method for calculation of the Born cross section for this process.

Then we found that the Coulomb corrections to this cross section are small while the unitarity corrections are large. This is in sharp contrast with the exclusive e^+e^- pair production where the Coulomb corrections to the Born cross section are large while the unitarity corrections are small.

We calculated also the cross section for the production of one $\mu^+\mu^-$ pair and several e^+e^- pairs in LLA. Using this cross section we found that the inclusive production of $\mu^+\mu^-$ pair coincides in this approximation with its Born value.

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