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TOTAL CROSS SECTIONS AND SOFT GLUON RESUMMATION*

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We discuss a model for total hadronic and photonic cross-sections which includes hard parton–parton scattering to drive the rise and soft gluon resummation to tame it. Unitarity is ensured by embedding the cross-section in the eikonal formalism. Predictions for LHC and ILC are presented.

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1. Introduction

Over three decades ago, the unexpected rise of the total proton–proton cross-section gave the first strong indications regarding the existence of hard scattering amongst the parton constituents of the proton [1]. And in the very near future, at the LHC, a new much higher energy window for parton parton scattering will open up for which a precise knowledge of total cross-section predictions will be necessary to disentangle background from other processes

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and perhaps detect new physics. In this note we shall give an overview of the physical role of the various parameters entering most phenomenological descriptions of the total cross-section and subsequently present a model — which incorporates them — that can be used to predict the total cross-section at the LHC and at the Photon Collider.

2. Understanding the parameters in total cross-section models

The proton data exhibit, and require explanation of, three basic features:

(i) the normalization of the cross section,

(ii) an initial decrease and

(iii) a subsequent rise with energy.

Many models are available in the literature regarding the above issues, their predictions depending upon a number of parameters which are usually fixed by comparison with the low energy data. Before discussing some of these models and their predictions, we shall provide phenomenomenological reasons for the approximate values of the parameters which are responsible towards a satisfactory description of the above behaviours *(i)–(iii)*.

Let us recall that the decrease and the subsequent rise are well understood within a number of models as due to the exchange of a Regge and a Pomeron trajectory, through the expression

$$\sigma_{\rm tot}(s) = X s^{\epsilon} + Y s^{-\eta} \,. \tag{1}$$

The two terms of Eq. (1) reflect the well known duality between resonance and Regge pole exchange on the one hand and background and Pomeron exchange on the other, established in the late 60's through FESR [3]. This correspondence meant that, while at low energy the cross-section could be written as due to a background term and a sum of resonances, at higher energy it could be written as a sum of Regge trajectory exchanges and a Pomeron exchange.

Before entering into the phenomenological analysis, it is good to ask *(i)* where the "two component" structure of Eq. (1) comes from and *(ii)* why the difference in the two powers (in s) is approximately a half.

Our present knowledge of QCD and its employment for a description of hadronic phenomena can and does provide some insight into the nature of these two terms. We shall begin answering the above two questions through considerations about the bound state nature of hadrons which necessarily transcends perturbative QCD. For hadrons made of light quarks (q) and glue (g), the two terms arise from $q\bar{q}$ and gg excitations. For these, the energy is given by a sum of three terms: *(i)* the rotational energy, *(ii)* the Coulomb energy and *(iii)* the "confining" energy. If we accept the Wilson area conjecture in QCD, *(iii)* reduces to the linear potential [4,5]. Explicitly, in the CM frame of two massless particles, either a $q\bar{q}$ or a gg pair separated by a relative distance r with relative angular momentum J, the energy is given by

$$E_i(J,r) = \frac{2J}{r} - \frac{C_i\bar{\alpha}}{r} + C_i\tau r\,, \tag{2}$$

where $i = 1$ refers to $q\bar{q}$, $i = 2$ refers to gg, τ is the "string tension" and the Casimir's are $C_1 = C_F = 4/3$, $C_2 = C_G = 3$. $\bar{\alpha}$ is the QCD coupling constant evaluated at some average value of r and whose precise value will disappear in the ratio to be considered. The hadronic rest mass for a state of angular momentum J is then computed through minimizing the above energy

$$M_i(J) = \mathrm{Min}_r \left[\frac{2J}{r} - \frac{C_i\bar{\alpha}}{r} + C_i\tau r\right]\,, \tag{3}$$

which gives

$$M_i(J) = 2\sqrt{(C_i\tau)[2J - C_i\bar{\alpha}]}\,. \tag{4}$$

The result may then be inverted to obtain the two sets of linear Regge trajectories $\alpha_i(s)$

$$\alpha_i(s) = \frac{C_i\bar{\alpha}}{2} + \left(\frac{1}{8C_i\tau}\right) s = \alpha_i(0) + \alpha_i' s\,. \tag{5}$$

Thus, the ratio of the intercepts is given by

$$\frac{\alpha_{gg}(0)}{\alpha_{q\bar{q}}(0)} = \frac{C_G}{C_F} = \frac{9}{4}\,. \tag{6}$$

Employing our present understanding that resonances are $q\bar{q}$ bound states while the background, dual to the Pomeron, is provided by gluon–gluon exchanges [6], the above equation can be rewritten as

$$\frac{\alpha_P(0)}{\alpha_R(0)} = \frac{C_G}{C_F} = \frac{9}{4}\,. \tag{7}$$

If we restrict our attention to the leading Regge trajectory, namely the degenerate $\rho - \omega - \phi$ trajectory, then $\alpha_R(0) = \eta \approx 0.48 - 0.5$, and we obtain for $\epsilon \approx$ 0.08–0.12, a rather satisfactory value. The same argument for the slopes gives

$$\frac{\alpha'_{gg}}{\alpha'_{q\bar{q}}} = \frac{C_F}{C_G} = \frac{4}{9}\,, \tag{8}$$

so that if we take for the Regge slope $\alpha'_R \approx 0.88$–0.90, we get for $\alpha'_P \approx 0.39$–0.40, in fair agreement with lattice estimates [7].

We now have good reasons for a break up of the amplitude into two components. To proceed further, it is necessary to realize that precisely because massless hadrons do not exist, Eq. (1) violates the Froissart bound and thus must be unitarized. To begin this task, let us first rewrite Eq. (1) by putting in the "correct" dimensions

$$\bar{\sigma}_{\rm tot}(s) = \sigma_1 \left(\frac{s}{\bar{s}}\right)^{\epsilon} + \sigma_2 \left(\frac{\bar{s}}{s}\right)^{1/2}, \tag{9}$$

where we have imposed the nominal value $\eta = 1/2$. In the following, we shall obtain rough estimates for the size of the parameters in Eq. (9).

A minimum occurs in $\bar{\sigma}_{\rm tot}(s)$ at $s = \bar{s}$, for $\sigma_2 = 2\epsilon\sigma_1$. If we make this choice, then Eq. (9) has one less parameter and it reduces to

$$\bar{\sigma}_{\rm tot}(s) = \sigma_1 \left[\left(\frac{s}{\bar{s}}\right)^{\epsilon} + 2\epsilon \left(\frac{\bar{s}}{s}\right)^{1/2}\right]. \tag{10}$$

We can isolate the rising part of the cross-section by rewriting the above as

$$\bar{\sigma}_{\rm tot}(s) = \sigma_1 \left[1 + 2\epsilon \left(\frac{\bar{s}}{s}\right)^{1/2}\right] + \sigma_1 \left[\left(\frac{s}{\bar{s}}\right)^{\epsilon} - 1\right]. \tag{11}$$

Eq. (11) separates cleanly the cross-section into two parts: the first part is a "soft" piece which shows a saturation to a constant value (but which contains no rise) and the second a "hard" piece which has all the rise. Moreover, $\bar{s}$ naturally provides the scale beyond which the cross-sections would begin to rise. Thus, our "Born" term assumes the generic form

$$\sigma^B_{\rm tot}(s) = \sigma_{\rm soft}(s) + \vartheta(s - \bar{s})\sigma_{\rm hard}(s), \tag{12}$$

with $\sigma_{\rm soft}$ containing a constant (the "old" Pomeron with $\alpha_P(0) = 1$) plus a (Regge) term decreasing as $1/\sqrt{s}$ and with an estimate for their relative magnitudes ($\sigma_2/\sigma_1 \sim 2\epsilon$). We shall assume that the rising part of the cross-section $\sigma_{\rm hard}$ is provided by jets which are calculable by perturbative QCD, obviating (at least in principle) the need of an arbitrary parameter ϵ.

An estimate of σ_1 may also be obtained through the hadronic string picture. Eq. (3) gives us the mean distance between quarks or the "size" of a hadronic excitation of angular momentum J in terms of the string tension

$$\bar{r}(J)^2 = \frac{2J - C_F\bar{\alpha}}{\tau}. \tag{13}$$

Thus, the size R_1 of the lowest hadron (which in this Regge string picture has $J = 1$, since $\alpha_R(0) = 1/2$) is given by

$$R_1^2 = \frac{1}{\tau} = 8\alpha' \,. \tag{14}$$

If two hadrons each of size R_1 collide, their effective radius for scattering would be given by

$$R_{\rm eff} = \sqrt{R_1^2 + R_1^2} = \sqrt{2}R_1 \,, \tag{15}$$

and the constant cross-section may be estimated (semi-classically) to be roughly

$$\sigma_1 = 2\pi R_{\rm eff}^2 = 4\pi R_1^2 \approx \frac{4\pi}{\tau} = 32\pi\alpha' \,, \tag{16}$$

which is about 40 mb, a reasonable value. In the later sections, for the "soft" cross-section we shall take a value of this order of magnitude as the nominal value.

The last remaining parameter is the scale $\bar{s}$, the jet production threshold in the hadronic cross-section. In e^+e^- annihilation, the threshold for jet production can be estimated from the appearance of multihadronic production in e^+e^- scattering first observed at ADONE around 3 GeV. For scattering of two hadrons, this should translate into $\sqrt{\bar{s}} \approx 12$ GeV. Thus, from Eq. (11), we have

$$\sigma_1 \left[1 + 2\epsilon\left(\frac{\bar{s}}{s}\right)^{1/2}\right] \approx \sigma_1 \left(1 + \frac{2}{\sqrt{s}}\right) \,. \tag{17}$$

The above phenomenological estimate holds for proton–antiproton scattering, whereas for proton–proton, which has no resonances in the s-channel, no Regge exchange is expected to contribute and only the (approximately) constant term remains.

3. The Bloch–Nordsieck model for LHC

In the past, several authors realized that QCD offers an elegant explanation of the rise of the cross-section via the minijets and hence suggested that the rise of $\sigma_{\rm tot}$ with energy was driven by the rapid rise with energy of the inclusive jet cross-section

$$\sigma_{\rm jet}^{ab}(s) = \int_{p_{\rm tmin}}^{\sqrt{s}/2} dp_t \int_{4p_t^2/s}^{1} dx_1 \int_{4p_t^2/(x_1 s)}^{1} dx_2 \sum_{i,j,k,l} f_{i|a}(x_1) f_{j|b}(x_2) \frac{d\hat{\sigma}_{ij\to kl}(\hat{s})}{dp_t} \,. \tag{18}$$

In an eikonal minijet model (EMM), the total cross-section then reads

$$\sigma_{\rm tot} \simeq 2 \int d^2\vec{b} \left[1 - e^{-n(b,s)/2}\right] \tag{19}$$

wherein

$$n(b,s) = 2{\rm Im}\,\chi(b,s) = n_{\rm soft} + n_{\rm hard} = A_{\rm soft}(b)\sigma_{\rm soft}(s) + A_{\rm jet}(b)\sigma_{\rm jet}(s) \tag{20}$$

and ${\rm Re}\,\chi(b,s) = 0$. In the Bloch–Nordsieck (BN) model [8], the overlap functions $A_i(b)$ are s-dependent and given by

$$A_{\rm BN} = \frac{e^{-h(b,s)}}{\int d^2\vec{b} e^{-h(b,s)}}\,, \tag{21}$$

$$h(b,s) = \frac{8}{3\pi} \int\limits_0^{q_{\rm max}} \frac{dk}{k} \alpha_{\rm s}(k^2) \ln\left(\frac{q_{\rm max} + \sqrt{q_{\rm max}^2 - k^2}}{q_{\rm max} - \sqrt{q_{\rm max}^2 - k^2}}\right) [1 - J_0(kb)] \tag{22}$$

and $q_{\rm max}$ depends on energy and the kinematics of the process [9].

The eikonal formalism which we use to describe the total cross-section, incorporates multiple parton–parton collisions, accompanied by soft gluon emission from the initial valence quarks, to leading order. Notice that in this model, we consider emissions only from the external quark legs. In the impulse approximation — on which the parton model itself is based — the valence quarks are free, external particles. In this picture, emission of soft gluons from the gluons involved in the hard scattering, is non leading. As the energy increases, more and more hard gluons are emitted but there is also a larger and larger probability of soft gluon emission: the overall effect is a rise of the cross-section, tempered by the soft emission, *i.e.* the violent mini-jet rise due to semi-hard gluon–gluon collisions is tamed by soft gluons. Crucial in this model, are the scale and the behaviour of the strong coupling constant which is present in the integral over the soft gluon spectrum. While in the jet cross-section $\alpha_{\rm s}$ never plunges into the infrared region, as the scattering partons are by construction semi-hard, in the soft gluon spectrum, the opposite is true and a regularization is mandatory. We notice however that here, as in other problems of soft hadron physics [10], what matters most is not the value of $\alpha_{\rm s}(0)$, but rather its integral. Thus, all that we need to demand, is that $\alpha_{\rm s}$ be integrable, even if singular [11]. We employ the same phenomenological expression for $\alpha_{\rm s}$ as used in our previous works, namely

$$\alpha_{\rm s}(k_\perp) = \frac{12\pi}{(33 - 2N_f)} \frac{p}{\ln[1 + p(\frac{k_\perp}{\Lambda})^{2p}]}\,. \tag{23}$$

Through the above, we were able to reproduce the effect of the phenomenologically introduced intrinsic transverse momentum of hadrons [11], and more recently obtained a very good description of the entire region where the total cross-section rises [8]. This expression for α_s coincides with the usual one-loop expression for large values of $k_\perp$, while going to a singular limit for small $k_\perp$. For $p = 1$ this expression corresponds to the Richardson potential [12] used in bound state problems. We see from Eq. (22) that $p = 1$, leads to a divergent integral, and thus cannot be used. Notice that, presently, in the expression for $h(b,s)$, the masses of the emitting particles are put to zero as is usual in perturbative QCD. Thus, for a convergent integral, one requires $p < 1$ and the successful phenomenology indicated in [8] gave $p = 3/4$.

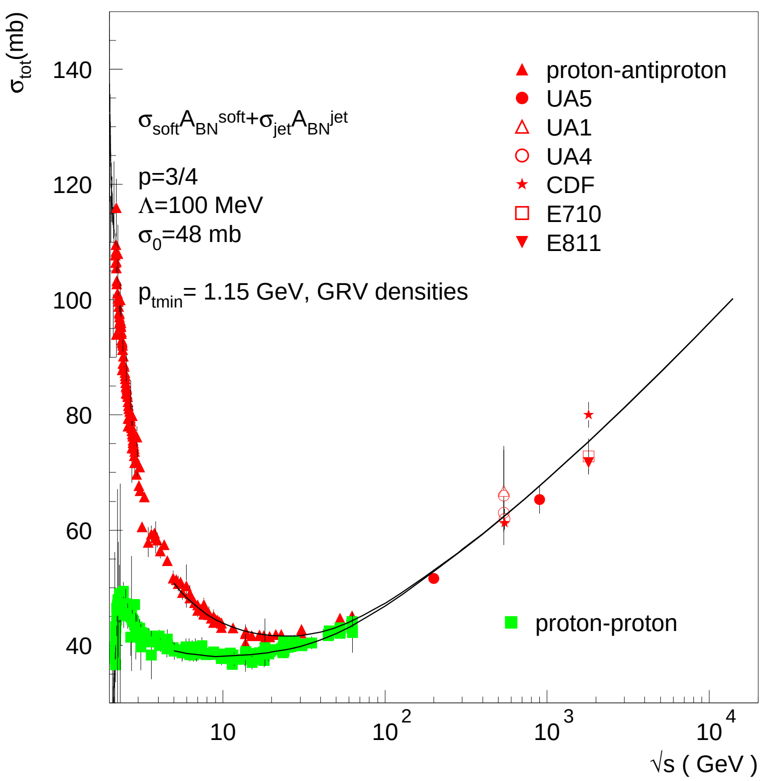

Fig. 1. Comparison of pp and $p\bar{p}$ total cross-section data [13–19] with predictions for the total pp and $p\bar{p}$ cross-section from the QCD model described in the text for an optimal choice of parameters.

4. The photon–photon total cross-section

We now show an application of the above model [21] to photon–photon scattering and its comparison with present data [20, 22] and with another model [23].

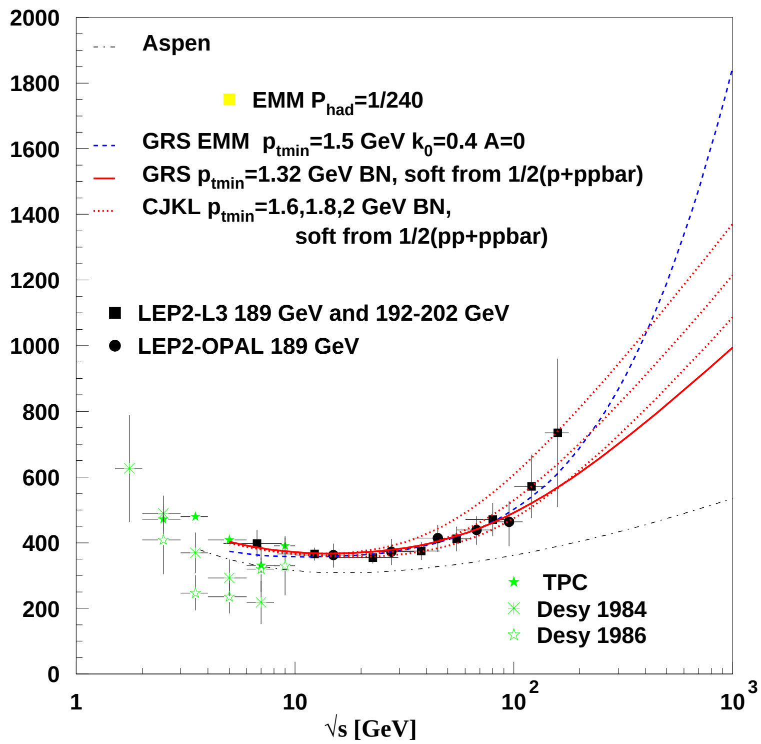

Fig. 2. Predictions for the total photon–photon cross-section based on the QCD model described in the text a parameters set consistent with those for proton and proton–antiproton scattering.

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