SUM RULES OF POLARIZED PHOTON STRUCTURE FUNCTIONS REVISITED*

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The target mass dependence of the sum rule for the polarized virtual photon structure function $g_1^\gamma(x,Q^2,P^2)$ is studied when P^2, the mass squared of the target photon, changes from on-shell to far off-shell. Also the sum rule for another polarized structure function $g_2^\gamma(x,Q^2,P^2)$ is analyzed in Parton Model (PM). It is found that the first moment of g_2^γ calculated in PM vanishes independent of Q^2, P^2 and quark mass.

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1. Introduction

In the electron–positron collision experiments in the future International Linear Collider, we can study the structure of the photon. When e^+ and e^- beams are polarized, we can measure the spin-dependent structure functions $g_1^\gamma(x,Q^2,P^2)$ and $g_2^\gamma(x,Q^2,P^2)$ of photon (Fig. 1), where $-Q^2(-P^2)$ is the mass squared of the probe (target) photon. We have studied the sum rules of g_1^γ and g_2^γ, especially focusing on the dependence of these sum rules on the photon mass parameters P^2 and Q^2. In this talk we will report our result.

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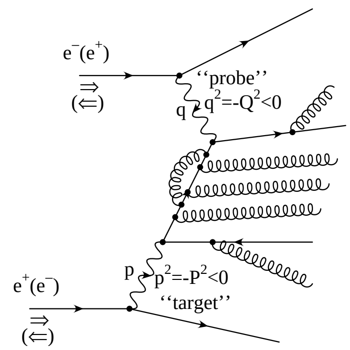

Fig. 1. Deep inelastic scattering on a virtual photon in e^+e^- collision.

2. Sum rule of g_1^γ

The polarized structure function g_1^γ of the real photon satisfies a remarkable sum rule [1–3]

$$\int_0^1 dx g_1^\gamma(x,Q^2) = 0\,. \tag{1}$$

Actually, applying the Drell–Hearn–Gerasimov sum rule to the case of virtual photon target and using the fact that the photon has zero anomalous magnetic moment, the authors of Ref. [3] showed that the first moment of $g_1^\gamma(x,Q^2)$ vanishes independent of Q^2 and that the sum rule holds to all orders in perturbation theory in both QED and QCD.

When the target photon becomes off-shell, *i.e.*, $P^2 \neq 0$, the sum rule for the corresponding photon structure function $g_1^\gamma(x,Q^2,P^2)$ does not vanish any more. In fact, it has been calculated up to the next-to-leading order (NLO) ($O(\alpha\alpha_\mathrm{s})$) in QCD for the case $Q^2 \gg P^2 \gg \Lambda^2$, where Λ is the QCD scale parameter [2, 4];

$$\begin{aligned}\int_0^1 dx g_1^\gamma(x,Q^2,P^2) \;=\; & -\frac{3\alpha}{\pi}\Bigg[n_\mathrm{f}\langle e^4\rangle\Big(1-\frac{\alpha_\mathrm{s}(Q^2)}{\pi}\Big) \\ & -\frac{2}{\beta_0}\left(n_\mathrm{f}\langle e^2\rangle\right)^2\Big(\frac{\alpha_\mathrm{s}(P^2)}{\pi}-\frac{\alpha_\mathrm{s}(Q^2)}{\pi}\Big)\Bigg]+\mathcal{O}(\alpha\alpha_\mathrm{s}^2)\,,\end{aligned} \tag{2}$$

with $\beta_0 = 11-2n_\mathrm{f}/3$ being the one-loop QCD β function. Here α ($\alpha_\mathrm{s}(Q^2)$) is the QED (QCD running) coupling constant, $n_\mathrm{f}\langle e^4\rangle = \sum_{i=1}^{n_\mathrm{f}} e_i^4$ and $n_\mathrm{f}\langle e^2\rangle = \sum_{i=1}^{n_\mathrm{f}} e_i^2$ with e_i being the electric charge of the active quark and n_f the number of active quarks. The first term in the square brackets of the r.h.s.

of Eq. (2) is coming from the QED axial anomaly while the second term is from the QCD axial anomaly.

Now the question is how the sum rule of g_1^γ changes when the target photon shifts from on-shell ($P^2 = 0$) to far off-shell ($P^2 \gg \Lambda^2$) [5]. Recall that for the operator product expansion (OPE) of two electromagnetic (and thus gauge-invariant) currents, only gauge-invariant operators need to be included with their renormalization basis [6]. Since there is no gauge-invariant twist-two gluon and photon operators with spin one, we need to consider only quark operators, *i.e.*, the flavor singlet $R_{\rm S}^\sigma = \overline{\psi}\gamma^\sigma\gamma_5 1\psi$ and nonsinglet $R_{\rm NS}^\sigma = \overline{\psi}\gamma^\sigma\gamma_5(Q_{\rm ch}^2 - \langle e^2\rangle 1)\psi$ axial currents, where 1 is an $n_{\rm f}\times n_{\rm f}$ unit matrix and $Q_{\rm ch}^2$ is the square of the $n_{\rm f}\times n_{\rm f}$ quark-charge matrix. Then the first moment of g_1^γ is expressed as

$$\int_0^1 dx g_1^\gamma(x,Q^2,P^2) \;=\; C_{\rm S}\left(\frac{Q^2}{\mu^2},\bar{g}(\mu^2),\alpha\right)\langle\gamma(p)|R_{\rm S}(\mu^2)|\gamma(p)\rangle + C_{\rm NS}\left(\frac{Q^2}{\mu^2},\bar{g}(\mu^2),\alpha\right)\langle\gamma(p)|R_{\rm NS}(\mu^2)|\gamma(p)\rangle\,. \quad (3)$$

Here $C_{\rm S}$ and $C_{\rm NS}$ are the coefficient functions corresponding to the axial currents $R_{\rm S}^\sigma$ and $R_{\rm NS}^\sigma$, respectively, and μ is the renormalization point.

Since we are interested in P^2-dependence of the sum rule and the range of P^2 covers from the region $P^2 \gg \Lambda^2$ to the on-shell $P^2 = 0$, we take the renormalization point at $\mu^2 = Q_0^2 \gg \Lambda^2$. Then the photon matrix element of the axial current R_i ($i =$S, NS) may be divided into two pieces:

$$\frac{\langle\gamma(p)|R_i(\mu^2=Q_0^2)|\gamma(p)\rangle}{(\alpha/4\pi)} \;=\; A_i(Q_0^2;P^2) \;=\; \widetilde{A}_i(Q_0^2;P^2) + \widehat{A}_i\,. \quad (4)$$

The second term $\widehat{A}_i$ is the matrix element with the photon state taken to be far off-shell $P^2 = Q_0^2 \gg \Lambda^2$, and thus considered to be a point-like piece. We can calculate it perturbatively and in the leading order (LO) we find

$$\widehat{A}_{\rm S} \;=\; -12 n_{\rm f}\langle e^2\rangle\,,$$
$$\widehat{A}_{\rm NS} \;=\; -12 n_{\rm f}\Big(\langle e^4\rangle - \langle e^2\rangle^2\Big)\,, \quad (5)$$

which are related to the Adler–Bell–Jackiw anomaly. The first term $\widetilde{A}_i(Q_0^2;P^2)$ in Eq. (4) is the rest which contains a nonperturbative contribution. Since we have $C_{\rm S}=\langle e^2\rangle$ and $C_{\rm NS}=1$ in the LO, we obtain from Eqs. (3)–(5)

$$\int_0^1 dx g_1^\gamma(x,Q^2,P^2) = -\frac{3\alpha}{\pi}n_{\rm f}\langle e^4\rangle + \frac{\alpha}{4\pi}\left\{\langle e^2\rangle\widetilde{A}_{\rm S}(Q_0^2;P^2)+\widetilde{A}_{\rm NS}(Q_0^2;P^2)\right\}\,. \quad (6)$$

The last two terms vanish as we go to higher $P^2 \gg \Lambda^2$. But for the real photon target, these terms turn out to be the hadronic contributions which cancel the first term arising from the pure QED point-like interaction.

In order to estimate the magnitude of these hadronic terms $\widetilde{A}_{\rm S}(Q_0^2;P^2)$ and $\widetilde{A}_{\rm NS}(Q_0^2;P^2)$, we adopt the viewpoint of the Vector Dominance Model. Using the current-field identity [7], $J_\mu^{\rm e.m.}=(m_V^2/f_V)V_\mu$, where $J_\mu^{\rm e.m.}$ is the electromagnetic current, V_μ and m_V are the field and mass of the relevant vector boson[1], we obtain

$$\widetilde{A}_{\rm S}(Q_0^2;P^2) = \langle e^2\rangle\left(\frac{m_V^2}{f_V}\right)^2\left(\frac{1}{m_V^2+P^2}\right)^2\langle V|J^5|V\rangle\,, \quad (7)$$

$$\widetilde{A}_{\rm NS}(Q_0^2;P^2) = \left(\langle e^4\rangle-\langle e^2\rangle^2\right)\left(\frac{m_V^2}{f_V}\right)^2\left(\frac{1}{m_V^2+P^2}\right)^2\langle V|J^5|V\rangle\,. \quad (8)$$

The condition that the sum rule should vanish at $P^2=0$ (see Eq. (1)) leads to $\langle V|J^5|V\rangle = 12 n_{\rm f} f_V^2$.

Putting all together, we finally obtain the LO expression for the sum rule of g_1^γ whose target photon mass squared ranges from $P^2=0$ (on-shell) to the region $\Lambda^2 \ll P^2 \ll Q^2$:

$$\int_0^1 dx g_1^\gamma(x,Q^2,P^2) = -\frac{3\alpha}{\pi}n_{\rm f}\langle e^4\rangle\left\{1-\left(\frac{m_V^2}{m_V^2+P^2}\right)^2\right\}\,. \quad (9)$$

The extension of the formula to the NLO (to the order $O(\alpha\alpha_{\rm s})$) is straightforward. In Fig. 2 we plot the first moment of g_1^γ as a function of P^2. We have taken $m_V = 0.7\,{\rm GeV}$. We see that the first moment quickly approaches the value $-3\alpha n_{\rm f}\langle e^4\rangle/\pi$ as P^2 goes to $1\,{\rm GeV}^2$.

[1] For simplicity, we consider here the contribution of only one vector boson. Refinement of the analysis introducing more vector bosons is straightforward.

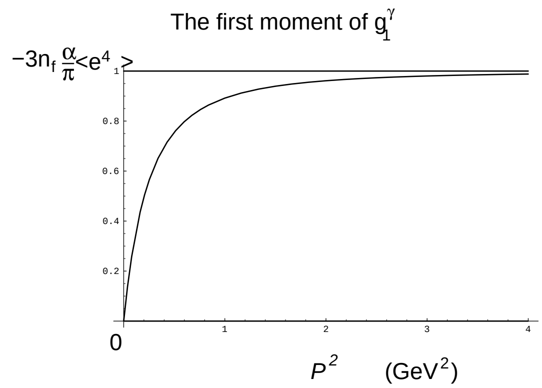

Fig. 2. The first moment of g_1^γ as a function of P^2.

3. Sum rule of g_2^γ

It is known that the *nucleon* structure function $g_2^{\rm nucl}$ satisfies the Burkhardt–Cottingham (BC) sum rule [8]

$$\int_0^1 dx\, g_2^{\rm nucl}(x,Q^2) = 0\,. \tag{10}$$

Its derivation relies on the assumption of the Regge theory. As for the virtual photon structure function g_2^γ, the general OPE analysis leads to the moment sum rule

$$\int_0^1 dx x^{n-1} g_2^\gamma(x,Q^2,P^2) = \frac{n-1}{n}\left\{ -\sum_i a^n_{(2)i} E^n_{(2)i}(Q^2) + \sum_i a^n_{(3)i} E^n_{(3)i}(Q^2)\right\}$$

which holds in the kinematical region $Q^2 \gg P^2$. The first and second terms in the curly brackets are the twist two and three contributions, respectively. Thus the BC sum rule also holds for g_2^γ:

$$\int_0^1 dx\ g_2^\gamma(x,Q^2,P^2) = 0, \qquad \text{for} \quad Q^2 \gg P^2\,. \tag{11}$$

Does g_2^γ satisfy the BC sum rule for other kinematical region? To answer this question, we analyze the sum rule of g_2^γ in the Parton Model (PM) for arbitrary Q^2 and P^2, and a quark mass m. Actually, we have the PM result for g_2^γ at hand [9–11], which was obtained from evaluating the box diagrams for $\gamma\gamma \to q\bar{q}$ in Fig. 3

$$g_2^\gamma(x,Q^2,P^2)\Big|_{\rm PM} = -\frac{\alpha}{\pi}\delta_\gamma\Bigg\{\frac{L}{\widetilde{\beta}^5}\left[x(3-\widetilde{\beta}^2)\frac{P^2}{Q^2}+\widetilde{\beta}^2(2-x)+3(x-1)\right] + \frac{\beta}{\widetilde{\beta}^2(1-\beta^2\widetilde{\beta}^2)}\Bigg[4x\frac{m^2}{Q^2}+\frac{3x\{(\beta^2+1)\widetilde{\beta}^2-2\}}{\widetilde{\beta}^2}\frac{P^2}{Q^2} + \frac{(\widetilde{\beta}^2+3x-3)\{(\beta^2+1)\widetilde{\beta}^2-2\}}{\widetilde{\beta}^2}\Bigg]\Bigg\}, \tag{12}$$

where

$$\widetilde{\beta}=\sqrt{1-\frac{p^2q^2}{(p\cdot q)^2}}, \qquad \beta=\sqrt{1-\frac{4m^2}{(p+q)^2}}, \qquad L=\ln\frac{1+\beta\widetilde{\beta}}{1-\beta\widetilde{\beta}}\,. \tag{13}$$

The BC sum rule of $g_2^\gamma(x,Q^2,P^2)\Big|_{\rm PM}$ has been analyzed for the limiting case $Q^2\to\infty$ with the ratio $r=P^2/m^2$ fixed in Ref. [10]. It was found that the BC sum rule is satisfied in this limit with all values of r.

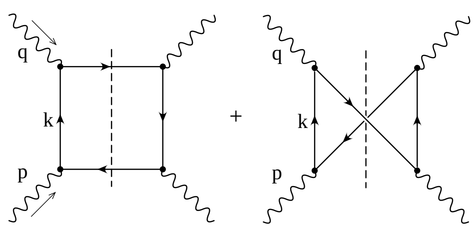

Fig. 3. The box diagrams contributing to g_2^γ.

We pursue further to see if the PM result (12) still satisfies the sum rule for arbitrary Q^2, P^2 and m^2. Now Q^2 is finite, the maximal value of the Bjorken variable x is not 1 but $x_{\rm max}=1/(1+P^2/Q^2+4m^2/Q^2)$. Using the expression given in Eq. (12) we perform the integration and find

$$\int_0^{x_{\rm max}} dx\ g_2^\gamma(x,Q^2,P^2)\Big|_{\rm PM}=0, \qquad \text{for arbitrary}\quad Q^2,P^2,m^2\,. \tag{14}$$

Thus the first moment of the box graph contribution to g_2^γ vanishes independent of the virtuality of both the probe and target photons and also irrespective of the produced quark mass. It is an unexpected result *at least to the present authors.* It may be due to superconvergence or to some symmetry. We need a simple explanation. Recalling the fact that the PM also gives the sum rule $\int_0^{x_{\max}} dx g_1^\gamma(x,Q^2)|_{\rm PM}=0$ for the real photon target, and that we actually have the sum rule Eq. (1), it is well expected that the BC sum rule indeed holds for the actual structure function $g_2^\gamma(x,Q^2,P^2)$ with arbitrary Q^2 and P^2.

4. Summary

The sum rules for the polarized photon structure functions g_1^γ and g_2^γ are studied. The first moment of g_1^γ is predicted to change quickly from null to the value $-3\alpha\, n_{\rm f}\langle e^4\rangle/\pi$ as P^2 goes from 0 to 1 GeV^2. Also it is found that the first moment of the box graph contribution to g_2^γ vanishes independent of Q^2, P^2 and the produced quark mass.

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