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SYMMETRIES OF 2HDM, DIFFERENT VACUA, CP VIOLATION AND POSSIBLE RELATION TO AN EVOLUTION OF UNIVERSE*

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The same physical reality in Two Higgs doublet model (2HDM) can be described by different Lagrangians. We study this property called by the reparametrization invariance (in space of Lagrangians). We consider the Z_2-symmetry of the Lagrangian, which prevents a $\phi_1 \leftrightarrow \phi_2$ transitions, and the different levels of its violation, soft and hard. We argue that softly Z_2 violated 2HDM is a natural model in the description of EWSB. We consider different vacua in the 2HDM. We find simple condition for a CP violation in the Higgs sector. In the Model II for Yukawa interactions we obtain the set of relations among the couplings to gauge bosons and to fermions which allows one to analyse different physical situations (including CP violation) in terms of these very couplings, instead of the parameters of Lagrangian. We discuss possible interaction of Higgs fields of the SM or 2HDM with the inflatory scalar field describing an exponential expansion of Universe after Big Bang.

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1. Lagrangian

A spontaneous electroweak symmetry breaking (EWSB) via the Higgs mechanism is described by the Lagrangian

$$\mathcal{L} = \mathcal{L}_{gf}^{\mathrm{SM}} + \mathcal{L}_H + \mathcal{L}_Y \qquad \text{with} \qquad \mathcal{L}_H = T - V\,. \tag{1a}$$

Here $\mathcal{L}_{gf}^{\mathrm{SM}}$ describes the SU(2)× U(1) Standard Model interaction of gauge bosons and fermions, $\mathcal{L}_Y$ describes the Yukawa interactions of fermions with Higgs scalars and $\mathcal{L}_H$ is the Higgs scalar Lagrangian; T is the Higgs kinetic term and V is the Higgs potential.

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In the minimal Standard Model (SM) one scalar isodoublet with hypercharge $Y=1$ is implemented, $\mathcal{L}_H=(D_\mu\phi)^\dagger D_\mu\phi - V$, $V=\lambda\phi^4/2-m^2\phi^2/2$ *etc.* In Ref. [1] we study in detail Two-Higgs-Doublet Model (2HDM) — the simplest extension of the SM , with two scalar fields ϕ_i being weak isodoublets ($T=1/2$) with hypercharges $Y=1$ (see [2] for earlier references). The kinetic term of the most general renormalizable Higgs Lagrangian is

$$T=(D_\mu\phi_1)^\dagger(D^\mu\phi_1)+(D_\mu\phi_2)^\dagger(D^\mu\phi_2)+\Big[\varkappa(D_\mu\phi_1)^\dagger(D^\mu\phi_2)+\text{h.c.}\Big] \tag{1b}$$

and the Higgs potential, containing operators of dimension y2 (in mass term) (1c) and of dimension 4 (1d), is

$$\begin{aligned} V \;=\; & -\Big\{m_{11}^2(\phi_1^\dagger\phi_1)+m_{22}^2(\phi_2^\dagger\phi_2)+\Big[m_{12}^2(\phi_1^\dagger\phi_2)+\text{h.c.}\Big]\Big\}\Big/2 & (1c)\\ & +\frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2+\frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2+\lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2)+\lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)\\ & +\frac{1}{2}\Big[\lambda_5(\phi_1^\dagger\phi_2)^2+\text{h.c.}\Big]+\Big\{\Big[\lambda_6(\phi_1^\dagger\phi_1)+\lambda_7(\phi_2^\dagger\phi_2)\Big](\phi_1^\dagger\phi_2)+\text{h.c.}\Big\}. & (1d)\end{aligned}$$

2. Reparametrization (RPa) invariance

Our model contains two fields with identical quantum numbers. Therefore, it can be described in similar way both in terms of fields ϕ_k ($k=1,2$), used in (1), and in terms of fields ϕ'_k obtained from ϕ_k by a global unitary transformation $\hat{\mathcal{F}}$ of SU(2)× U(1) *general reparametrization (RPa) group*:

$$\begin{pmatrix}\phi'_1\\ \phi'_2\end{pmatrix}=\hat{\mathcal{F}}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix},\quad \hat{\mathcal{F}}=e^{-i\rho_0}\begin{pmatrix}\cos\theta\, e^{i\rho/2} & \sin\theta\, e^{i(\tau-\rho/2)}\\ -\sin\theta\, e^{-i(\tau-\rho/2)} & \cos\theta\, e^{-i\rho/2}\end{pmatrix}. \tag{2}$$

This group splits into a proper SU(2) RPa group with parameters θ, ρ, τ and U(1) group describing *overall phase freedom*, with parameter ρ_0.

The transformation $\mathcal{F}$ induces the changes of coefficients of Lagrangian, $\lambda_i\to\lambda'_i$ and $m_{ij}^2\to(m')_{ij}^2$, $\varkappa\to\varkappa'$ with renormalization of fields ϕ'_i (RPa transformation of parameters). The Lagrangian of the form (1) with coefficients λ_i, m_{ij}^2 and that with new coefficients λ'_i, $(m')_{ij}^2$ describe the same physical reality. We call this property a *RPa invariance in a space of Lagrangians* (with coordinates given by its parameters).

The set of RPa transformations for parameters of Lagrangian forms representation of RPa group in the *16-dimensional space of Lagrangians* with coordinates given by λ_{1-4}, Reλ_{5-7}, Imλ_{5-7}, $m_{11,22}^2$, Re(m_{12}^2), Im(m_{12}^2), Re$\varkappa$, Im$\varkappa$. The parameters of Lagrangian can be determined from measurements in principle only with accuracy up to the RPa freedom.

All observable quantities are invariants of the RPa group (IRPa). These are, for example, masses of observable Higgs bosons, related to the eigenvalues of the mass-squared matrix (20) and (17), and eigenvalues of Higgs–Higgs scattering matrices [13].

By writing the Higgs potential as a sum of terms such as $Y_{ab}(\phi_a^\dagger\phi_b) + Z_{abcd}(\phi_a^\dagger\phi_b)(\phi_c^\dagger\phi_d)$, with a, $b = 1, 2$, one can construct IRPa's at $\varkappa = 0$ as combinations of products of Y and Z summed over a, b. In this way large series of (generally not independent) IRPa's was obtained [3]. The group–theoretical analysis of RPa group gives the **complete** set of 11 *independent* IRPa's [4].

3. Rephasing (RPh) invariance

It is useful to consider a particular case of the transformations (2) with $\theta = 0$ — a global *rephasing (RPh) transformation of the fields*:

$$\phi_k \to e^{-i\rho_i}\phi_k, \qquad \rho_1 = \rho_0 - \rho/2\,, \quad \rho_2 = \rho_0 + \rho/2\,, \quad \rho = \rho_2 - \rho_1\,. \quad \text{(3a)}$$

This transformation leads to *a RPh transformation of the Lagrangian*:

$$\begin{aligned} &\lambda_{1-4} \to \lambda_{1-4}\,, \quad m_{11}^2 \to m_{11}^2\,, \quad m_{22}^2 \to m_{22}^2\,, \\ &\lambda_5 \to \lambda_5\, e^{-2i\rho}\,,\ \lambda_{6,7} \to \lambda_{6,7}\, e^{-i\rho}\,,\ m_{12}^2 \to m_{12}^2 e^{-i\rho}\,,\ \varkappa \to \varkappa\, e^{-i\rho}\,. \quad \text{(3b)} \end{aligned}$$

The Lagrangian of the form (1) with coefficients λ_i, m_{ij}^2 and that with coefficients given by Eq. (3) describe the same physical reality. We call this property a *RPh invariance*. The transformations (3) represent U(1) *RPh transformation group* being a subgroup of the SU(2) RPa group.

4. Lagrangian and Z_2 symmetry

The CP violation and the flavour changing neutral currents (FCNC) can be naturally suppressed by imposing a Z_2 symmetry on the Lagrangian [5]. This symmetry inhibits the $\phi_1 \leftrightarrow \phi_2$ transitions, and therefore it leads to the invariance under the interchange

$$\phi_1 \leftrightarrow \phi_1, \phi_2 \leftrightarrow -\phi_2 \text{ or } \phi_1 \leftrightarrow -\phi_1, \phi_2 \leftrightarrow \phi_2\,. \quad (4)$$

• **The case of exact Z_2 symmetry** is described by the Lagrangian $\mathcal{L}_H$ (1) with $\lambda_6 = \lambda_7 = \varkappa = m_{12}^2 = 0$; only one parameter λ_5 can be complex. The RPh transformation (3) with a suitable phase ρ allows one to get Lagrangian with a real λ_5, within the RPh invariant space.

• **In case of soft violation of Z_2 symmetry** one adds to the Z_2 symmetric Lagrangian the term of operator dimension 2, $m_{12}^2(\phi_1^\dagger\phi_2)$+h.c.

This type of violation respects the Z_2 symmetry at small distances (much smaller than $1/M$) in all orders of perturbative series, *i.e.* the amplitudes for $\phi_1 \leftrightarrow \phi_2$ transitions disappear at virtuality $k^2 \sim M^2 \to \infty$.

The general RPa transformation converts the Lagrangian with exact or softly violated Z_2 symmetry $\mathcal{L}_{\rm s}$ to a *hidden soft Z_2 violation* form, $\mathcal{L}_{\rm hs}$, with λ_6, $\lambda_7 \neq 0$, $\varkappa = 0$. 14 parameters of $\mathcal{L}_{\rm hs}$ are constrained since they can be obtained from 9 independent parameters of an initial Lagrangian $\mathcal{L}_{\rm s}$ plus 3 RPa group parameters (nondiagonal $\varkappa$ kinetic term does not arise from loop corrections). For such physical system $\mathcal{L}_{\rm s}$ *is a preferable RPa representation.*

• In general case the terms of the operator dimension 4, with generally complex parameters λ_6, λ_7 and $\varkappa$, are added to the Lagrangian with a softly violated Z_2 symmetry. In the case of **the true hard violation of $\boldsymbol{Z_2}$ symmetry** this Lagrangian cannot be transformed to the exact or softly violated Z_2 symmetry form by any RPa transformation, the Z_2 symmetry is broken *at both large and small distances* in any scalar basis.

The mixed kinetic terms (1b) can be eliminated by the nonunitary transformation (rotation + renormalization), *e.g.*

$$(\phi_1', \phi_2') \to \left(\frac{\sqrt{\varkappa^*}\phi_1 + \sqrt{\varkappa}\phi_2}{2\sqrt{|\varkappa|(1+|\varkappa|)}} \pm \frac{\sqrt{\varkappa^*}\phi_1 - \sqrt{\varkappa}\phi_2}{2\sqrt{|\varkappa|(1-|\varkappa|)}} \right). \tag{5}$$

However, in presence of the λ_6 and λ_7 terms, the renormalization of quadratically divergent, non-diagonal two-point functions leads anyway to the mixed kinetic terms (*e.g.* from loops with $\lambda_6^*\lambda_{1,3-5}$ and $\lambda_7^*\lambda_{2-5}$). It means that $\varkappa$ becomes nonzero at the higher orders of perturbative theory (and *vice versa* a mixed kinetic term generates counter-terms with $\lambda_{6,7}$). Therefore all of these terms should be included in Lagrangian (1a) on the same footing, *i.e.* the treatment of the true hard violation of Z_2 symmetry without $\varkappa$ terms is inconsistent. The parameter $\varkappa$ is running like λ's. (This term does not appear if parameters λ_i are constrained by relations of hidden soft violation of Z_2 symmetry.) Therefore, the diagonalization (5) is scale dependent, and the Lagrangian *remains off-diagonal* in fields $\phi_{1,2}$ even at very small distances in any RPa representation. Such theory seems to be *unnatural.*

Although in [1] and in this paper we present relations for the case of hard violation of Z_2 symmetry at $\varkappa = 0$, the loop corrections can change results significantly. *Such treatment of the case with true hard violation of Z_2 symmetry is as incomplete as in most of the papers considering this "most general 2HDM potential".*

5. Vacua

The extremes of the potential define the vacuum expectation values (v.e.v.'s) $\langle\phi_{1,2}\rangle$ of the fields $\phi_{1,2}$ via equations:

$$\partial V/\partial\phi_i|_{\phi_i=\langle\phi_i\rangle} = 0\,. \tag{6}$$

These equations have trivial electroweak symmetry conserving solution $\langle\phi_i\rangle = 0$ and electroweak symmetry violating solutions, discussed below. With accuracy to the choice of z axis in the weak isospin space, and using the overall phase freedom of the Lagrangian to choose one v.e.v. real, in the minimal SM such equation has a single EWSB solution: $\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$, $v = m/\sqrt{2\lambda}$. With the same choice the most general electroweak symmetry violating solution of (6) can be written in a form

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ v_2 e^{i\xi} \end{pmatrix}. \tag{7}$$

To describe these extremes it is useful to denote

$$x_1 = (\phi_1^\dagger\phi_1)\,, \quad x_2 = (\phi_2^\dagger\phi_2)\,, \quad x_3 = (\phi_1^\dagger\phi_2)\,, \quad y_i = \langle x_i\rangle\,. \tag{8}$$

It is easy to check that $\partial x_1/\partial\phi_2 = \partial x_2/\partial\phi_1 = 0$ and

$$\begin{aligned}
&x_3\left(\frac{\partial x_1}{\partial\phi_1}\phi_1\right) - x_1\left(\frac{\partial x_1}{\partial\phi_1}\phi_2\right) = x_3^*\left(\frac{\partial x_3^*}{\partial\phi_1}\phi_2\right) - x_2\left(\frac{\partial x_3^*}{\partial\phi_1}\phi_1\right) = 0\,,\\
&x_3\left(\frac{\partial x_3^*}{\partial\phi_1}\phi_1\right) - x_1\left(\frac{\partial x_3^*}{\partial\phi_1}\phi_2\right) = x_3^*\left(\frac{\partial x_1}{\partial\phi_1}\phi_2\right) - x_2\left(\frac{\partial x_1}{\partial\phi_1}\phi_1\right) = x_3 x_3^* - x_1 x_2\,.
\end{aligned}$$

Now, introducing $Z = y_3^* y_3 - y_1 y_2$, the extremum condition (6) can be rewritten as

$$\begin{aligned}
&\left\langle x_3\left(\frac{\partial V}{\partial\phi_1}\phi_1\right) - x_1\left(\frac{\partial V}{\partial\phi_1}\phi_2\right)\right\rangle = Z\left(\lambda_4 y_3 + \lambda_5^* y_3^* + \lambda_6^* y_1 + \lambda_7^* y_2 - \frac{m_{12}^{*2}}{2}\right) = 0,\\
&\left\langle x_3^*\left(\frac{\partial V}{\partial\phi_1}\phi_2\right) - x_2\left(\frac{\partial V}{\partial\phi_1}\phi_1\right)\right\rangle = Z\left(\lambda_1 y_1 + \lambda_3 y_2 + \lambda_6^* y_3^* + \lambda_6 y_3 - \frac{m_{11}^2}{2}\right) = 0,\\
&\left\langle x_3\left(\frac{\partial V}{\partial\phi_2}\phi_1\right) - x_1\left(\frac{\partial V}{\partial\phi_2}\phi_2\right)\right\rangle = Z\left(\lambda_2 y_2 + \lambda_3 y_1 + \lambda_7^* y_3^* + \lambda_7 y_3 - \frac{m_{22}^2}{2}\right) = 0.
\end{aligned} \tag{9}$$

Therefore, two opportunities can be realized, for zero and nonzero value of $Z = y_3^* y_3 - y_1 y_2$. Depending on the parameters of potential, these solutions describe either saddle point or a minimum of the potential. The condition for minimum is that all eigenvalues of Higgs mass matrix are positive, and vacuum energy of one of these states is smaller than of the other one.

5.1. $\boldsymbol{u} \neq \mathbf{0}$ *solution, charged vacuum*

We denote by *charged vacuum* a solution which appears for

$$Z = y_3^* y_3 - y_1 y_2 \neq 0 \quad \Rightarrow \quad u \neq 0 \,. \tag{10}$$

In this case the v.e.v.'s are given by equations followed directly from (9), namely

$$\begin{aligned}
\lambda_1 y_1 + \lambda_3 y_2 + \lambda_6^* y_3^* + \lambda_6 y_3 \;&=\; m_{11}^2/2 \,,\\
\lambda_2 y_2 + \lambda_3 y_1 + \lambda_7^* y_3^* + \lambda_7 y_3 \;&=\; m_{22}^2/2 \,,\\
\lambda_4 y_3^* + \lambda_5 y_3 + \lambda_6 y_1 + \lambda_7 y_2 \;&=\; m_{12}^2/2 \,.
\end{aligned} \tag{11}$$

With these y_i the Higgs potential (1) can be written using $\bar{x}_i = x_i - y_i$ as ($E^c_{\rm vac}$ is a vacuum energy)

$$\begin{aligned}
V \;=\; & \lambda_1 \bar{x}_1^2/2 + \lambda_2 \bar{x}_2^2/2 + \lambda_3 \bar{x}_1 \bar{x}_2 + \lambda_4 \bar{x}_3^* \bar{x}_3\\
& + \left[\lambda_5 \bar{x}_3^2/2 + (\lambda_6 \bar{x}_1 + \lambda_7 \bar{x}_2)\bar{x}_3 + \text{h.c.}\right] + E^c_{\rm vac} \,.
\end{aligned} \tag{12}$$

In this case it is not possible to split the gauge boson mass matrix into a neutral and charged sector, the interaction of gauge bosons with fermions will not preserve electric charge, photon become massive, *etc.* [6]. Certainly, this case is not realized in our World.

5.2. $\boldsymbol{u} = \mathbf{0}$ *solution, physical (neutral) vacuum*

We consider below (except for a final section) only solution of extremum condition (6), obeying a condition for U(1) symmetry of electromagnetism,

$$Z = y_3^* y_3 - y_1 y_2 = 0 \;\Rightarrow \langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0\\ v_1\end{pmatrix}, \;\langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0\\ v_2 e^{i\xi}\end{pmatrix}. \tag{13}$$

There is other standard notations: $v_1 = v\cos\beta$, $v_2 = v\sin\beta$, with SM constraint $v = (\sqrt{2}G_{\rm F})^{-1/2} = 246\,\text{GeV}$.

The rephasing of fields (3a) shifts the phase difference ξ as $\xi \to \xi - \rho$.

Let us take some Lagrangian describing our model and calculate v.e.v.'s. Then, by making the RPh transformation with $\rho = \xi$, we get *the real vacuum Lagrangian* with real v_2 and with parameters, given by (3) with $\rho = \xi$ (supplied for a moment by subscript rv).

The following combinations of parameters and new quantities are useful:

$$\begin{aligned}
&\lambda_{3,\rm rv} + \lambda_{4,\rm rv} + {\rm Re}\lambda_{5,\rm rv} = \lambda_{345,\rm rv}\,,\\
&\frac{v_1}{v_2}\lambda_{6,\rm rv} \pm \frac{v_2}{v_1}\lambda_{7,\rm rv} = \lambda^{\pm}_{67,\rm rv}\,,\\
&m^2_{12,\rm rv} = 2v_1 v_2(\nu + i\delta)\,.
\end{aligned} \tag{14}$$

The minimum conditions (8) for this form of Lagrangian are written as

$$
\begin{aligned}
&(m_{11}^2 - \mathrm{Re}\, m_{12}^2 v_2/v_1)/2 - \lambda_1 v_1^2 + \lambda_{345} v_2^2 + \mathrm{Re}\left(3\lambda_6 v_1 v_2 + \lambda_7 v_2^3/v_1\right) = 0\,,\\
&(m_{22}^2 - \mathrm{Re}\, m_{12}^2 v_2/v_1)/2 - \lambda_2 v_2^2 + \lambda_{345} v_1^2 + \mathrm{Re}\left(3\lambda_7 v_1 v_2 + \lambda_6 v_1^3/v_2\right) = 0,\\
&\mathrm{Im}\,(m_{12}^2) \equiv 2 v_1 v_2 \delta = v_1 v_2 \mathrm{Im}\,\left(\lambda_5 + \lambda_{67}^+\right)\,. \qquad (15)
\end{aligned}
$$

Below we perform calculations for a real vacuum potential, describing it in terms of v_1, v_2, ν instead of three parameters $m_{11,22}^2$, $\mathrm{Re}(m_{12}^2)$, which are coefficients of the quadratic terms. In this way $\delta \propto \mathrm{Im}\, m_{12,\mathrm{rv}}^2$ is expressed via $\mathrm{Im}(\lambda_{5-7,\mathrm{rv}})$ (15).

6. Physical Higgs representation

A standard decomposition of the fields $\phi_{1,2}$ in the component fields is

$$
\phi_1 = \begin{pmatrix} \varphi_1^+ \\ (v_1 + \eta_1 + i\chi_1)/\sqrt{2} \end{pmatrix},\quad \phi_2 = \begin{pmatrix} \varphi_2^+ \\ (v_2 + \eta_2 + i\chi_2)/\sqrt{2} \end{pmatrix}. \qquad (16)
$$

At $\varkappa = 0$ such decomposition preserves a diagonal form of kinetic terms for fields φ_i^+, χ_i, η_i. The mass-squared matrix is transformed to the block diagonal form by a separation of the massless Goldstone boson fields, $G^0 = \cos\beta\,\chi_1 + \sin\beta\,\chi_2$ and $G^\pm = \cos\beta\,\varphi_1^\pm + \sin\beta\,\varphi_2^\pm$, and the charged Higgs boson fields $H^\pm$ with mass $M_{H^\pm}$,

$$
H^\pm = -\sin\beta\,\varphi_1^\pm + \cos\beta\,\varphi_2^\pm,\quad M_{H^\pm}^2 = \left[2\nu - \lambda_4 - \mathrm{Re}\lambda_5 - \mathrm{Re}\lambda_{67}^+\right] v^2/2. \qquad (17)
$$

6.1. Neutral Higgs sector

By definition $\eta_{1,2}$ are the standard C- and P-even (scalar) fields. The field

$$
A = -\sin\beta\,\chi_1 + \cos\beta\,\chi_2\,, \qquad (18)
$$

is C-odd (which in the interactions with fermions behaves as a P-odd particle, *i.e.* a pseudoscalar). In other words, the $\eta_{1,2}$ and A are fields with opposite CP parities (see *e.g.* [2] for details).

The decomposition (16) results in the symmetric mass-squared matrix $\mathcal{M}$ in the η_1, η_2, A basis

$$
\mathcal{M} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{12} & M_{22} & M_{23} \\ M_{13} & M_{23} & M_{33} \end{pmatrix}, \qquad (19)
$$

$$
\begin{aligned}
M_{11} &= \left[c_\beta^2\,\lambda_1 + s_\beta^2\,\nu + s_\beta^2 \mathrm{Re}\,(\lambda_{67}^+/2 + \lambda_{67}^-)\right] v^2\,,\\
M_{22} &= \left[s_\beta^2\,\lambda_2 + c_\beta^2\,\nu + c_\beta^2 \mathrm{Re}\,(\lambda_{67}^+/2 - \lambda_{67}^-)\right] v^2\,,\\
M_{33} &= \left[\nu - \mathrm{Re}\,(\lambda_5 - \lambda_{67}^+/2)\right] v^2 \equiv M_A^2\,,\\
M_{12} &= -\left[\nu - \lambda_{345} - \mathrm{Re}\,3\lambda_{67}^+/2\right] c_\beta s_\beta v^2\,,\\
M_{13} &= -\left[\delta + \mathrm{Im}\lambda_{67}^-/2\right] s_\beta v^2\,,\\
M_{23} &= -\left[\delta - \mathrm{Im}\lambda_{67}^-/2\right] c_\beta v^2\,,
\end{aligned}
\tag{20}
$$

where $c_\beta = \cos\beta$, $s_\beta = \sin\beta$. Note that M_{33} is equal to the mass squared of the CP-odd Higgs boson in the CP conserving case M_A^2.

The masses squared M_i^2 of the physical neutral states h_{1-3} are eigenvalues of the matrix $\mathcal{M}$. These states are obtained from fields η_1, η_2, A by a unitary transformation R which diagonalizes the matrix $\mathcal{M}$:

$$
\begin{pmatrix} h_1\\ h_2\\ h_3\end{pmatrix} = R\begin{pmatrix}\eta_1\\ \eta_2\\ A\end{pmatrix}\,, \qquad R = \begin{pmatrix} R_{11} & R_{12} & R_{13}\\ R_{21} & R_{22} & R_{23}\\ R_{31} & R_{32} & R_{33}\end{pmatrix}\,,
\tag{21}
$$

with $R\mathcal{M}R^T = \mathrm{diag}(M_1^2,\, M_2^2,\, M_3^2)$. All observable Higgs fields h_i, $H^\pm$, their masses and couplings are RPa independent, in contrast to the original fields $\phi_{1,2}$. The useful 2-step diagonalization procedure is described in [1].

Criterium for CP violation. In general, the Higgs eigenstates h_i (21) have no definite CP parity since they are mixtures of fields $\eta_{1,2}$ and A having opposite CP parities. *Just this mixing provides a CP nonconservation within the Higgs sector since the interaction of these Higgs bosons with matter explicitly violates the CP-symmetry.*

Eq. (20) shows that such mixing is absent and the CP is not violated if and only if $M_{13} = M_{23} = 0$. The explicit form for these terms (20) shows that these two conditions can be valid if and only if λ_{67}^- and m_{12}^2 are real. In accordance with (14) it means that **the CP violation is absent if all coefficients in potential of a real vacuum form are real**. Vice versa, the complexity of some parameters of the potential in *a real vacuum form* is a sufficient condition for CP violation in the Higgs sector. For an arbitrary form of Lagrangian **the sufficient condition for CP violation in the Higgs sector** can be written as complexity at least one of combinations

$$
\lambda_5^*(m_{12}^2)^2\,, \quad (\lambda_6^* + \lambda_7^*)m_{12}^2\,, \quad \lambda_6^*\lambda_7\,.
\tag{22}
$$

Each quantity written above is not RPa invariant one, however these conditions are very simple. (For the soft Z_2 violated potential the condition is simply $\mathrm{Im}\,\lambda_5^*(m_{12}^2)^2 \neq 0$ — *cf.* [7].) The RPa invariant conditions for CP violation [3, 8] are more complex.

7. Couplings to gauge bosons and fermions

Below we use *in principle measurable relative couplings* — ratios of the couplings of each neutral Higgs boson h_i to the corresponding SM couplings

$$\chi_j^{(i)} = g_j^{(i)}/g_j^{\rm SM}\,, \tag{23}$$

for the gauge bosons W or Z and the quarks or leptons ($j = W, Z, u, d, \ell$...).

• The gauge bosons V (W and Z) couple only to the CP-even fields η_1, η_2. For the physical Higgs bosons h_i (21) one obtains

$$\chi_V^{(i)} = \cos\beta\, R_{i1} + \sin\beta\, R_{i2}, \;\; V = W \text{ or } Z. \tag{24}$$

• **Yukawa interaction**

The general form of Yukawa interaction couples 3-family vector of the left-handed quark isodoublets $Q_{\rm L}$ with 3-family vectors of the right-handed field singlets $d_{\rm R}$ and $u_{\rm R}$ and Higgs fields ϕ_i. It allows large FCNC effects and leads to true hard violation of Z_2 symmetry via loop effect (see [1] for details).

To have only the soft violation of Z_2 symmetry, each right-handed fermion should couple to only one scalar field, either ϕ_1 or ϕ_2 [5, 9].

7.1. Model II

We consider first most popular opportunity (realized also in the MSSM) called Model II. In this Model the physical reality allows the description in which the fundamental scalar field ϕ_1 couples to d-type quarks and charged leptons ℓ, while ϕ_2 couples to u-type quarks, and this interaction is diagonal (or almost diagonal) in family index k:

$$-\mathcal{L}_Y^{\rm II} = \sum g_{dk} \bar{Q}_{{\rm L}k}\phi_1 d_{{\rm R}k} + \sum g_{uk}\bar{Q}_{{\rm L}k}\tilde{\phi}_2 u_{{\rm R}k} + \sum g_{\ell k}\bar{\ell}_{Lk}\phi_1\ell_{Rk} + \text{h.c.} \tag{25}$$

The suitable choice of phases in the RPh transformations makes all Yukawa parameters real. The RPa transformation makes Model II property of Lagrangian hidden and changes $\tan\beta$. For the Lagrangian with explicit Model II property we supply quantity β by a subscript II, $\beta \to \beta_{\rm II}$.

The relative Yukawa couplings of the physical neutral Higgs bosons h_i (23) are identical for all u-type and for all d-type quarks (and charged leptons). They are expressed via elements of the rotation matrix R (21) as

$$(\text{MII}): \qquad \chi_u^{(i)} = \frac{R_{i2} - i\cos\beta_{\rm II}\, R_{i3}}{\sin\beta_{\rm II}}, \quad \chi_d^{(i)} = \frac{R_{i1} - i\sin\beta_{\rm II}\, R_{i3}}{\cos\beta_{\rm II}}\,. \tag{26}$$

The unitarity of the mixing matrix R allows one to obtain a number of **relations between the relative couplings of neutral Higgs particles**. These relations, listed below, are very useful in phenomenological analyses.

First, the quantity $\tan\beta_{\rm II}$ (coincident with the ratio v_2/v_1 only in a Model II form of Lagrangian) is described via the basic couplings to h_i as

$$\cot^2\beta_{\rm II}=\frac{(\chi_V^{(i)}-\chi_u^{(i)})}{\left(\chi_d^{(i)}-\chi_V^{(i)}\right)^*}=\frac{1-|\chi_u^{(i)}|^2}{|\chi_d^{(i)}|^2-1}=\frac{{\rm Im}\chi_u^{(i)}}{{\rm Im}\chi_d^{(i)}}=\sum_i({\rm Im}\chi_u^{(i)})^2. \qquad (27)$$

1. *The pattern relation for each neutral Higgs particle h_i* (for CP conserving case see [10]):

$$(\chi_u^{(i)}+\chi_d^{(i)})\chi_V^{(i)}=1+\chi_u^{(i)}\chi_d^{(i)}\,. \qquad (28)$$

2. *A vertical sum rule for* **all three** *neutral Higgs bosons h_i* [11]:

$$\sum_{i=1}^{3}(\chi_j^{(i)})^2=1 \qquad (j=V,d,u)\,. \qquad (29)$$

3. *A horizontal sum rule* [11] for each neutral Higgs boson h_i:

$$|\chi_u^{(i)}|^2\sin^2\beta_{\rm II}+|\chi_d^{(i)}|^2\cos^2\beta_{\rm II}=1\,. \qquad (30)$$

4. Besides, the *linear relation* follows directly from Eqs. (21), (26):

$$\begin{aligned}&{\rm Re}\left(\cos^2\beta_{\rm II}\chi_d^{(i)}+\sin^2\beta_{\rm II}\chi_u^{(i)}\right)=\chi_V^{(i)},\\ &{\rm Im}\left(\cos^2\beta_{\rm II}\chi_d^{(i)}-\sin^2\beta_{\rm II}\chi_u^{(i)}\right)=0\,.\end{aligned} \qquad (31)$$

5. *The relation between CP violated parts of Yukawa couplings* is obtained by exclusion of $\beta_{\rm II}$ from the equations (30), (31)

$$(1-|\chi_d^{(i)}|^2)\,{\rm Im}\chi_u^{(i)}+(1-|\chi_u^{(i)}|^2)\,{\rm Im}\chi_d^{(i)}=0\,. \qquad (32)$$

7.2. Model I

In this model all right handed fermions are coupled to one scalar field ϕ_1. The corresponding Yukawa Lagrangian is similar to that given by (25) with only change $\phi_2\to\phi_1$. We supply the parameter β for the explicit Model I case by subscript I. In this case $\chi_u^{(i)}=\chi_d^{(i)}\equiv\chi_f^{(i)}=[R_{i2}-i\cos\beta_I\,R_{i3}]/\sin\beta_I$. Besides, $\cot^2\beta_I=\sum_i({\rm Im}\chi_u^{(i)})^2$ and vertical sum rules (29) are valid. Other relations written for Model II do not apply.

8. Possible relation to an evolution of Universe

Scalar, Higgs–like field ϕ plays cardinal role in the wide-spread description of an early Universe. While primitive medium is hot, the mass term in effective potential for ϕ is equal to the sum of standard term $-m^2\phi^2/2$ and the temperature dependent term $cT^2\phi^2/2$. Immediately after Big Bang temperature T is very high, so that effective potential has minimum at $\langle\phi\rangle = 0$. In this period Universe expands very fast (*inflation*). At the temperature $T_{\rm c} \approx m/\sqrt{c}$ phase transition occurs, v.e.v. of Higgs field $\langle\phi\rangle \propto \sqrt{m^2 - cT^2}$ appears, inflation stops, the expansion of Universe becomes slower. The v.e.v. $\langle\phi\rangle$ increases with time simultaneously with cooling of Universe. If this Higgs field is that of SM, during inflation EW symmetry is not broken, all particles are massless. After phase transition (at $T < T_{\rm c}$) EW symmetry becomes broken, particles acquire masses growing with time $\propto \langle\phi\rangle$, nonzero vacuum energy $E_{\rm vac}$ arises.

If 2HDM is realized, possible existence of two vacua opens new opportunities in the evolution of Universe. We discuss briefly some possible features of this evolution for two distinct interrelations of inflatory Higgs field and those, responsible for EWSB.

• **If inflation is caused by Higgs field, responsible for EWSB**, the mass term of effective potential (1c) is enlarged in the hot primitive medium by terms

$$\left[c_{11}(\phi_1^\dagger\phi_1)+c_{12}(\phi_1^\dagger\phi_2)+c_{12}^*(\phi_2^\dagger\phi_1)+c_{22}(\phi_2^\dagger\phi_2)\right]\frac{T^2}{2}\,. \tag{33}$$

With this new mass term immediately after Big Bang the Universe expands inflatory, with $\langle\phi_1\rangle = \langle\phi_2\rangle = 0$, in the same manner as in the minimal SM. At some critical temperature $T_{\rm c}$ the EW symmetry breaks and the inflation stops. The subsequent fate of Universe depends on values of parameters.

In one case, at decreasing of temperature below $T_{\rm c}$ the neutral vacuum (13) appears. In this case evolution of Universe is similar to that as in the minimal SM, with possible dependence of the parameter $\tan\beta_{\rm II}$ on time.

The other possibility is that phase transition at $T = T_{\rm c}$ transforms Universe in the state with the charged vacuum (10) for Higgs subsystem and only later, at some temperature $T_{\rm c1} < T_{\rm c}$, the charged vacuum is transformed to well known neutral vacuum (13). The properties of Universe in the period when $T_{\rm c1} < T < T_{\rm c}$ are quite unusual. In this stage the medium is non-transparent for light (photon is massive), the interactions of particles differ from modern ones, the C violation for particles interaction (vacuum is charged) can remain after second phase transition, track in a form of residual CP violation, baryon asymmetry, *etc.* Besides, some small domains of charged phase arisen from fluctuations in one of phase transitions can leave long at $T < T_{\rm c1}$, influencing modern observations. Some of these opportunities can be excluded immediately, others must be studied in detail.

• **If inflation is caused by a specific inflanton Higgs field $\boldsymbol{\phi_0}$ with v.e.v. $\boldsymbol{\langle\phi_0\rangle = U_0(t)}$ varying in time**, this field interacts with Higgs field responsible for EWSB like (1d). In the mean field approximation the effective Higgs potential is enlarged by both terms (33) and terms

$$\left[a_{11}(\phi_1^\dagger\phi_1)+a_{12}(\phi_1^\dagger\phi_2)+a_{12}^*(\phi_2^\dagger\phi_1)+a_{22}(\phi_2^\dagger\phi_2)\right]\frac{U_0^2(t)}{2}\,. \tag{34}$$

Therefore, during inflation effective mass term of the EWSB Higgs field varies with time as $m_{ij}^2 \to m_{ij}^2 - c_{ij}T^2 - a_{ij}U_0^2(t)$. It can result in even more complex sequence of phase transitions than that discussed above (*e.g.*, with restoration of SU(2)× U(1) symmetry in some intermediate period).

Both these opportunities should be analysed in future.

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