PROBING CP VIOLATION AND THE MAJORANA NATURE OF NEUTRALINOS*

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Based on the possibility of having highly polarized neutralinos in reconstructed rest frames, we show in a systematic way a method of verifying the Majorana nature of neutralinos and testing the CP properties of the neutralino sector of the MSSM in three-body leptonic decays of neutralinos.

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1. Introduction

All SUSY theories contain neutralinos, the spin-1/2 Majorana superpartners of neutral gauge bosons and Higgs bosons, that are expected to be among the lightest supersymmetric particles and can be produced at future colliders — the LHC and the ILC. It is of great importance to confirm that the discovered particles are indeed partners of Standard Model (SM) particles and measure their quantum numbers, masses, mixing angles, couplings and CP violating phases, with great precision. This would allow us to reconstruct fundamental SUSY parameters and give an insight of physics at very high energy scales.

In this talk we report on the results obtained in [1], where we focus on the Majorana nature of neutralinos and the CP properties of the neutralino sector of the Minimal Supersymmetric Standard Model (MSSM) through the charge self-conjugate three body decays of polarized neutralinos into the lightest neutralino and a lepton pair:

$$\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \ell^+ \ell^-, \tag{1}$$

where $\ell = e$ or μ. Throughout this work we assume that SUSY is a well established theory and that the masses of particles are known very precisely.

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Our method is based on two crucial observations: neutralinos produced in $\tilde{e}^{\pm}_{\rm L}$ decays are 100% polarized, having positive/negative helicity in $\tilde{e}^{\pm}_{\rm L} \to e^{\pm}\tilde{\chi}^0_i$ [2]. Furthermore, as it was shown in [3], the rest frame of the neutralino $\tilde{\chi}^0_2$ can be reconstructed in some cascade decay processes, *e.g.* $e^+e^- \to \tilde{e}^+_{\rm L}\tilde{e}^-_{\rm L} \to e^+\tilde{\chi}^0_1 e^-\tilde{\chi}^0_2$, followed by the three-body decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1\mu^+\mu^-$. Exploiting these two possibilities we show a method of probing the Majorana nature of neutralinos and measuring CP properties of the neutralino sector of the MSSM; for alternative methods see *e.g.* [4].

If at the initial phase of the e^+e^- collider the selectron pair production turns out to be kinematically shut, the photon collider in the $e\gamma$ mode might supply polarized neutralinos from selectrons produced in $e^-\gamma \to \tilde{e}^-_{\rm L}\tilde{\chi}^0_1$, however with reduced ability to reconstruct their decay rest frame.

2. Three-body leptonic neutralino decays

The three-body leptonic decay of neutralino $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1\ell^+\ell^-$ is mediated by Z boson and slepton exchanges. We neglect the exchange of neutral Higgs bosons, since their couplings to e and μ are suppressed by the small lepton masses. After a simple Fierz transformation of slepton exchange diagrams the decay matrix element has a vector-current product form:

$$\mathcal{D}\left(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1\ell^+\ell^-\right) = \frac{e^2}{m_2^2} D_{\alpha\beta}\Big[\bar{u}(\tilde{\chi}^0_1)\gamma^\mu P_\alpha u(\tilde{\chi}^0_2)\Big]\Big[\bar{u}(\ell^-)\gamma_\mu P_\beta v(\ell^+)\Big], \qquad (2)$$

where $m_2 = m_{\tilde{\chi}^0_2}$, and the bilinear charges $D_{\alpha\beta}$ $(\alpha,\beta = {\rm L},{\rm R})$ contain internal propagators and couplings [1].

As a reference frame for our discussion we choose the rest frame of the decaying neutralino. The neutralino $\tilde{\chi}^0_2$ spin vector $\hat{n} = (0,0,1)$ defines the direction of the z-axis. The x-z plane and the angle θ are then fixed

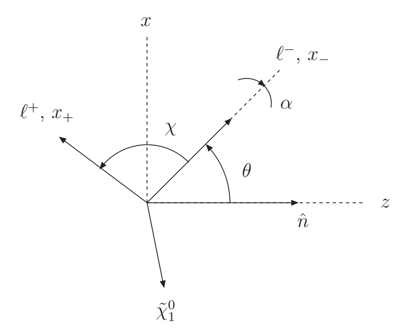

Fig. 1. Kinematic configuration of momenta and the spin vector in the neutralino $\tilde{\chi}^0_2$ rest frame.

by the momentum vector of the negative lepton. The angle α determines the neutralino decay plane (NDP), so that by rotating the NDP by $-\alpha$ around ℓ^- momentum direction it is brought to the x-z plane, as shown in Fig. 1. After neglecting lepton masses, we can write the differential decay distribution in terms of two dimensionless energy variables, $x_- = 2E_{e^-}/m_2$ and $x_+ = 2E_{e^+}/m_2$, and angles θ and α as

$$\frac{d^4\Gamma}{dx_- dx_+ d(\cos\theta) d\alpha} = \frac{\alpha^2\, m_2}{16\pi^2}\Big[F_0(x_-,x_+) + (\hat{q}_-\cdot\hat{n})\,F_1(x_-,x_+) + (\hat{q}_+\cdot\hat{n})\,F_2(x_-,x_+) + \hat{n}\cdot(\hat{q}_-\times\hat{q}_+)\;F_3(x_-,x_+)\Big], \quad (3)$$

where $\hat{q}_\pm = \vec{q}_\pm/|\vec{q}_\pm|$, and $\vec{q}_\pm$ are the leptons momenta in the $\tilde{\chi}^0_2$ rest frame. The four kinematic functions $F_i(x_-,x_+)$ $(i = 0$–$3)$ depend on the dimensionless energy variables, x_- and x_+, and the bilinear charges, but not on the orientation angles θ and α.

By applying the CP transformation to the decay matrix element (2) we can derive relations between bilinear charges and kinematic functions (3), which are consequences of the Majorana nature of neutralinos:

$$\begin{array}{l} D_{\rm LR} = \eta_1\eta_2 D_{\rm RR}(t\leftrightarrow u) \\ D_{\rm RL} = \eta_1\eta_2 D_{\rm LL}(t\leftrightarrow u) \end{array} \Longrightarrow \begin{array}{l} F_0(x_-,x_+) = +F_0(x_+,x_-) \\ F_1(x_-,x_+) = -F_2(x_+,x_-) \\ F_3(x_-,x_+) = -F_3(x_+,x_-)\,, \end{array} \quad (4)$$

where $\eta_{1,2} = \pm i$ are the intrinsic CP parities of $\tilde{\chi}^0_{1,2}$, respectively [6]. On the other hand, applying the CP$\tilde{\rm T}$[1] transformation results in:

$$\begin{array}{l} D_{\rm LR} = -D^*_{\rm RR}(t\leftrightarrow u) \\ D_{\rm RL} = -D^*_{\rm LL}(t\leftrightarrow u) \end{array} \Longrightarrow \begin{array}{l} F_0(x_-,x_+) = +F_0(x_+,x_-) \\ F_1(x_-,x_+) = -F_2(x_+,x_-) \\ F_3(x_-,x_+) = +F_3(x_+,x_-) \end{array} \quad (5)$$

in the approximation of neglecting particle widths.

3. Numerical analyses

In order to show practical consequences of the results derived in Sec. 2 we adopt an MSSM scenario defined at the electroweak scale by the following set of parameters:

$$|M_1| = 80\,\text{GeV}, \quad M_2 = 158\,\text{GeV}, \quad \mu = 415\,\text{GeV}, \quad \tan\beta = 10. \quad (6)$$

[1] The naive time reversal transformation $\tilde{\rm T}$ reverses directions of all 3-momenta and spins, but does not exchange the initial and final states.

In this analysis we take μ to be real and the phase Φ_1 of M_1 is the only source of the CP violation. In the scenario (6), approximately 2×10^5 events of $\tilde{e}_R^{\pm}\tilde{e}_L^{\mp}$ and $\tilde{e}_L^{+}\tilde{e}_L^{-}$ production at an integrated luminosity of 1000 fb^{-1} are expected and after combining with the branching ratios a sufficient number of events for the decays $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 e^+e^-$ and $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \mu^+\mu^-$ can be selected. In our Monte Carlo analysis we conservatively assume that at least 1000 neutralino decay events can be reconstructed.

The spin averaged differential decay distribution

$$\frac{d^2\Gamma}{dx_- dx_+} \propto F_0(x_-, x_+) \tag{7}$$

due to the Majorana nature of neutralinos, *cf.* Eqs. (4), has to be symmetric with respect to the energy variables x_+ and x_- in the CP invariant case (and to a good approximation in the CP non-invariant case) [5]. The left panel of Fig. 2 shows the Dalitz plot of the decay $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \ell^+\ell^-$ for the parameter set (6) with $\Phi_1 = 0$. We find that the asymmetry in the number of events with $\mathrm{sign}(x_- - x_+) = -$ and $\mathrm{sign}(x_- - x_+) = +$ is $\Delta N_{\rm ev} = 24$, which is within the statistical error of $\Delta N_{\rm stat} = \sqrt{N_{\rm ev}} \simeq 32$ for $N_{\rm ev} = 1000$.

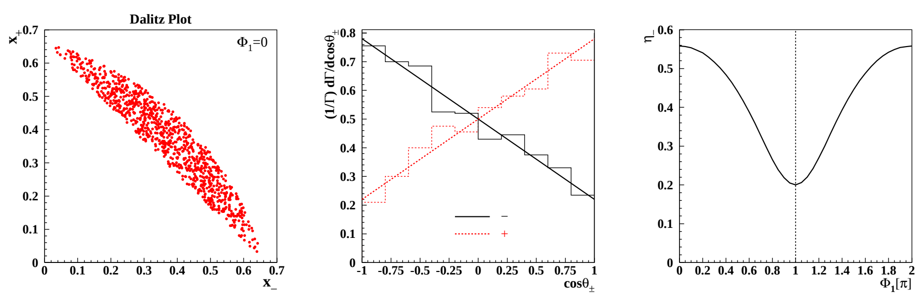

Fig. 2. Left: the Dalitz plot for the neutralino $\tilde{\chi}_2^0$ decay. Middle: the normalized lepton angle distribution (8); the solid (dashed) line is for ℓ^- (ℓ^+). Right: the Φ_1 dependence of the slope parameter η_- for the parameter set (6).

A complementary test of the Majorana nature of neutralinos is provided by the lepton angle distribution with respect to the neutralino polarization vector. Defining $\theta_\pm$ to be the polar angle between the $\ell^\pm$ momentum and the polarization vector $\hat{n}$, the normalized lepton angle distribution can be written as

$$\frac{1}{\Gamma}\frac{d\Gamma}{dz_\pm} = \frac{1}{2}\left(1 \pm \eta_\pm\, z_\pm\right), \tag{8}$$

with $z_\pm = \hat{q}_\pm \cdot \hat{n} = \cos\theta_\pm$. As a result of the CP$\tilde{\rm T}$ invariance and the Majorana nature of neutralinos we get $\eta_- = \eta_+$, irrespective of whether the theory is CP invariant or not. The middle panel of Fig. 2 shows the lepton angle distribution for the parameter set (6) with the phase $\Phi_1 = 0$. A simple numerical analysis based on $N_{\rm ev} = 1000$ events shows that the CP$\tilde{\rm T}$ relation and the Majorana nature of neutralinos can be confirmed within 1-σ statistical uncertainty of about 10% for the range[2] of $|\cos\theta_\pm| < 0.8$. The dependence of the slope parameter η_- on the Φ_1 phase for the parameter set (6) is shown in the right panel of Fig. 2.

The next interesting observables are the lepton invariant mass and the lepton opening angle distributions. They give us the possibility to check the relative CP parities of two neutralinos involved in the decay (1). Near the end point of the lepton invariant mass distribution the neutralino $\tilde{\chi}^0_1$ is produced nearly at rest. In this case we can expand the squared matrix element in powers of neutralino velocity β. If neutralinos $\tilde{\chi}^0_1$ and $\tilde{\chi}^0_2$ are of the same (opposite) parity then all bilinear charges are purely real (purely imaginary) and the invariant mass distribution exhibits a characteristic steep S-wave (slow P-wave) decrease proportional to β (β^3) near the maximum of $m_{\ell\ell}$ [1], as can be seen in the left panel of Fig. 3.

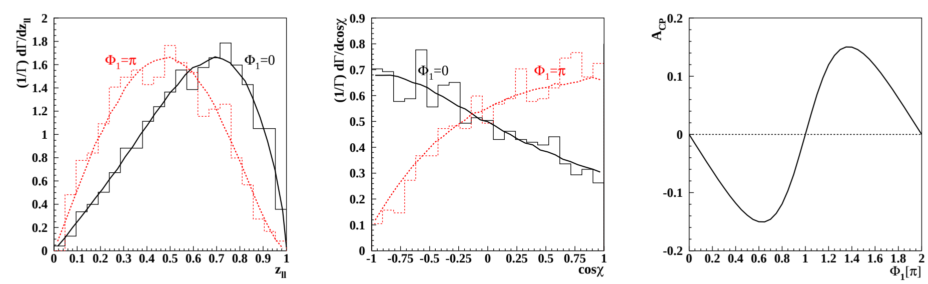

Fig. 3. The lepton invariant mass distribution (the left panel) and the opening angle distribution (the middle panel). The solid (dashed) lines are for $\Phi_1 = 0$ ($\Phi_1 = \pi$), *i.e.* for neutralinos with the same (opposite) CP parities. The right panel: the Φ_1 dependence of the $A_{\rm CP}$ (11).

The relative CP parity can also be read from the opening angle distribution. The invariant mass of two leptons with respect to the opening angle of the lepton pair χ is given by

$$m^2_{\ell\ell} = \frac{m^2_2}{2}\, x_+ x_- \, (1 - \cos\chi)\,. \tag{9}$$

[2] The cut may be necessary to avoid distortions of the $\ell^\pm$ distributions by experimental selection criteria [7].

At its maximum, for $\cos\chi = -1$, the directions of lepton momenta are opposite. Because the helicities of leptons coupled to a vector current are opposite, the angular momentum conservation forces the orbital angular momentum to be zero. On the other hand, since the selection rule of the orbital angular momentum L by the CP symmetry reads: $1 = -\eta_1\eta_2(-1)^L$, for neutralinos of the same (opposite) parity, the opening angle distribution is enhanced (suppressed) near $\cos\chi = -1$, as can be seen in the middle panel of Fig. 3.

From the CP and CP$\tilde{\mathrm{T}}$ relations (4) and (5) it follows that the distribution:

$$F_{\mathrm{CP}}(x_-, x_+) = \frac{1}{2}\left[\, F_3(x_-, x_+) + F_3(x_+, x_-)\,\right] \tag{10}$$

is CP-odd but CP$\tilde{\mathrm{T}}$-even. This distribution is connected with a triple neutralino spin and leptons momenta product: $O_{\mathrm{CP}} = \hat{n}\cdot(\hat{q}_+ \times \hat{q}_-)$, which allows us to construct a CP-odd asymmetry:

$$A_{\mathrm{CP}} = \frac{N(O_{\mathrm{CP}} > 0) - N(O_{\mathrm{CP}} < 0)}{N(O_{\mathrm{CP}} > 0) + N(O_{\mathrm{CP}} < 0)}. \tag{11}$$

The right panel of Fig. 3 shows the Φ_1 dependence of the asymmetry A_{CP}. With 1000 events this would enable us to measure the CP violation in the neutralino system with a 1-σ statistical uncertainty of

$$\sqrt{\frac{1 - A_{\mathrm{CP}}^2}{N_{\mathrm{ev}}}} \simeq 3.1\%.$$

4. Summary

We have showed that a sample of 100% polarized neutralinos in their decay rest frames can provide us with a powerful tool for probing the Majorana nature of neutralinos and their CP properties. The Majorana nature of neutralinos can be checked through: the lepton energy distribution and the lepton angle distribution with respect to the neutralino polarization vector. The relative CP parity of two neutralinos can be identified using the threshold behavior of the lepton invariant mass distribution and the opening angle distribution of the lepton pair. Finally, the CP violating phases in the neutralino system can be measured using the CP-odd quantity built from the neutralino spin vector and two leptons momenta.

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