FORMULA $E = mc^2$ IN THE YEAR OF PHYSICS*

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The "famous formula" $E = mc^2$ and the concept of "relativistic mass" increasing with velocity, which follows from it, are historical artifacts, contradicting the basic symmetry of Einstein's Special Relativity. The relation discovered by Einstein is not $E = mc^2$, but $E_0 = mc^2$, where E_0 is the energy of a free body at rest. The source of the longevity of the "famous formula" is irresponsible attitude of relativity theory experts to the task of explaining it to the non-experts.

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1. Year of Physics

2005 is the first year of physics in the history of humankind. It celebrates the papers on relativity, quanta, and atoms, published in 1905 by Einstein.

The whole building of modern physics has these papers at its basis.

2. Rest energy E_0

1905: Einstein introduced [1] the notion of rest energy E_0 of a body and established the connection between its change and the change of the mass of the body

$$\Delta mc^2 = \Delta E_0 \,.$$

Later on he generalized it [2] to:

$$mc^2 = E_0 \,.$$

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3. Energy, momentum and mass

According to Special Relativity,

$$(mc^2)^2 = E^2 - \boldsymbol{p}^2c^2 = p^2\,,$$

where E — energy, $\boldsymbol{p}$ — momentum, p — 4-momentum, m — mass of a free body.

$p = (E, \boldsymbol{p}c)$ — Lorentz 4-vector, m — Lorentz scalar, relativistically invariant.

E and $\boldsymbol{p}$ can be expressed through mass m and the velocity of a body, $\boldsymbol{v} = \boldsymbol{p}c^2/E$:

$$E = mc^2\gamma\,, \quad \boldsymbol{p} = m\boldsymbol{v}\gamma\,, \quad \gamma = \frac{1}{\sqrt{1 - \boldsymbol{v}^2/c^2}}\,.$$

For a free body at rest $\boldsymbol{p} = 0$, $\boldsymbol{v} = 0$. Hence $E_0 = mc^2$, where E_0 — rest energy.

$$E_0 = mc^2\,.$$

Mass m — relativistically invariant!

4. The ship of Galileo

Why do I stress the properties of various physical quantities under Lorentz transformations? Because symmetry is the heart of physics in general and of relativity theory in particular. The principle of relativity was formulated by Galileo [3], who insisted that there is no experiment inside a ship that can tell whether the ship is at rest or is moving uniformly and rectilinearly. At the turn of 19th and 20th century it became clear, that experiments with light are not better in this respect than with any other objects. This led to Lorentz transformations of spatial and time coordinates instead of Galilean transformations according to which time is not transformed.

5. Origin of $E = mc^2$

1900: Poincaré "proved" that mass of a pulse of the light is proportional to its energy [4].

Poincaré misused non-relativistic Newton's relation

$$\boldsymbol{p} = m\boldsymbol{v}$$

at $|\boldsymbol{v}| = c$. And from Poynting relation

$$|\boldsymbol{p}| = \frac{E}{c}$$

derived $E = mc^2$.

"The famous formula" $E = mc^2$ was wrong in 1905 and even more so in 2005! But it is still very popular.

6. Longitudinal and transverse masses

1899: Lorentz [5] introduced two masses of the electron, which depend on the angle between velocity and force by using non-relativistic connection between force and acceleration

$$\boldsymbol{F} = m_t \boldsymbol{a} \quad \text{for} \quad \boldsymbol{F} \perp \boldsymbol{v}\,,$$

$$\boldsymbol{F} = m_l \boldsymbol{a} \quad \text{for} \quad \boldsymbol{F} \| \boldsymbol{v}\,,$$

$$m_t = m\gamma\,,$$

$$m_l = m\gamma^3\,,$$

$$\gamma = \frac{1}{\sqrt{1 - \boldsymbol{v}^2/c^2}}\,.$$

These transverse and longitudinal masses are almost forgotten today, unlike the "relativistic mass" $m = E/c^2$.

7. $E = mc^2$ in the first half of the 20th century

The relativistic mass was strongly promoted by many prominent physicists (Lewis and Tolman [5, 6], Born [7], Fock [8]). In the first half of the 20th century this could be justified by the wish to preserve the role of mass in Newtonian physics, first of all as a measure of inertia. Another aim was to preserve the additivity: the mass of a system of free particles is equal to the sum of their masses. As we know today, both these properties are lost in the case of relativistic particles.

As for Einstein, he "oscillated" between $E_0 = mc^2$ and $E = mc^2$ till 1921. In 1921 he definitely chose the former [2]. But even when signing the letter to Roosvelt on atomic bomb he used the "famous formula".

8. Evolution of concept of matter

It was Newton who introduced the notion of mass. He defined mass as quantity of matter. This definition is not valid today.

At present the concept of matter includes not only atoms, but also massless and extremely light particles: the photon and neutrinos.

The masses of relativistic particles are not additive, while their energies are.

For instance, mass of the system of two photons: $m^2 = p^2 = (p_1 + p_2)^2$, where $p = (E, \boldsymbol{p})$, $p_1 = (E_1, \boldsymbol{p}_1)$, $p_2 = (E_2, \boldsymbol{p}_2)$, and we use c as a unit of velocity. The value of this mass depends on the relative momenta of photons:

$$m = 0\,, \quad \text{if} \quad \boldsymbol{p}_1 = \boldsymbol{p}_2\,;$$

$$m = 2E\,, \quad \text{if} \quad \boldsymbol{p}_1 = -\boldsymbol{p}_2\,.$$

9. Photosynthesis

The light from the sun is absorbed by vegetation on earth to produce carbohydrates via reaction of photosynthesis:

$$\text{light} + 6CO_2 + 6H_2O = 6O_2 + C_6H_{12}O_6\,.$$

The total energy of light required to produce one molecule of $C_6H_{12}O_6$ is about 4.9 eV. This does not mean that the photons are massive. They are massless, but the kinetic energy of photons is transformed into the rest energy of carbohydrates.

10. $E = mc^2$ in the Year of Physics

The Year of Physics is marked by revival of "famous equation" $E = mc^2$ and of "relativistic mass": $m_r = m_0/\sqrt{1 - v^2/c^2}$, where m_0 is "rest mass". The champion in this campaign is "Scientific American", the September 2004 issue of which is full with these notions.

Many other magazines, journals and books could be mentioned, *e.g.* "Physics World", January 2005, September 2005, October 2005. "New York Times" has recently joined the bandwagon [9]

In a brand new thousand pages thick book Penrose writes [10]:

"In a clear sense mass and energy become completely equivalent to one another, according to Einstein's most famous equation $E = mc^2$".

This book is addressed both to physicists and to "pedestrians". It is interesting to compare it with a classical monograph [11] coauthored by Penrose and Rindler and addressed to theoretical physicists, where photons

and neutrinos were referred to as massless particles and which had no trace of $E = mc^2$.

11. Who is guilty?

Of course, light-minded journalists.

But first of all, renowned professors of physics, who promote $E = mc^2$ and relativistic mass as authors, lecturers, and members of editorial boards.

They try to conform the prevailing opinions of ignorant readers, instead of educating them.

12. Niels Bohr on truth and clarity

Niels Bohr once said that truth and clarity are complementary. A true statement cannot be clear, and a clear one cannot be true. This maxim is valid for the deepest truths at the front line of science, but it should be applied with some reservation in the fields, such as Special Relativity, behind the front line, where everything is firmly established.

Still many authors consider that to be clear is "politically incorrect".

It seems that this belief strengthens the longstanding confusion which surrounds the relation between energy and mass.

13. What to do?

To reach a consensus in the community of experts in Relativity Theory on the concept of unique relativistically invariant mass, m.

Experts should discard from their writings the terms "rest mass" and "relativistic mass" and the famous but wrong formula

$$E = mc^2 \,.$$

The rest energy should be promoted:

$$E_0 = mc^2 \,.$$

14. Those who are indifferent

The ongoing struggle for and against $E = mc^2$ is considered by many of physicists as a kind of lilliputian war described by J. Swift in "Gulliver's Travels" [12]. They do not consider seriously both big-endians and small-endians. Their motto is: "All true believers break their eggs at the convenient end". This attitude with respect to $E = mc^2$ is sharply criticized by Gary Oas [13]. A quasi indifferent stand of Max Jammer [14] is in fact a slightly disguised propaganda of $E = mc^2$.

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