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CALIBRATION OF THE MULTI-FACTOR HJM MODEL FOR ENERGY MARKET*

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The purpose of this paper is to show that using the toolkit of interest rate theory, already well known in financial engineering as the HJM model [D. Heath, R. Jarrow, A. Morton, *Econometrica* **60**, 77 (1992)], it is possible to derive explicite option pricing formula and calibrate the theoretical model to the empirical electricity market. The analysis is illustrated by numerical cases from the European Energy Exchange (EEX) in Leipzig. The multi-factor *versus* one-factor HJM models are compared.

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1. Introduction

The environment surrounding world financial markets is changing drastically. Fluctuations are now so complex that they are beyond the scope of conventional economic theories. For example, the price of electricity is far more volatile than of other commodities normally noted for extremes volatility. The possibility of extreme price movements increases the risk of trading in electricity markets. A growing number of countries worldwide, including the US, have recently undertaken restructuring processes in their electric power sectors. Although the speed and scope of the reforms varies across countries, such liberalization process have been based on opening the

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electricity systems to competition wherever it was consider to be feasible, notably generation and retailing activities. The deregulating process have been accompanied by the introduction of competitive wholesale electricity markets, and power derivatives contracts, both OTC (over-the-counter) and exchange-traded, providing a variety of contract provisions to meet the needs of the electricity market participants [2, 3].

The oldest European Energy Exchange is the IPE, which was established in 1980 and already offered in 1981 gas and oil futures contracts. In 1990, were introduced options on gas and oil, then in 1997 natural gas futures contracts. As far as electricity is concerned, the first exchange to be established was the Nordpool, in 1993. Right from the start, it benefited from the fact that electricity in Scandinavia is in great part hydroelectricity, hence has the very valuable property of being storable. The non-storability of the other forms of electricity is an important explanatory factor of the spikes as those which were observed in the US in the ECAR market (Midwestern region close to Chicago) in June 1998 when the prices jumped up to the record value 7500 USD/MWh, at a smaller extent, in the North East during Summer 1997, and during the famous California's electricity crisis, December 2000–January 2001 [4]. Obviously, another reason for electricity price spikes is the lack of capacity. Real or strategically organized by major. Today, the Nordpool is a successful exchange, where the electricity players in Europe feel they can place their orders safely. In 1996 the SWEP (Swiss Electricity Price) index started being quoted by Dow Jones on the basis of the transactions taking place in Lufenburg, at the border of Germany and Switzerland. A fair amount of trading in Europe still uses the SWEP as the reference price index but the most two active exchanges are the Amsterdam Power Exchange (APX) created in early 1999 and the Leipzig Power Exchange (LPX) which was established during Summer 2000 and consolidates today the activities of the LPX and what used to be in 2001 the European Energy Exchange (EEX) in Frankfurt [5].

The energy derivatives market began with natural gas swaps, followed by basis swaps. Next it was time for electricity. For example, large power companies Cinergy and Energy were major players in the US on the New York Mercantile Exchange (NYMEX) futures and option market. Electric energy is not the only new commodity — other cutting edge contracts involve sulphur oxide (SO_2 or SO_x) emissions, nitrogen oxide (NO_2 or NO_x) emissions and weather [4]. Exchanges provide the infrastructure for price discovery and heading — to shift risk to those who want to take it. The exchange creates standard products and brings liquidity and more players into the market. On the other hand most OTC deals in the US are 25 MWh, which require 11–12 futures contracts and, therefore, the electronic trading community has replaced the futures market. Another reason is that the OTC market is also better able to meet a customer's specific needs. Therefore, it is clear that the OTC market presents challenges to exchanges.

Historical time series, implicit volatilities of quoted option prices, as well as the experience of professional traders and brokers, clearly indicate the presence of a volatility term structure in the electricity derivatives markets [6]. Our starting point is to represent the electricity forward market at date t by a forward price function $p(t,T)$, which may be interpreted as the forward price at date t of a hypothetical contract with delivery at date T *i.e.*, with an infinitesimal delivery period. In the electricity forward market, the underlying quantity is delivered as a flow during a specific future time period. This contract may be interpreted as a portfolio of hypothetical single-delivery contracts, hence the forward price follows from the function $f(t,T)$ by no-arbitrage. The main problem associated with the pricing of those derivatives is that the fundamental financial models were established for stocks and bonds and do not capture the unique features of electricity, in particular the non-storability (except for hydroelectricity), the seasonality and spikes of prices [5], the difficulties of transportation (existence of high voltage lines, constraints at the hubs imposed by the Kirchoff laws), not to mention the necessity for the European Community to define clear rules for cross-border electricity transmission.

2. HJM model for energy market

Suppose that on our energy market we have two kinds of discounting instruments: power forward contracts and savings bank account. For all $0 \le t \le T \le T^*$, $p(t,T)$ is the market price of 1 MWh at time t for power forward maturing at T and $N(t)$ is a savings account in EURO paying a constant interest rate $r > 0$ (which show us how the EURO price of 1MWh changes in time). It means that $e^{-rt}N(t)$ is the reciprocal EURO price at time t for electricity delivered within $[t, t+\Delta]$. In other words, we have to chosen a "domestic currency" unit at t as 1 MWh, delivered within a short interval immediately after t.

There exists a measure $\mathbb{P}$ [7], for which the discounted process $Z(t,T) = p(t,T)N^{-1}(t)$ is the martingale with respect to this measure, $p(t,T) = N(t)E_{\mathbb{P}}(N^{-1}(T)|\mathcal{F}_t) \quad \forall t \in [0,T]$. Now according to Heath–Jarrow–Morton (HJM) model [8, 9], we assume that the forward rate dynamics $f(t,T)$ for $t < T$ is given by the following stochastic differential equation

$$df(t,T) = \alpha(t,T)dt + \sum_{i=1}^{p} \sigma_i(t,T)dW_t^i\,,$$

and the price process of forward contract is $p(t,T) = e^{-\int_t^T f(t,s)ds}$, where $W_t = (W_t^1, \ldots, W_t^p)$ is a p-dimensional Brownian motion with the history of its dynamics $\mathcal{F}_t$ described by the past values of $W_s, s \in [0,t]$. We introduce a second discounting process $\Lambda(t) = e^{\int_0^t f(u,u)du}$. Choosing this process as a new *numeraire* we have to find an equivalent martingale (no-arbitrage) measure $\mathbb{Q}$. This measure is given by $d\mathbb{Q} = \frac{N(0)\Lambda(T)}{N(T)\Lambda(0)}\, d\mathbb{P}$, so for the above measure the discounted processes $\tilde{p}(t,T) = p(t,T)\Lambda^{-1}(t)$ and $\tilde{N}(t) = N(t)\Lambda^{-1}(t)$ are martingales.

According to the HJM framework, [10–13] we know that, when there is no possibility of arbitrage, function α is determined by function σ in following way $\alpha(t,T) = \sum_{i=1}^{p} \sigma_i(t,T) \int_t^T \sigma_i(t,u)du$ and the process $\tilde{p}(t,T)$ can be described as

$$d\tilde{p}(t,T) = \tilde{p}(t,T) \sum_{i=1}^{p} s_i(t,T)\, dW_t^i\,, \tag{1}$$

where $s_i(t,T) = -\int_t^T \sigma_i(t,u)du$. The second $\mathbb{Q}$-martingale will be defined later after formula (5) as:

$$d\tilde{N}(t) = \tilde{N}(t) \sum_{i=1}^{p} v_i(t)\, dV_t^i\,. \tag{2}$$

Suppose that we want to evaluate the European call option with strike price K in EURO and maturity date T, where power forward contract with maturity date $U > T$ is an underlying instrument. The crucial point is that option strike price is given in EURO, which is a "foreign currency" in the electricity market.

The option price at time point $t \leq T$ is defined by the following conditional expectation [8, 9]

$$C_t = \Lambda(t) \frac{E_{\mathbb{Q}}\left(\Lambda^{-1}(T)\left(p(T,U) - Ke^{-rT}N(T)\right)^{+} \Big|\, \mathcal{F}_t\right)}{e^{-rt}N(t)}.$$

Observe, that here $Ke^{-rT}N(T)$ is the strike price in MWh and the division by $e^{-rt}N(t)$ transforms from MWh to EURO.

Suppose that $\sum_{i=1}^{p}[s_i(t,T) - v_i(t)]$ is deterministic for all $t \in [0,T]$ and $T \in [0,T^*]$, then EURO price C_t at the time $t \in [0,T^*]$ for European call option with strike price K in EURO and time to maturity $T \in [t,T^*]$ written on power forward with time to maturity $U \in [T,T^*]$ is given by the following price formula:

$$C_t = P(t,U)\Phi(d_+) - e^{-r(T-t)}K\Phi(d_-)\,, \tag{3}$$

where $\Phi(d)$ is the normal distribution function and $P(t,U) = \frac{\tilde{p}(t,U)}{e^{-rt}\tilde{N}(t)} = \frac{p(t,U)}{e^{-rt}N(t)}$ is the EURO price at time t for the underlying forward and

$$d_{\pm} = \frac{\ln(P(t,U)/K) + r(T-t) \pm \frac{1}{2}\Sigma^2}{\Sigma},$$

$$\Sigma^2 = \int_t^T \sum_{i=1}^p \|s_i(u,U) - v_i(u)\|^2 \, du\,.$$

In order to check the price formula (3) we will follow [7], where the case $p = 1$ was considered. Using the new measure $d\mathbb{Q}' = \frac{\tilde{N}(T)}{\tilde{N}(t)} d\mathbb{Q}$ we obtain easily

$$C_t = E_{\mathbb{Q}'}\left(\left(e^{rt}\frac{\tilde{p}(T,U)}{\tilde{N}(T)} - e^{-r(T-t)}K\right)^+ |\mathcal{F}_t\right).$$

The process $\mathcal{P}(u,U) = e^{-ru}P(u,U) = \frac{p(u,U)}{N(u)}$ for $u \in [0,T]$, which is interpreted as the discounted EURO price at time u for electricity delivered at time U, posses the stochastic differential

$$d\mathcal{P}(u,U) = \mathcal{P}(u,U)\left(a(u)du + \sum_{i=1}^p b_i(u)dW_u^i\right), \tag{4}$$

where by the Ito formula $a(u) = \sum_{i=1}^p (\|v_i(u)\|^2 - v_i(u)s_i(u,U))$ and $b_i(u) = s_i(u,U) - v_i(u)$. If we write the solution of (4) as $\mathcal{P}(t,U)e^{L_u - L_t - \frac{1}{2}([L]_u - [L]_t)}$ for all $u \in [t,T]$, then

$$L_u = \int_0^u a(q)dq + \int_0^u \sum_{i=1}^p b_i(q)dW_q^i\,, \qquad [L]_u = \int_0^u \sum_{i=1}^p \|b_i(q)\|^2 \, dq\,,$$

so

$$C_t = E_{\mathbb{Q}'}\left(\left(P(t,U)e^{L_T - L_t - \frac{1}{2}([L]_T - [L]_t)} - e^{-r(T-t)}K\right)^+ |\mathcal{F}_t\right).$$

The variable $G = L_T - L_t$ is by the Girsanov theorem normally distributed with variance $\mathrm{Var}(G) = [L]_T - [L]_t = \Sigma^2$, so we obtain

$$C_t = E_{\mathbb{Q}'}\left(\left(P(t,U)e^{G - \frac{1}{2}\Sigma^2} - e^{-r(T-t)}K\right)^+ |\mathcal{F}_t\right),$$

where Q' is the equivalent to Q measure. Now the price formula (3) follows by a straight-forward derivation.

3. Calibration of the model

At the beginning we remind that the forward contract price in EURO is given by

$$P(t,T)=\frac{p(t,T)}{e^{-rt}N(t)}=\frac{\tilde{p}(t,T)}{e^{-rt}\tilde{N}(t)}, \tag{5}$$

where $\tilde{N}(t)$ is described as

$$d\tilde{N}(t)=\tilde{N}(t)\sum_{i=1}^{p}v_i(t)dV_t^i=\tilde{N}(t)\sum_{i=1}^{p}v_i(t)\left(\rho\, dW_t^{i,1}+\sqrt{1-\rho^2}\, dW_t^{i,2}\right)$$

and $W_t^1=(W_t^{1,1},\ldots,W_t^{p,1})$, $W_t^2=(W_t^{1,2},\ldots,W_t^{p,2})$ are independent p-dimensional Brownian motions.

Let us fix times $0=t_0<t_1<\ldots<t_m$ where $t_{j+1}-t_j=\Delta t$ and maturing times $0<T_1<\ldots<T_n$ where $T_{k+1}-T_k=\Delta T$ for some fixed Δt and ΔT.

In this section we assume that functions $\sigma_i(t,T)$ depend only from time to maturity $\sigma_i(t,T)=\sigma_i(T-t)$. It means that volatility functions should be estimated using forward contract prices with constant time to maturity $T_k=t+k\Delta T$ and it is easy to see that we can use only data in time points $t_j=j\Delta T$. Denote $\Delta f(t,T_k)=f(t+\Delta T,T_k)-f(t,T_k)$, so we can write that

$$\Delta f(t,t+k\Delta T)=\alpha(t,t+k\Delta T)\Delta T+\sum_{i=1}^{p}\sigma_i(t,t+k\Delta T)\Delta W_t^{i,1}\,.$$

The estimators of functions $\alpha(T-t)$ and $\sigma_i(T-t)$ are given by [14]

$$\begin{bmatrix}\hat{\alpha}(1\Delta T)\\ \hat{\alpha}(2\Delta T)\\ \vdots\\ \hat{\alpha}(p\Delta T)\end{bmatrix}=\frac{1}{m}\sum_{i=0}^{m-1}\frac{\boldsymbol{X}_j}{\Delta T}\,,$$

and

$$\begin{bmatrix}\hat{\sigma}_i(1\Delta T)\\ \hat{\sigma}_i(2\Delta T)\\ \vdots\\ \hat{\sigma}_i(p\Delta T)\end{bmatrix}=\frac{\vec{u}_i\cdot\sqrt{\lambda_i}}{\sqrt{\Delta T}}\,,$$

where $i=1,\ldots,p$, $j=0,\ldots,m-1$ and

$$\boldsymbol{X}_j=\begin{bmatrix}x_{j,1}\\ x_{j,2}\\ \vdots\\ x_{j,p}\end{bmatrix}=\begin{bmatrix}\Delta f(t_j,T_1)\\ \Delta f(t_j,T_2)\\ \vdots\\ \Delta f(t_j,T_p)\end{bmatrix}=\begin{bmatrix}f(t_j,t_j+1\Delta T)-f(t_{j-1},t_{j-1}+1\Delta T)\\ f(t_j,t_j+2\Delta T)-f(t_{j-1},t_{j-1}+2\Delta T)\\ \vdots\\ f(t_j,t_j+p\Delta T)-f(t_{j-1},t_{j-1}+p\Delta T)\end{bmatrix}$$

and for covariance matrix of observations vectors $\boldsymbol{X}_0, \boldsymbol{X}_1, \dots, \boldsymbol{X}_m$, values $\lambda_1, \lambda_2, \dots, \lambda_p$ are eigenvalues and $\vec{u}_1, \vec{u}_2, \dots, \vec{u}_p$ are eigenvectors of this matrix.

The estimators of functions $v_i(t)$ are given by

$$\begin{bmatrix} \hat{v}_i(1\Delta T) \\ \hat{v}_i(2\Delta T) \\ \vdots \\ \hat{v}_i(m\Delta T) \end{bmatrix} = \frac{\vec{w}_i \cdot \sqrt{\lambda_i}}{\sqrt{\Delta T}},$$

where $i, l = 1, \dots, p$, $k = 0, \dots, n-1$,

$$\boldsymbol{Y}_k = \begin{bmatrix} y_{1,k} \\ y_{2,k} \\ \vdots \\ y_{m,k} \end{bmatrix} = \begin{bmatrix} \frac{P(t_1,T_k)a_{1,k}}{e^{-r\Delta T}P(t_2,T_k)} \\ \frac{P(t_2,T_k)a_{2,k}}{e^{-r\Delta T}P(t_3,T_k)} \\ \vdots \\ \frac{P(t_{m-1},T_k)a_{m,k}}{e^{-r\Delta T}P(t_m,T_k)} \end{bmatrix},$$

and

$$a_{j,k} = \exp\left[\frac{1}{2}\sum_{i=1}^{p}\left(\sum_{l=1}^{k}\hat{\sigma}_i(l\Delta T)\Delta T\right)^2 \Delta T - \sum_{l=1}^{k}\left(\Delta f(t_j, T_l) - \hat{\alpha}(l\Delta T)\Delta T\right)\Delta T\right].$$

For covariance matrix of observations vectors $\boldsymbol{Y}_1, \boldsymbol{Y}_2, \dots, \boldsymbol{Y}_n$, values $\lambda_1, \lambda_2, \dots, \lambda_p$ are eigenvalues and $\vec{w}_1, \vec{w}_2, \dots, \vec{w}_p$ are eigenvectors of this matrix.

In order to justify the above formulas for estimators we begin with the fraction

$$\frac{\Delta\tilde{N}(t_j)}{\tilde{N}(t_j)} = \frac{\tilde{N}(t_j + \Delta T) - \tilde{N}(t_j)}{\tilde{N}(t_j)} = \frac{\tilde{N}(t_j + \Delta T)}{\tilde{N}(t_j)} - 1.$$

If we combine above equation with equation (5) we obtain

$$\begin{aligned} 1 + \frac{\Delta\tilde{N}(t_j)}{\tilde{N}(t_j)} &= \frac{\tilde{p}(t_j + \Delta T, T_k)e^{-rt_j}P(t_j, T_k)}{e^{-r(t_j+\Delta T)}P(t_j + \Delta T, T_k)\tilde{p}(t_j, T_k)} \\ &= \frac{\tilde{p}(t_{j+1}, T_k)}{\tilde{p}(t_j, T_k)} \frac{P(t_j, T_k)}{e^{-r\Delta T}P(t_{j+1}, T_k)}. \end{aligned}$$

Knowing that equation (1) have the solution

$$\tilde{p}(t, T) = \tilde{p}(0, T)\exp\left(-\frac{1}{2}\sum_{i=1}^{p}\int_0^t s_i^2(u, T)du + \sum_{i=1}^{p}\int_0^t s_i(u, T)dW_u^{i,1}\right),$$

we may write

$$\ln \frac{\tilde{p}(t_{j+1}, T_k)}{\tilde{p}(t_j, T_k)} = \sum_{i=1}^{p} \int_{t_j}^{t_{j+1}} s_i(u, T_k)\, dW_u^{i,1} - \frac{1}{2} \sum_{i=1}^{p} \int_{t_j}^{t_{j+1}} s_i^2(u, T_k)\, du\,.$$

Using the Euler scheme for approximating integrals we obtain a discretized form of this equation

$$\begin{aligned}
\ln \frac{\tilde{p}(t_{j+1}, T_k)}{\tilde{p}(t_j, T_k)} \;&\approx\; \sum_{i=1}^{p} s_i(t_j, T_k)\Delta W_t^{i,1} - \frac{1}{2}\sum_{i=1}^{p} s_i^2(t_j, T_k)\Delta T \\
&= -\sum_{i=1}^{p} \int_{t_j}^{T_k} \sigma_i(t_j, u)\Delta W_t^{i,1} du + \frac{1}{2}\sum_{i=1}^{p} [\int_{t_j}^{T_k} \sigma_i(t_j, u) du]^2 \Delta T \\
&= \frac{1}{2}\sum_{i=1}^{p} (\int_{t_j}^{T_k} \sigma_i(t_j, u) du)^2 \Delta T - \int_{t_j}^{T_k} (\Delta f(t_j, u) - \alpha(t_j, u)\Delta T) du \\
&\approx \frac{1}{2}\sum_{i=1}^{p} \left(\sum_{l=1}^{k} \hat{\sigma}_i(l\Delta T)\Delta T\right)^2 \Delta T - \sum_{l=1}^{k} (\Delta f(t_j, T_l) - \hat{\alpha}(l\Delta T)\Delta T)\Delta T\,.
\end{aligned}$$

From equation (2) we conclude that $y_k^j = 1 + \sum_{i=1}^{p} v_i(t_j)\Delta V_t^i$. So for every k vector $\boldsymbol{Y}_k$ is normally distributed with covariance matrix Σ. Notice that every covariance matrix is symmetrical and it could be decomposed in following way $\Sigma = W \Lambda W'$, where the matrix W is a matrix of eigenvectors and the matrix Λ is a diagonal matrix of eigenvalues. Using PCA (Principal Component Analysis) we calculate volatility functions for p components and it is described as $\hat{v}_i(j\Delta T) = (w_{i,l}\sqrt{\lambda_i})/\sqrt{\Delta T}$, see [14].

The moment estimator $\hat{\rho}$ of the correlation parameter is given by the formula

$$\hat{\rho} = \frac{1}{mp} \sum_{k=1}^{p} \sum_{j=0}^{m-1} \frac{(y_{j,k} - 1)(a_{j,k} - 1)}{\sum_{i=1}^{p} s_i(t_j, T_k) v_i(t_j)\Delta T}\,, \tag{6}$$

where

$$y_{j,k} = \frac{P(t_j, T_k) a_{j,k}}{e^{-r\Delta T} P(t_{j+1}, T_k)}\,,$$

$$a_{j,k} = \exp\left[\frac{1}{2}\sum_{i=1}^{p} \left(\sum_{l=1}^{k} \hat{\sigma}_i(l\Delta T)\Delta T\right)^2 \Delta T - \sum_{l=1}^{k} (\Delta f(t_j, T_l) - \hat{\alpha}(l\Delta T)\Delta T)\Delta T\right]$$

and $s_i(t_j, T_k) = -\sum_{l=1}^{k} \hat{\sigma}_i(l\Delta T)\Delta T$.

We consider two equations

$$d\tilde{N}(t) = \tilde{N}(t)\sum_{i=1}^{p} v_i(t)\left(\rho dW_t^{i,1} + \sqrt{1-\rho^2}dW_t^{i,2}\right),$$

and

$$d\tilde{p}(t,T) = \tilde{p}(t,T)\sum_{i=1}^{p} s_i(t,T)dW_t^{i,1}.$$

Let us take the fraction

$$\frac{\Delta\tilde{N}(t_j)\Delta\tilde{p}(t_j,T_k)}{\tilde{N}(t_j)\tilde{p}(t_j,T_k)} = \frac{(\tilde{N}(t_{j+1})-\tilde{N}(t_j))(\tilde{p}(t_{j+1},T_k)-\tilde{p}(t_j,T_k))}{\tilde{N}(t_j)\tilde{p}(t_j,T_k)}.$$

We easily see that for every j,k

$$\boldsymbol{E}\left(\frac{\Delta\tilde{N}(t_j)\Delta\tilde{p}(t_j,T_k)}{\tilde{N}(t_j)\tilde{p}(t_j,T_k)\Delta T\sum_{i=1}^{p} s_i(t_j,T_k)v_i(t_j)}\right) = \rho,$$

and

$$\frac{\Delta\tilde{N}(t_j)\Delta\tilde{p}(t_j,T_k)}{\tilde{N}(t_j)\tilde{p}(t_j,T_k)} = (y_{j,k}-1)(a_{j,k}-1).$$

4. Simulation for the EEX data

Let us estimate functions $\sigma_i(T-t)$, $v_i(t)$ and ρ for data from European Energy Exchange (EEX) in Leipzig. We consider 34 forward contracts with a monthly delivery period from January 2003 to October 2005. Our observations start 02.01.2003 and finish 08.04.2005. We assume that ΔT equals one month. Having historical contract prices $P(t,T)$ in EURO, we may calculate the forward rate curve $f(t,T)$ in points $0 = t_0 < t_1 < \ldots < t_m$ and $0 < T_1 < \ldots < T_n$ from the formula $f(t,T) = -(\Delta \ln P(t,T))/\Delta T$ and next we can estimate and interpolate all volatility functions.

All volatility functions are estimated only for historical data, so if we want to know the future value of volatility $v_i(t)$ we must observe its historical shape and fit some deterministic function [15]. We should remember that energy market is periodic. The results of MATLAB calculation are showed below in Table I and Table II. The estimated value of correlation parameter is $\rho = -0.3474$.

In Table I and Table II we have estimated values of multifactor parameters. For volatility function σ we have six factors and for function v we have only five significant factors.

TABLE I

Estimated values of multi-factor volatility function $\sigma_i(T-t)$ for different times to maturity.

	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$
$\sigma_i(1\Delta T)$	−0.2011	−1.1385	0.8284	3.5067	1.5739	0.9480
$\sigma_i(2\Delta T)$	−0.6899	−0.5585	−1.9587	0.5678	−3.2151	2.6061
$\sigma_i(3\Delta T)$	−1.1149	−1.2068	−0.2591	−1.4772	1.2165	−3.3488
$\sigma_i(4\Delta T)$	−0.8365	0.1591	2.2509	−1.3157	−0.1959	3.7407
$\sigma_i(5\Delta T)$	−0.8619	1.4194	0.6159	1.7857	−1.5967	−2.9155
$\sigma_i(6\Delta T)$	−0.5783	1.0433	−1.6256	0.1932	3.6058	1.9518

TABLE II

Estimated values of multi-factor volatility function $v_i(t)$ for different time points.

	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$
$v_i(t_{m-6})$	−0.1610	−0.0527	−0.0473	−0.1593	−0.1443
$v_i(t_{m-5})$	0.0935	−0.1991	0.0842	0.0995	0.0564
$v_i(t_{m-4})$	0.0516	0.0682	0.1102	−0.1621	0.0850
$v_i(t_{m-3})$	0.0768	0.0569	−0.0041	0.0708	−0.1592
$v_i(t_{m-2})$	0.0934	0.0698	−0.1887	0.0214	0.2057
$v_i(t_{m-1})$	0.0127	−0.1019	−0.1418	−0.1605	−0.1897
$v_i(t_m)$	−0.1382	−0.0602	0.2699	0.2926	0.0921

Finally, we could compare our results with results which were also calculated for data from EEX (Table III), but for one-factor HJM model and constant parameters σ, v, ρ.

TABLE III

Estimated values of constant parameters for one-factor HJM model (they were obtained in [7]).

Parameter	Estimate
σ	2.063
v	0.5177
ρ	−0.2497

5. Conclusions

Looking at all results we can draw a simple conclusion that the use of the multi-factor model is preferable because it much better describes reality. If there have been only one factor, then the PCA analysis would show us that it is true. But in Table I and Table II we clearly see that there is more than one significant factor.

In comparison with [7], we do not assume that parameters are constant because values of parameters σ and v depend on the time. In Fig. 1 we see that we could fit some deterministic function to estimated values of volatility v. It is interesting that this function is periodic with period one year. This fact remind us of periodic nature of electricity market and suggests that in presented HJM model the volatility v may be described by some periodic function.

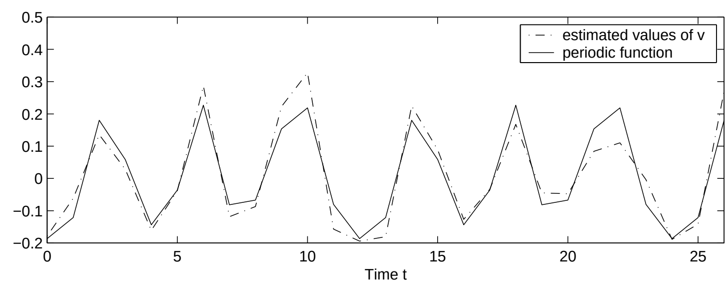

Fig. 1. Estimated values of function $v_3(t)$ and fitted periodic function with period one year.

Both one-factor and multi-factor models exhibit a term structure of the implied volatility (plug-in volatility [7]). As we can see in Fig. 2 the distance between two volatilities grows up with time to maturity. This difference appears also in Fig. 3. All results indicate that using multi-factor model we could better describe market. The difference between two models is significant what indicates that extended multi-factor HJM model could be more useful.

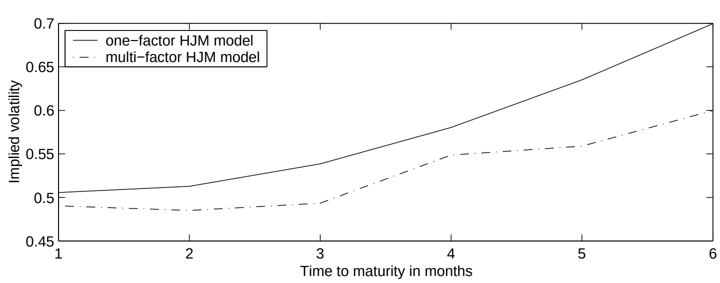

Fig. 2. Implied volatility (plug-in volatility) for estimated parameters ($T = U$).

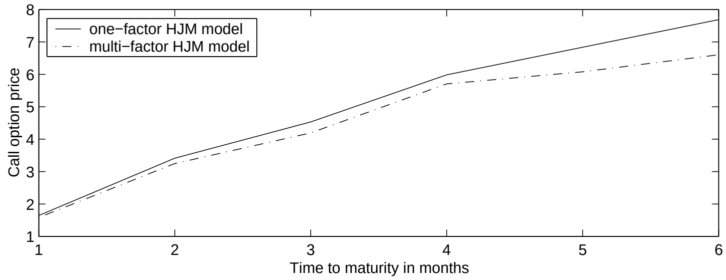

Fig. 3. Call option price under different models with strike price $K = 35$.

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REFERENCES

[1] D. Heath, R. Jarrow, A. Morton, *Econometrica* **60**, 77 (1992).

[2] A. Eydeland, H. Geman, *Risk* **11**, 71 (1998).

[3] E.S. Schwartz, E.S. Lucia, *Review Derivatives Res.* **5**, 5 (2002).

[4] A. Weron, R. Weron, *Power Exchange; Tools for Risk Management*, Wrocław, CIRE, 2000 (in Polish).

[5] R. Weron, *Modeling and Forecasting Loads and Prices in Deregulated Electricity Markets*, Warsaw, ARE, 2006.

[6] R. Weron, I. Simonsen, P.Wilman, *The Application of Econophysics*, Ed. H. Takeyasu, Springer-Verlag, Tokyo 2004, p. 182.

[7] J. Hinz, L. Grafenstein, M. Verschuere, M. Wilhelm, *Pricing Electricity Risk by Interest Rate Methods*, submitted to *Quantitative Finance*.

[8] M. Baxter, A. Rennie, *Financial Calculus*, Cambridge, University Press, 1996.

[9] M. Musiela, M. Rutkowski, *Martingale Methods in Financial Modeling*, Berlin 2005, Springer-Verlag.

[10] A. Brace, M. Musiela, *Math. Finance* **4**, 259 (1994).

[11] A. Brace, D. Gatarek, M. Musiela, *Math. Finance* **7**, 127 (1997).

[12] F. Jamshidian, *Finance Stochast.* **1**, 293 (1997).

[13] M. Musiela, M. Rutkowski, *Finance Stochast.* **1**, 261 (1997).

[14] K. Sulzyk, A. Palczewski, *Rynek Terminowy* **19**, 124 (2003) (in Polish).

[15] P. Sztuba, A. Weron, *Appl. Math. (Warsaw)* **28**, 211 (2001).