FEATURES OF THE RIGIDITY SPECTRUM OF GALACTIC COSMIC RAY INTENSITY DURING THE RECURRENT FORBUSH EFFECT*

A. Wawrzynczak[a], M.V. Alania[b,c], R. Modzelewska[b]

[a]Institute of Computer Science, University of Podlasie
Sienkiewicza 51, 08-110 Siedlce, Poland

[b]Institute of Mathematics and Physics, University of Podlasie
3 Maja 54, 08-110 Siedlce, Poland

[c]Institute of Geophysics, Georgian Academy of Sciences, Tbilisi, Georgia

(Received February 21, 2006)

Temporal changes of the rigidity spectrum of the recurrent Forbush effect (16–30 June, 2003) have been studied using the data of the worldwide network of neutron monitors. The rigidity spectrum is soft at the beginning and at the end phases of the recurrent Forbush effect, and it is hard in the minimum phase. The steady-state model based on the Parker's transport equation is able to explain the changes of the rigidity spectrum of the recurrent Forbush effect of the galactic cosmic ray intensity. An increase of the power spectral density in the lower frequency area of the energy range of the interplanetary magnetic field turbulence ($\sim 10^{-6}$– 10^{-5} Hz) causes the hardening of the rigidity spectrum of the recurrent Forbush effect of the GCR intensity.

PACS numbers: 96.40.Cd, 96.50.Ci, 96.50.Fm

1. Introduction

The transient disturbances in the interplanetary space, usually are accompanied by the short period decreases — Forbush effects of the galactic cosmic ray (GCR) intensity [1–5]. Generally, two types of the Forbush effects of the GCR intensity are distinguished — sporadic and recurrent. The sporadic Forbush effects are characterized with a rapid decrease phase during the one–two days and by a subsequent recovery phase lasting for few days. They are caused by the transient disturbances associated with the powerful solar flares. Recurrent Forbush effects have the approximately symmetric

* Presented at the XVIII Marian Smoluchowski Symposium on Statistical Physics, Zakopane, Poland, September 3–6, 2005.

decrease and recovery phases and last 8–12 days; they are caused by the rotating disturbances in the interplanetary space associated with the long-lived active heliolongitudinal zones on the Sun. Normally, the amplitudes of the recurrent Forbush effects are lower than the amplitudes of the sporadic Forbush effect. The Forbush effects give an information about the processes in the Sun's atmosphere and in the interplanetary space both for the short periods (few days) and for the 11-year cycles [2,5]. It was assumed [2] that the 11-year variation of the GCR intensity could be the result of the accumulation of a few Forbush decreases caused by the transient disturbances in the interplanetary space. The Forbush effect of the GCR intensity is a non stationary process and for its modeling the Parker's time-dependent transport equation should be used. Several theoretical papers are devoted to the problems of the Forbush effect's time profile, dependence of the amplitudes and the duration of the Forbush effect *versus* the distance. Nishida [6] solved one dimensional (1D) non stationary model without adiabatic cooling suggesting an enhanced scattering of GCR particles behind the moving interplanetary shock. 1D time-dependent model with an adiabatic cooling was considered by Perko [7] who used a radial independence diffusion coefficient in the transient disturbances. On the other hand, to model the Forbush effect Chih and Lee, [8] and Lockwood *et al.* [9] suggested weakening disturbances in the interplanetary space. Kadakura and Nishida [10] solved two dimensional time-dependent model with the enhanced scattering and convection, and the enhanced magnitude of the interplanetary magnetic field behind the shock front. Le Roux and Potgieter [11] developed an axially-symmetric time dependent drift model with a simulated wavy neutral sheet assuming that Forbush effects were caused by propagating regions of enhanced scattering and diminished drift. An important information is obtained from the temporal changes of the rigidity spectrum of the Forbush effect. Elsewhere, [12–15] it was shown that the change of the rigidity R spectrum $\delta D(R)/D(R)$ of the Forbush effects of the GCR intensity is related with the changes in the energy range of the interplanetary magnetic field (IMF) turbulence. The similar dependence of the rigidity spectrum on the IMF's turbulence was obtained in papers [16] for the 11-year variation of the GCR intensity. Put differently, a quasi-linear theory of cosmic ray propagation is a satisfactory approximation for the interplanetary space for both the short and long period modulation of the GCR intensity. In this paper we model the recurrent Forbush effect of the GCR intensity supposing that the diffusion coefficient alternates only due to the variations of the IMF's turbulence during the recurrent Forbush effect. To show this relationship uniquely we consider the model of the isotropic diffusion of the GCR in the interplanetary space, *i.e.* a modulation of GCR is determined by the processes — convection, diffusion and energy changes of the GCR particles.

We assume that the diffusion coefficient changes *versus* the heliolongitudes and heliolatitudes only due to the variations in the energy range of the IMF turbulence in the vicinity of the interplanetary space, where the recurrent Forbush effect takes place. We compare theoretically expected results with the experimental data of the recurrent Forbush effect of the GCR intensity registered by neutron monitors.

2. Experimental data and methods

To study the temporal changes of the rigidity spectrum of the recurrent Forbush effect of the GCR intensity the daily average experimental data of the neutron monitors for the period of 16–30 June 2003 were used.

The daily average intensity for each neutron monitor during 14–16 June was accepted as a reference level (100%). The amplitudes $\delta J_i/J_i$ of the GCR intensity variation were calculated, as: $\delta J_i/J_i = (N_i - N_0)/N_0$, where N_i is daily average count rate of the i-th neutron monitor. In Fig. 1 the temporal changes of the GCR intensities, smoothed over three days running period (amplitudes of the Forbush effect) as observed by Irkutsk (I), Kiel (K) and Rome (R), neutron monitors for 16–30 June 2003, are presented. The rigidity spectrum of the Forbush effect was assumed [3,17].

$$\frac{\delta D(R)}{D(R)} = \begin{cases} AR^{-\gamma}\,, & R \le R_{\max}\,, \\ 0\,, & R > R_{\max}\,, \end{cases} \tag{1}$$

where $R_{\max}$ is the upper limiting rigidity beyond which the Forbush effect of GCR intensity vanishes and A is the amplitude of the Forbush effect in the heliosphere. The amplitude $\delta J_i/J_i$ of the GCR intensity variation at any point of observation (by i-th neutron monitor) with the geomagnetic cut off rigidity R_i and the average atmospheric depth h_i is defined as [3,17].

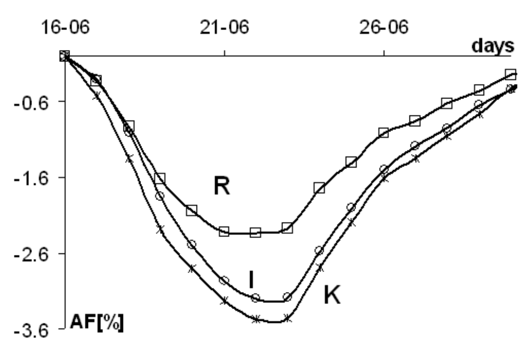

Fig. 1. Temporal changes of the GCR intensities for period 16–30 June 2003, Irkutsk (I), Kiel (K) and Rome (R).

$$\frac{\delta J_i}{J_i} = \int\limits_{R_i}^{R_{\max}} \frac{\delta D(R) W_i(R, h_i)\, dR}{D(R)}\,, \tag{2}$$

where $W_i(R, h_i)$ is the coupling coefficient. The expression (2) for the power type rigidity spectrum type (1) can be rewritten in the form of

$$\frac{\delta J_i}{J_i} = A_i \int\limits_{R_i}^{R_{\max}} R^{-\gamma} W_i(R, h_i)\, dR\,. \tag{3}$$

The amplitude A_i of the Forbush effect in the heliosphere must be the same for the arbitrary neutron monitor in the scope of the accuracy of calculations.

In order to find the temporal changes of the rigidity spectrum exponent γ of the recurrent Forbush effect a minimization of the expression $\phi = \sum_{i=1}^{n} (A_i - A)^2$ was performed [13–15]. Here A represents an average amplitude, $A = \frac{1}{n}\sum_{i=1}^{n} A_i$ calculated for n neutron monitors with different cut-off rigidities R_i. The values of the integral $\int_{R_i}^{R_{\max}} R^{-\gamma} W_i(R, h_i)\, dR$ for different magnitudes of $R_{\max}$ (from 30 GV up to 200 GV with a step of 10 GV) and γ (from 0 to 2 with a step of 0.05) were found using the method presented by Yasue *et al.*, [17]. The minimization of ϕ were carried out with respect to γ and $R_{\max}$ using the daily amplitudes of the recurrent Forbush effect for eight($n = 8$) neutron monitors with different geomagnetic cut-off rigidities. These were: Apatity of rigidity 0.65 GV, Climax — 3.03 GV, Irkutsk — 3.66 GV, Kiel — 2.29 GV, Moscow — 2.46 GV, Oulu — 0.81 GV, Thule — 0 GV and Yakutsk — 1.7 GV. We found that a minimum of the function ϕ for each day's amplitude of the recurrent Forbush effect (16–30 June, 2003) for the particular value of γ is reached at $R_{\max} \geq$ 90–100 GV.

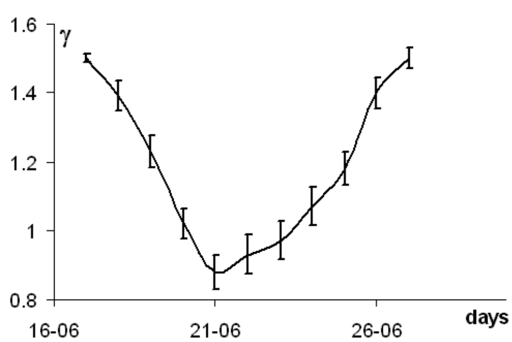

Fig. 2. Temporal changes of the rigidity spectrum exponent γ of the recurrent Forbush effect of 16–30 June 2003.

Based on that, the upper limiting rigidity beyond which the Forbush effect of GCR intensity vanishes is accepted, $R_{\rm max} = 100\,{\rm GV}$. Results of calculations of $\gamma_{\rm exp}$ for $R_{\rm max} = 100\,{\rm GV}$ are presented in Fig. 2.

Fig. 2. shows that the rigidity spectrum is softer at the beginning and at the end phases of the Forbush effect ($\gamma \approx 1.5$) in comparison with the minimum phase of the Forbush effect ($\gamma \approx 0.9$).

3. Theoretical model and discussion

The amplitudes of the recurrent Forbush effects of GCR intensity are rather small ($\leq$ 3–4% in the energy range of 10 GeV) and their duration is relatively long (10–12 days). Therefore, in order to describe the recurrent Forbush effect of the GCR intensity we use the steady-state approximation ($\partial N/\partial t = 0$) of Parker's transport equation [18]:

$$\nabla_i(K_{ij}\nabla_j N) - \nabla_i(U_i N) + \frac{1}{3}\frac{\partial}{\partial R}(NR)(\nabla_i U_i) = 0\,, \tag{4}$$

where N and R are density and rigidity of cosmic ray particles, respectively; K_{ij} is the anisotropic diffusion tensor of cosmic rays, U_i — solar wind velocity. We set up the dimensionless density $f = N/N_0$ and distance $\rho = r/r_0$. N and N_0 are density in the interplanetary space and in the local interstellar medium (LISM), respectively; r is the distance from the Sun and r_0 the region of the modulation. The density N_0 of GCR is $N_0 = 4\pi I_0$, where the intensity I_0 in the LISM [19] has the form:

$$I_0 = \frac{21.1T^{-2.8}}{1 + 5.85T^{-1.22} + 1.18T^{-2.54}}.$$

Here T stands for kinetic energy in GeV, $T = \sqrt{R^2 + 0.938^2} - 0.938$.

Eq. (4) for the dimensionless variables f and ρ in the spherical coordinate system (ρ, θ, φ) can be written as:

$$\begin{aligned} A_1\frac{\partial^2 f}{\partial\rho^2} \;+\; A_2\frac{\partial^2 f}{\partial\theta^2} + A_3\frac{\partial^2 f}{\partial\varphi^2} + A_4\frac{\partial^2 f}{\partial\rho\partial\theta} + A_5\frac{\partial^2 f}{\partial\theta\partial\varphi} + A_6\frac{\partial^2 f}{\partial\rho\partial\varphi} \\ +\; A_7\frac{\partial f}{\partial\rho} + A_8\frac{\partial f}{\partial\theta} + A_9\frac{\partial f}{\partial\varphi} + A_{10}f + A_{11}\frac{\partial f}{\partial R} = 0\,. \end{aligned} \tag{5}$$

The coefficients $A_1, A_2, \ldots, A_{11}$ are functions of the spherical coordinates (ρ, θ, φ) and rigidity R of GCR particles.

The disturbances responsible for the recurrent Forbush effect rotate around the Sun during 27 days with the linear velocity of $\approx 400\,{\rm km/s}$ at the Earth orbit, which is much greater than the linear orbital velocity of the Earth, $\approx 30\,\mathrm{km/s}$. Both velocities have the same directions. In effect, the disturbances, due to the rotation, reach the Earth and a relative motion of the Earth in the disturbances is a reason of the recurrent Forbush effect. Duration of the recurrent Forbush effect depends on the size of the disturbances. The change of the density of GCR *versus* the heliolongitudes at 1 AU (in the disturbed vicinity of the interplanetary space) is a supposed steady-state model of the recurrent Forbush effect of GCR (one day corresponds to 13.33 degrees of φ). In papers [12–15] we have shown that the temporal changes of the rigidity R spectrum $\delta D(R)/D(R)$ of the Forbush effect, found based on the theoretical modeling and the neutron monitors experimental data, are related to the temporal changes in the energy range of the IMF turbulence ($10^{-6} < F < 10^{-5}\,\mathrm{Hz}$). In order to delineate the relationship between the changes of the rigidity spectrum and the IMF's turbulence we consider the model of the isotropic diffusion of GCR in the interplanetary space, *i.e.* a modulation of GCR is determined by the processes — convection, diffusion and energy changes of GCR particles. We assume that the diffusion coefficient changes *versus* the heliolongitudes and heliolatitudes only due to the changes in the energy range of the IMF turbulence in the vicinity of the interplanetary space, where the recurrent Forbush effect takes place. According to the quasi-linear theory [20, 21] the diffusion coefficient K among the spatial coordinates depends on the rigidity R of the cosmic ray particles as, $K \propto R^{\alpha}$, where $0 \leq \alpha \leq 2$. The parameter α is equal to $2-\nu$; ν is an exponent of the power spectral density (PSD) of the IMF turbulence (PSD$\propto F^{-\nu}$, F is a frequency). We assume the dependence of the diffusion coefficient on the spatial coordinates to be:

$$
\begin{aligned}
&K = K_0 K(r) K(R,\varphi)\,,\\
&K_0 = 4.5\times 10^{22}\mathrm{cm}^2/\mathrm{s}\,,\\
&K(r) = \left(1+\frac{r}{1\mathrm{AU}}\right)\,,\\
&K(R,\alpha(r,\varphi)) = R^{\alpha(r,\varphi)}\,,\\
&\alpha(r,\varphi) = 1.2 + 0.35\,(\cos\varphi - 0.2)\exp -\frac{\rho-0.01}{0.03}\,,\\
&U = 400\,\mathrm{km/s}\,.
\end{aligned}
$$

The expression $\alpha(r,\varphi)$ describes the changes in the energy range of the IMF's turbulence in the vicinity of the interplanetary space being responsible for the recurrent type of the Forbush effect of the GCR intensity. In the adopted model the disturbed vicinity in the interplanetary space is restricted by the angles $\varphi \in (80^\circ, 280^\circ)$ and $\theta \in (60^\circ, 120^\circ)$ with respect to heliolongitudes and heliolatitudes, correspondingly, and gradually vanishes with the distance $r < 10\,\mathrm{AU}$.

In Fig. 3 the changes of the diffusion coefficient K *versus* the radial distance r and heliolongitudes φ are presented. Fig. 3 shows that the change of the diffusion coefficient *versus* the heliolongitudes is about 3-times and according to our model it is responsible for the recurrent Forbush effect of GCR intensity.

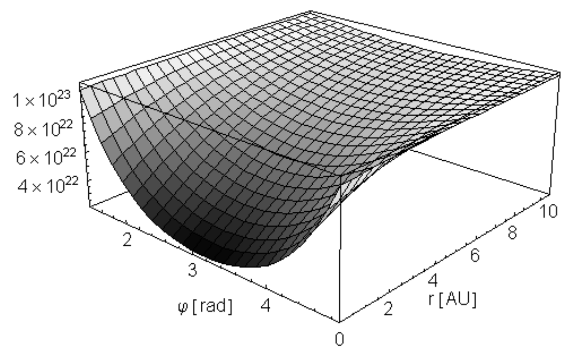

Fig. 3. Changes of the diffusion coefficient K for the rigidity of 10 GV *versus* the radial distance and heliolongitudes.

The differential Eq. (5) was transformed to the algebraic system of equations using the finite difference scheme; then this algebraic system was solved numerically by the iteration method. The number of equations in the linear algebraic system is equal to $N \times M \times L = 57 \times 79 \times 360 = 1621080$, *i.e.* there are: 57 steps in the distance r ($r = i \times h_1$, h_1 is an alternating step and $i = 1, 2, \ldots, N$), 79 steps in the zenith (θ) angle ($\theta = j \times h_2$, h_2 is an alternating step and $j = 1, 2, \ldots, M$) and 360 steps in the azimuthally (φ) direction ($\varphi = k \times h_3$, h_3 is the constant step of one degree and $k = 1, 2, \ldots, L$). A grid is denser in the distances less than 10 AU and near the helioequatorial regions. The algebraic system was solved for the boundary conditions as:

$$
\begin{aligned}
&N|_{r=100\,\mathrm{AU}} = 1\,,\\
&\frac{\partial N}{\partial \theta}|_{\theta=0^\circ} = \frac{\partial N}{\partial \theta}|_{\theta=180^\circ} = 0\,,\\
&N|_{R=100\mathrm{GV}} = 1\,,\\
&\frac{\partial N}{\partial \varphi}|_{\varphi=\varphi_1} = \frac{\partial N}{\partial \varphi}|_{\varphi=\varphi_{L+1}}\,,\\
&\frac{\partial N}{\partial \varphi}|_{\varphi=\varphi_{-1}} = \frac{\partial N}{\partial \varphi}|_{\varphi=\varphi_{L-1}}\,,
\end{aligned}
$$

In Fig. 4 the expected change of the density of GCR *versus* the heliolongitudes at the distance of 1 AU (Earth orbit) is presented. The expected amplitude of the Forbush effect is about 3% for rigidity 10 GV of GCR and is comparable with the neutron monitors experimental data (Fig. 1).

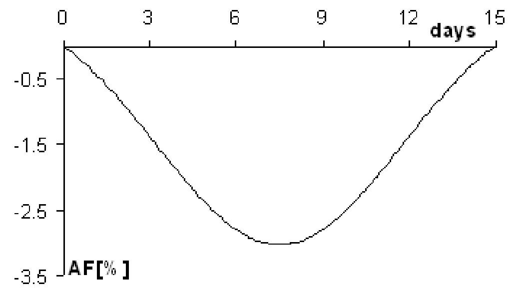

Fig. 4. Changes of the expected amplitude of the Forbush effect (AF[%]) for rigidity 10 GV.

In Fig. 5 are shown the changes of the $\alpha(r,\varphi)$ at the distance of 1 AU, the expected rigidity spectrum exponent γ calculated by the experimental data (dashed line, the same results shown in Fig. 2) and based on the numerical solution according to the expression:

$$\frac{\delta D(R)}{D(R)} = \frac{1}{f}\frac{df}{dR} \propto R^{-\gamma}\,.$$

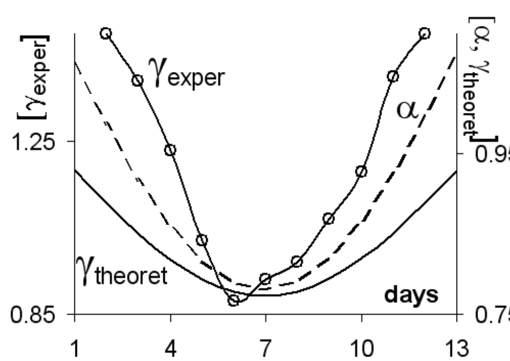

Fig. 5. Changes of the expected exponent $\gamma_{\rm theor}$ (solid line) and calculated, from the neutron monitor experimental data, exponent $\gamma_{\rm exp}$ (spotted line, the same results shown in Fig. 2) of the rigidity spectrum of the recurrent Forbush effect and the function $\alpha(r,\varphi)$ (dashed line) *versus* the heliolongitudes.

There is a good correlation between $\alpha(r,\varphi)$, $\gamma_{\rm theor}$ and $\gamma_{\rm exp}$. The dashed curve (changes of the exponent $\gamma_{\rm exp}$ corresponding to the experimental data of neutron monitors) shows the similar changes as the exponent $\gamma_{\rm theor}$. In other words, an expected rigidity spectrum exponent γ is proportional to the $\alpha(r,\varphi)$ when the recurrent type Forbush effect is formed by the changes of the IMF's turbulence; diffusion coefficient depends on the cosmic ray particles rigidity *versus* the heliolongitudes, as $R^{\alpha(r,\varphi)}$. Then, as far as $\alpha(r,\varphi) = 2 - \nu(r,\varphi)$, the increase of the exponent $\nu(r,\varphi)$ of the PSD of the IMF turbulence (in the range of $\sim 10^{-6} - 10^{-5}$ Hz) causes the decrease of the exponent $\gamma_{\rm theor}$, *i.e.* the rigidity spectrum of the Forbush effect is hardening. Thus, the proposed steady-state model is able to explain the relationship between the changes of the rigidity spectrum of the recurrent Forbush effect of GCR intensity and the condition of the IMF turbulence.

4. Conclusion

1. The rigidity spectrum of the recurrent Forbush effect of the GCR intensity (16–30 June, 2003) is soft at the beginning phase ($\gamma_{\rm exp} \approx 1.5 \pm 0.05$) and gradually becomes relatively hard ($\gamma_{\rm exp} \approx 0.9 \pm 0.1$) at the minimum phase; then it again steadily becomes soft ($\gamma_{\rm exp} \approx 1.5 \pm 0.09$) at the end phase of the Forbush effect.

2. The changes of the exponents $\gamma_{\rm theor}$ and $\gamma_{\rm exp}$ are in good correlation among themselves and with the changes of the parameter α. In effect, the proposed steady-state model describing the recurrent Forbush effect of the GCR intensity due to changes in the energy range of IMF turbulence is well-suited to explain the behavior of the rigidity spectrum found from the neutron monitors experimental data.

REFERENCES

[1] S.E. Forbush, *J. Geophys. Res.* **59**, 525 (1954).

[2] J.A. Lockwood, *J. Geophys. Res.* **65**, 19 (1960).

[3] L.I. Dorman, *Cosmic Rays Variations and Space Exploration*, *Nauka*, Moscow 1963.

[4] J.A. Lockwood, W.R. Webber, Proceedings of 20th International Cosmic Ray Conference, Moscow, 3, 283, 1987.

[5] H.V. Cane, *Space Sci. Rev.* **93**, 49 (2000).

[6] A. Nishida, *J. Geophys. Res.* **87**, 6003 (1982).

[7] J.S. Perko, *J. Geophys. Res.* **91**, 13 (1986).

[8] P.P. Chih, M.A. Lee, *J. Geophys. Res.* **91**, 2903 (1986).
[9] J.A. Lockwood, W.R. Webber, J.R. Jokipii, *J. Geophys. Res.* **91**, 2851 (1986).
[10] A. Kadokura, A. Nishida, *J. Geophys. Res.* **92**, 8502 (1987).
[11] J.A. le Roux, M.S. Potgieter, *Astron. Astrophys.* **243**, 531 (1991).
[12] M.V. Alania, A. Wawrzynczak, A. Bishara, A. Darwish, Proceedings of 29th ICRC, Pune, India, 2005.
[13] M.V. Alania, A. Wawrzynczak, *Acta Phys. Pol. B* **35**, 1551 (2004).
[14] A. Wawrzynczak, M.V. Alania, *Acta Phys. Pol. B* **36**, 1847 (2005).
[15] A. Wawrzynczak, M.V. Alania, *Adv. in Space Res.* **35**, 682 (2005).
[16] M.V. Alania, K. Iskra, *Adv. Space Res.* **16**, 241 (1995).
[17] S. Yasue, *et al.* "Coupling Coefficients of Cosmic Rays Daily Variations for Neutron Monitors", 7, Nagoya 1982.
[18] E.N. Parker, *Planet. Space. Sci.* **13**, 9 (1965).
[19] W.R. Webber, J.A. Lockwood, *J. Geophys. Res.* **106**, 323 (2001).
[20] J.R. Jokipii, *Rev. Geophys. Space Phys.* **9**, 21 (1971).
[21] I.N. Toptygin, "Cosmic Rays in Interplanetary Magnetic Fields", Redel Publishing Company, Dordrecht 1985.