LEPTON GENERATION-WEIGHTING FACTORS AND NEUTRINO MASS FORMULA: *Addendum**

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(Received December 16, 2005; revised version received February 2, 2006)

We continue the discussion of a very simple empirical neutrino mass formula, implying the mass proportion $m_1 : m_2 : m_3 = 1 : 4 : 24(1-\beta^2)$. For the value $\beta = (5-1)(5-3)/5^2 = 8/25$ the formula predicts precisely $\Delta m^2_{21} \sim 8.0 \times 10^{-5}\,\mathrm{eV}^2$ in consistency with the experiment, when the input of experimental estimate $\Delta m^2_{32} \sim 2.4 \times 10^{-3}\,\mathrm{eV}^2$ is applied.

PACS numbers: 12.15.Ff, 14.60.Pq, 12.15.Hh

The role of empirical spectral formulae in the history of atomic and nuclear physics cannot be overestimated. The Balmer formula for hydrogen spectrum is perhaps the most famous example. At the advent of quantum physics, it has inspired the semiclassical Bohr model of hydrogen atom and, in a further consequence, the quantum mechanics of Schrödinger and Heisenberg. In particle physics, attempts to construct empirical spectral formulae for leptons and quarks are not very popular (*cf.* Refs. [1,2] for some recent publications). The main reason may be the apprehension about the possibility of not controlled numerological speculations connected with the fact that, on the level of fundamental particles, the spectral formulae become actually mass formulae, while the very concept of mass has not been advanced essentially from Newton's time, in contrast to the enormous progress in particle dynamics operating with masses having still the status of free parameters (in Higgs mechanism in the Standard Model and its extensions, the corresponding Yukawa coupling constants are free parameters). But, paradoxically, it seems that in the present situation, where the mass concept waits to be developed, the search for empirical mass formulae for leptons and quarks ought to be natural.

* Work supported in part by the Polish Ministry of Education and Science, grant No. 1 PO3B 099 29 (2005-2007).

In a recent paper [3] (in Supplement there), we reported on a very simple empirical mass formula for three mass neutrinos ν_1, ν_2, ν_3 related to three active neutrinos ν_e, ν_μ, ν_τ through the unitary mixing transformation $\nu_\alpha = \sum_i U_{\alpha i} \nu_i$ ($\alpha = e, \mu, \tau$ and $i = 1, 2, 3$). It has the form

$$m_i = \mu\, \rho_i (1 - \beta^2 \delta_{i3}) \qquad (i = 1, 2, 3)\,, \tag{1}$$

where $\mu > 0$ and $\beta^2 < 1$ are two free parameters and

$$\rho_1 = 1/29\,, \quad \rho_2 = 4/29\,, \quad \rho_3 = 24/29\,, \tag{2}$$

three specific normalized fractions ($\sum_i \rho_i = 1$) that may be called generation-weighting factors.

Making use of the experimental estimates (best fits) $\Delta m^2_{21} \sim 8.0 \times 10^{-5}$ eV^2 and $|\Delta m^2_{32}| \sim 2.4 \times 10^{-3}$ eV^2 [4] giving the ratio $|\Delta m^2_{32}|/\Delta m^2_{21} \sim 30$, we evaluated from Eq. (1)

$$\mu \sim 6.7 \times 10^{-2} \text{ eV}\,, \quad \beta^2 \sim 0.10\,, \tag{3}$$

(Eq. (1) implied the normal hierarchy and so, $\Delta m^2_{32}/\Delta m^2_{21} \sim 30$). Then, from Eq. (1) we obtained the following neutrino masses:

$$m_1 \sim 2.3 \times 10^{-3} \text{ eV}\,, \quad m_2 \sim 9.3 \times 10^{-3} \text{ eV}\,, \quad m_3 \sim 5.0 \times 10^{-2} \text{ eV}\,, \tag{4}$$

(one of these three neutrino masses, say m_1, was our *prediction*). Thus, we could consider that $m_1^2 \ll m_2^2 \ll m_3^2$.

Now, it is interesting to observe that with the use of three first odd integers

$$N_1 = 1\,, \quad N_2 = 3\,, \quad N_3 = 5\,, \tag{5}$$

the term $\beta^2 \delta_{i3}$ in the neutrino mass formula (1) can be numerically reproduced as

$$\beta^2 \delta_{i3} \equiv \left[\frac{(N_i - 1)(N_i - 3)}{N_i^2} \right]^2 = \left(\frac{8}{25} \right)^2 \delta_{i3} = 0.1024\ \delta_{i3} \simeq 0.10\, \delta_{i3}\,, \tag{6}$$

implying

$$\beta^2 = \left(\frac{8}{25} \right)^2 = 0.1024 \simeq 0.10\,. \tag{7}$$

If we conjecture that $\beta^2 \delta_{i3}$ in the neutrino mass formula (1) is *defined* as in Eq. (6), we *predict* from the relation $\Delta m^2_{32}/\Delta m^2_{21} = (16/15)[36(1-\beta^2)^2 - 1]$ given by Eq. (1) that

$$\frac{\Delta m^2_{32}}{\Delta m^2_{21}} = 29.8717 \simeq 30 \tag{8}$$

and hence,

$$\Delta m_{21}^2 \sim 8.0 \times 10^{-5} \text{ eV}^2 \tag{9}$$

when the input $\Delta m_{32}^2 \sim 2.4 \times 10^{-3}$ eV^2 is applied in Eq. (8). The prediction (9) is precisely consistent with the experimental estimate (best fit) of Δm_{21}^2 [4] (now, two of three neutrino masses (4), say m_1 and m_2, are our prediction). In the case of the conjecture (6), the neutrino mass formula (1) takes the form

$$m_i = \mu\rho_i \left[1 - \frac{(N_i - 1)^2(N_i - 3)^2}{N_i^4}\right] \qquad (i = 1, 2, 3)\,. \tag{10}$$

Concluding, we can see that the *very simple* mass proportion $m_1 : m_2 : m_3 = 1 : 4 : 24(1 - \beta^2)$ implied by our neutrino mass formula (1) *predicts* precisely the correct ratio of the mass-squared scales appearing in the so different phenomena as the atmospheric and solar neutrino oscillations. For this correct prediction the choice of $\beta^2 \simeq 0.10$ is necessary. Such a choice is provided all right in the case of our conjecture (6).

It is exciting enough to observe that an efficient empirical mass formula for charged leptons found out previously [2] is built up of *the same* numerical elements N_i and ρ_i $(i = 1, 2, 3)$ as the neutrino mass formula (10). These numerical elements are interpreted in Ref. [2] on the ground of a Kähler-like extension of the Dirac equation (*i.e.* on an extension of Dirac's square-root procedure).

As far as the problem of uniqueness in the deduction of spectral formulae from experimental data is concerned, the situation of the empirical mass formula (1) is evidently much worse than that of the Balmer formula which could be derived from very complete and precise spectroscopic data already available then for hydrogen. In fact, in spite of the enormous progress in neutrino physics, data on neutrino masses are still rather imprecise. In particular, for the mass m_1 only upper limits are estimated from tritium β decay experiments or astrophysical observations, say, $m_1 < 2.3$ eV or $m_1 < 0.6$ eV, respectively, although Δm_{21}^2 and $|\Delta m_{32}^2|$ are reasonably well known. At any rate, we can expect a growing improvement in experimental estimation of Δm_{21}^2 and $|\Delta m_{32}^2|$ that will allow a more precise confrontation of these mass-squared scales with our prediction (8) consistent with the actual experimental best-fit values of Δm_{21}^2 and $|\Delta m_{32}^2|$. Generally, the neutrino physics is in a state of rapid development, and various surprises are possible.

REFERENCES

[1] For a recent discussion of the Koide equation for charged leptons *cf.* Y. Koide, `hep-ph/0506247`; `hep-ph/0509214` and references therein.

[2] For a recent presentation of an efficient charged-lepton mass formula *cf.* W. Królikowski, *Acta Phys. Pol. B* **33**, 2559 (2002) [`hep-ph/0203107`]; `hep-ph/0504256`; `hep-ph/0509043` and references therein.

[3] W. Królikowski, *Acta Phys. Pol. B* **36**, 2051 (2005) [`hep-ph/0503074`]. The present paper is *Addendum* to this reference. It is an abridged version of the e-print `hep-ph/0510355` (unpublished).

[4] *Cf. e.g.* G.L. Fogli, E. Lisi, A. Marrone, A. Palazzo, `hep-ph/0506083`.