Vol. 37 (2006) *ACTA PHYSICA POLONICA B* No 7

NEUTRINOLESS DOUBLE β-DECAY NUCLEAR MATRIX ELEMENTS WITHIN QRPA AND ITS VARIANTS*

P. Beneš[a], A. Bobyk[b], W.A. Kamiński[b], A. Faessler[c]
F. Šimkovic[c,d]

[a]IEAP, Czech Technical University
Zikova 4, 166-36 Prague 6, Czech Republic
[b]Dept. of Theor. Phys., Marie-Curie-Sklodowska University
Marii Curie-Skłodowskiej 1 20-031 Lublin, Poland
[c]Inst. of Theor. Phys., University of Tuebingen
Auf der Morgenstelle 14, 72076 Tuebingen, Germany
[d]Dept. of Nucl. Phys., Comenius University
Mlynska dolina, 842 48 Bratislava, Slovakia

(Received May 17, 2006)

The nuclear matrix elements associated with the light Majorana neutrino mass mechanism of the neutrinoless double beta decay of ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{128}Te, ^{130}Te, ^{136}Xe, and ^{150}Nd have been calculated within the self-consistent renormalized quasiparticle random phase approximation (SRQRPA). By following a recently proposed procedure, where data on the two-neutrino double β-decay half-lives are used to derive appropriate value of particle-particle strength of nuclear Hamiltonian, we have found that the SQRPA results are comparable with those of the QRPA and the RQRPA approaches. This constitutes an additional argument in favor of the convergence of the QRPA-like results.

PACS numbers: 21.60.Jz, 23.40.Bw, 23.40.Hc

1. Introduction

The discovery of neutrino oscillations has opened a new excited era in neutrino physics and represents a big step forward in our knowledge of neutrino properties. However, the most fundamental question whether neutrino is a Dirac or Majorana particle remains unsolved. In order to reveal the Majorana nature of neutrinos the observation of neutrinoless double β-decay

* Presented at the Cracow Epiphany Conference on Neutrinos and Dark Matter, Cracow, Poland, 5–8 January 2006.

($0\nu\beta\beta$-decay) is necessary. This would also allow to get important information on the neutrino mass scale, on the hierarchy and on the Majorana phases.

From the measurement of the half-life of the $0\nu\beta\beta$-decay only the product of the effective Majorana neutrino mass and the nuclear matrix element ($|m_{\beta\beta}|\ |M^{0\nu}(A,Z)|$) can be determined. Thus, accurate calculation of nuclear matrix elements (NMEs), is necessary to reach qualitative conclusions about neutrino masses and the type of neutrino mass spectrum [1].

Among other approaches, the Random Phase Approximation (RPA) and its variants has been considered a very powerful tool for studying the nuclear structure. In particular, the quasiparticle version of the theory (the Quasiparticle Random Phase Approximation — QRPA) has been successfully applied to the nuclei far from the closed shells, and consequently extended as the proton–neutron QRPA (pnQRPA) to the description of charge-changing transitions in nuclei [2–9].

The main drawback in the formulation of the QRPA theory, however, is the violation of the Pauli exclusion principle, connected with the usage of bosonic commutation relations for the QRPA phonon operators, that are in fact collective pairs of fermions. To overcome this shortcoming of the QRPA framework the renormalization technique has been proposed [10] and extended to include proton–neutron pairing [11]. This approach has been based on the early works by Rowe [19], Hara [12], Ikeda [13] and Schuck and Ethofer [14] in the context of RPA and QRPA. The main goal of the method, called in the literature the renormalized QRPA (RQRPA), is to take into account additional one-quasiparticle scattering terms in the commutation relations by a self-iteration of the QRPA equation.

On the other hand, the calculation of the $0\nu\beta\beta$-decay matrix elements is a difficult problem because ground and many excited states of open-shell nuclei with complicated nuclear structure have to be considered. Recently, important progress has been achieved in the QRPA evaluation of the $0\nu\beta\beta$-decay NMEs [17]. It was shown that when the strength of the particle-particle interaction is adjusted so that the two-neutrino double β-decay ($2\nu\beta\beta$-decay) rate is correctly reproduced, the dependence of $|M^{0\nu}(A,Z)|$ on the size of the singe-particle basis and other factors, that are not *a priori* fixed, is essentially removed.

In this contribution the $0\nu\beta\beta$-decay NMEs are evaluated within the self-consistent RQRPA (SRQRPA) by following the procedure of fixing nuclear structure parameter space of Ref. [17]. The SRQRPA is an extension to the RQRPA formalism, that tries to solve the problem of non-vanishing quasiparticle content of the ground state that in turn introduces some inconsistency between RQRPA and the BCS approach. Our method [18], called the self-consistent RQRPA (SRQRPA), is based on the reformulation of the

BCS equations [16] and further reiteration of the BCS+RQRPA calculation scheme. It is a more complex version of the renormalized QRPA (RQRPA), the latter, in contrary to the QRPA, implementing the Pauli exclusion principle in description of nuclear states. In the SRQRPA at the same time the mean field is changed by minimizing the energy and fixing the number of particles in the correlated ground state instead of uncorrelated BCS one as is done in the QRPA and the RQRPA.

The SRQRPA was first applied to the $0\nu\beta\beta$-decay of ^{76}Ge and a considerable reduction of the SRQRPA NME in comparison with the QRPA and RQRPA results was found [18]. Our present analysis shows that this suppression is due to a consideration of bare G-matrix elements of realistic nucleon–nucleon potential dictated by a numerical complexity of the problem. Here, we have avoided this simplification by introducing a schematic proton and neutron pairing interactions, which were fixed to fit the observed mass differences.

2. Calculation procedure

Since the formalism of the SRQRPA has been presented in detail in our previous publications [15, 18], here we present only the basics of the theory. Since we are interested in the charge-changing transitions only, from now on we restrict ourselves to the proton–neutron version of the theory. In the RQRPA and SRQRPA one introduces the so-called renormalization matrix D_{pn}, defined by the expectation value of the commutator of the angular-momentum coupled bi-quasifermion operators:

$$D_{pn} \equiv \left\langle 0 \left| \left[A_{(pn)J^\pi M}, A^\dagger_{(pn)J^\pi M} \right] \right| 0 \right\rangle = (1 - n_p - n_n)\,, \tag{1}$$

where n_p and n_n are the RPA ground state quasiparticle densities:

$$\begin{aligned} n_p &\equiv \hat{\jmath}_p^{-1} \left\langle 0 \left| \left[a_p^\dagger \tilde{a}_p \right]_{00} \right| 0 \right\rangle, \\ n_n &\equiv \hat{\jmath}_n^{-1} \left\langle 0 \left| \left[a_n^\dagger \tilde{a}_n \right]_{00} \right| 0 \right\rangle. \end{aligned} \tag{2}$$

With the help of the D_{pn} matrix, one can introduce the renormalized angular-momentum coupled two-quasiparticle creation operators [20]:

$$\mathcal{A}^\dagger_{(pn)J^\pi M} \equiv D_{pn}^{-1/2} \left[a_p^\dagger a_n^\dagger \right]_{J^\pi M}, \tag{3}$$

that behave as bosons, as far as the ground-state expectation value of their commutator is concerned. Assuming the harmonicity of the nuclear motion the excited-state creation phonon operators can be written as [19, 21]:

$$Q^{m\dagger}_{J^\pi M} = \sum_{pn} \left[\mathcal{X}^m_{(pn)J^\pi} \mathcal{A}^\dagger_{(pn)J^\pi M} - \mathcal{Y}^m_{(pn)J^\pi} \tilde{\mathcal{A}}_{(pn)J^\pi M} \right]. \tag{4}$$

Using *e.g.* the equation of motion (EOM) method [19], one gets the RQRPA equations in the usual form, with $\Omega^m_{J^\pi} \equiv E_{m,J^\pi} - E_{\rm gs}$ being the energy of the QRPA phonon:

$$\left(\begin{array}{cc} \mathcal{A} & \mathcal{B} \\ \mathcal{B} & \mathcal{A} \end{array} \right)_{J^\pi} \left(\begin{array}{c} \mathcal{X}^m \\ \mathcal{Y}^m \end{array} \right)_{J^\pi} = \Omega^m_{J^\pi} \left(\begin{array}{c} \mathcal{X}^m \\ -\mathcal{Y}^m \end{array} \right)_{J^\pi} \tag{5}$$

with the renormalized RPA matrices $\mathcal{A}$ and $\mathcal{B}$:

$$\begin{aligned} \mathcal{A}^{J^\pi}_{pn,p'n'} &= (E_p + E_n)\delta_{pp'}\delta_{nn'} \\ &- 2[g_{pp}G(pn,p'n';J^\pi)(u_p u_n u_{p'} u_{n'} + v_p v_n v_{p'} v_{n'}) \\ &+ g_{ph}F(pn,p'n';J^\pi)(u_p v_n u_{p'} v_{n'} + v_p u_n v_{p'} u_{n'})] \\ &\times \sqrt{D_{pn}D_{p'n'}}\,, \end{aligned} \tag{6}$$

$$\begin{aligned} \mathcal{B}^{J^\pi}_{pn,p'n'} &= 2[g_{pp}G(pn,p'n';J^\pi)(u_p u_n v_{p'} v_{n'} + v_p v_n u_{p'} u_{n'}) \\ &- g_{ph}F(pn,p'n';J^\pi)(u_p v_n v_{p'} u_{n'} + v_p u_n u_{p'} v_{n'})] \\ &\times \sqrt{D_{pn}D_{p'n'}}\,. \end{aligned} \tag{7}$$

The particle–particle (G) and the particle–hole (F) matrix elements of the two-body nucleon–nucleon interaction [22] are scaled by the factors g_{pp} and g_{ph} respectively, to account for the finite range of the nucleus and limited model space [2]. In the calculations we have chosen two values of the g_{ph} parameter (0.8 and 1.0) and leave g_{pp} as a free parameter of the theory [23, 24]. E_p and E_n are the proton and neutron quasiparticle energies and the u's and v's are the usual BCS occupation factors.

The crucial point of the RQRPA is the calculation of the renormalization matrix D_{pn}. This can be achieved with a help of the mapping [10]:

$$\begin{aligned} [a^\dagger_p \tilde{a}_p]_{00} &\mapsto \hat{\jmath}_p^{-1} \sum_{J^\pi M n} A^\dagger_{(pn)J^\pi M} A_{(pn)J^\pi M}\,, \\ [a^\dagger_n \tilde{a}_n]_{00} &\mapsto \hat{\jmath}_n^{-1} \sum_{J^\pi M p} A^\dagger_{(pn)J^\pi M} A_{(pn)J^\pi M} \end{aligned} \tag{8}$$

and inversion of (4). Equations (1)–(8) became coupled and can be solved by the iteration procedure, we call 'inner iteration': we start with $n_p = n_n = 0$, *i.e.* QRPA solution, calculate new quasiparticle densities and input them back again, till the convergence is achieved.

Now we proceed with the SRQRPA 'outer iteration'. This is necessary, since RQRPA ground state has a non-vanishing quasiparticle content, while the BCS ground state is the quasiparticle vacuum. We relax the latter requirement and rewrite the BCS equations, by recalculating the density matrix ρ and the pairing tensor κ:

$$\rho_a \equiv \left\langle 0 \left| c^\dagger_\alpha c_\alpha \right| 0 \right\rangle \; = \; v_a^2 + (u_a^2 - v_a^2) n_a \,, \tag{9}$$

$$\kappa_a \equiv \langle 0 \, | \tilde{c}_\alpha c_\alpha | \, 0 \rangle \; = \; u_a v_a (1 - 2n_a) \,. \tag{10}$$

The u and v coefficients and quasiparticle energies are then obtained by minimizing the BCS ground-state energy. To solve the SRQRPA equations we start with the ordinary BCS equations, putting $n_p = n_n = 0$, than proceed with the corresponding RQRPA problem (inner iteration), that gives us new quasiparticle densities and loop with them back to BCS until the convergence is achieved (outer iteration).

3. Results and conclusions

We calculated SRQRPA NMEs for nuclei of experimental interest. We essentially used the same approach, as described in [15, 18], except that $g_A = 1.25$ was adopted and the overlap matrix of the intermediate nuclear states was calculated more accurately in comparison with the previous study [17]. The experimental uncertainties in the measured $2\nu\beta\beta$-decay half-lives were not taken into account. The two-body matrix elements were calculated from the Bonn-B nucleon–nucleon one boson exchange potential [25] within the Brueckner theory [22]. The single-particle energies were calculated from the Coulomb-corrected Woods–Saxon potential with Bertsch parametrization [26]. As previously, we have found weak dependence of the RQRPA and the SRQRPA results on the dimension of the single-particle basis, the contrary to the QRPA behaviour (Fig. 1). The conclusion is, that the most suitable single-particle basis for all the nuclei in the mass range $100 < A \leq 150$ should contain 16 nlj shells (both for protons and neutrons) with ^{40}Ca as an inert core: $1p_{1/2}$, $1p_{3/2}$, $0f_{5/2}$, $0f_{7/2}$, $2s_{1/2}$, $1d_{3/2}$, $1d_{5/2}$, $0g_{7/2}$, $0g_{9/2}$, $2p_{1/2}$, $2p_{3/2}$, $1f_{5/2}$, $1f_{7/2}$, $0h_{9/2}$, $0h_{11/2}$, $0i_{13/2}$. The obtained results are displayed in Fig. 2 and compared with the RQRPA and SRQRPA NMEs. We see that if data on the $2\nu\beta\beta$-decay are used to extract a more accurate value for NMEs, very similar NMEs for all considered many-body approaches are obtained. We note that for the closed and partially closed shell nuclei (^{48}Ca, ^{116}Sn and ^{136}Xe) a further improvement in description of pairing interaction is needed. We can conclude that, the RQRPA and the SRQRPA are more stable with growing dimension of the single-particle model space and that the RQRPA reproduces the experimental data for higher values of the particle–particle force. The SRQRPA behaves like QRPA, but the collapse is pushed forward towards higher $g_{\rm pp}$ values. For all nuclei we have studied, $0\nu\beta\beta$ nuclear matrix elements can be accurately reproduced within QRPA, RQRPA and SQRPA by fixing the g_{pp} value using $2\nu\beta\beta$ experimental data.

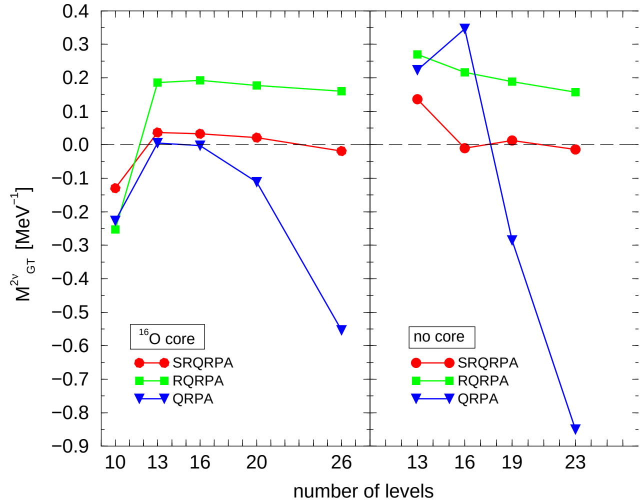

Fig. 1. Dependence of the ^{76}Ge $\rightarrow$ ^{76}Se two-neutrino nuclear matrix elements $M^{2\nu}$ on the dimension of the single-particle basis within the QRPA, the RQRPA and the SRQRPA.

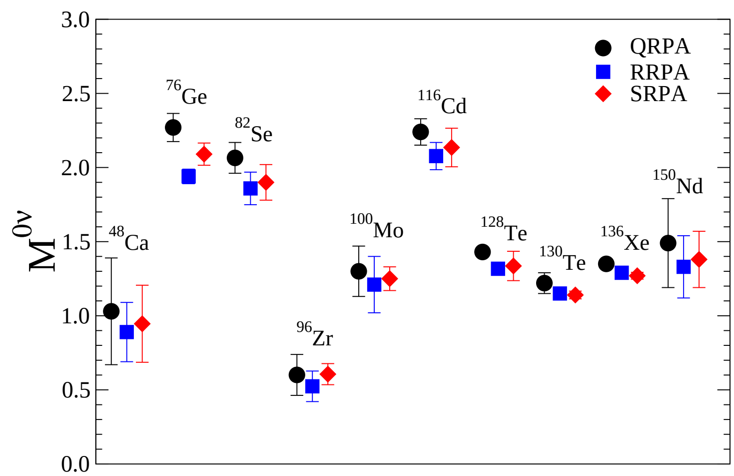

Fig. 2. Average neutrinoless nuclear matrix elements $M^{0\nu}$ and their variance within the QRPA, the RQRPA and the SRQRPA.

In summary, the $0\nu\beta\beta$-decay NMEs were systematically evaluated using the SRQRPA. The important role of pairing interaction was stressed. It was found that the SQRPA results agree suprisingly well with those of the QRPA and the RQRPA approaches, thus giving more confidence to the whole set of various QRPA-like methods.

This work was supported by the Deutsche Forschungsgemeinschaft (grant 436 SLK 17/298), Polish State Committee for Scientific Research (grant 2P03B 071 25) and by EU ILIAS project under contract RII3-CT-2004-506222.

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