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MODELLING CC NEUTRINO CROSS SECTIONS IN THE FEW GeV ENERGY REGION*

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Selected problems in modelling neutrino–nucleon and neutrino–nuclei cross sections in the neutrino energy region of the few GeV are reviewed.

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1. Introduction

The aim of this paper is to review some of recent developments in modelling the Charge Current (CC) neutrino interactions with both free nucleons and nuclei targets in the few GeV neutrino energy region [1]. This energy range is characteristic for atmospheric neutrinos and for several running or approved long-baseline experiments. The knowledge of the cross sections is necessary for future more precise measurements of neutrino oscillation parameters. In our discussion we adopt a practical approach and will always have in mind Monte Carlo (MC) implementation of presented models.

The few GeV energy region is rather complicated because three different dynamical formalisms are relevant: of quasi-elastic reactions, resonance excitation and more inelastic channels treated together in the DIS formalism. The significance of three dynamics is seen in Fig. 1 where the total CC cross section for the muon neutrino scattering off free *isoscalar* nucleon target (*i.e.* average from proton and neutron targets) is presented. The contributions from quasi-elastic, single pion production (SPP) and more inelastic channels (denoted as DIS) are also shown separately. It is seen that in the few GeV energy region all three contributions are important. The cross section from SPP channels in Fig. 1 is restricted by the condition on the invariant hadronic mass $W < W_{\rm cut} = 2\,{\rm GeV}$.

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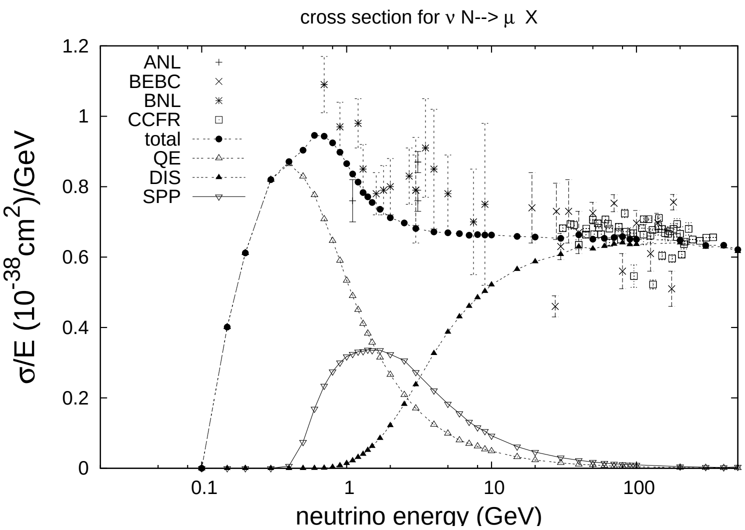

Fig. 1. Total cross section for ν_μ CC scattering on isoscalar target as predicted by the WROCLAW MC generator. The contributions from quasi-elastic, single pion production (SPP) and more inelastic channels (denoted as DIS) are also shown separately.

The plan of the paper is the following. We start in Sec. 2 from a description of quasi-elastic reaction. In Secs. 3 and 4 we review models of SPP and the formalism of Deep Inelastic Scattering. Secs. 5 and 6 deal with nuclear effects.

2. Quasi-elastic reactions

There are two CC $\Delta S = 0$ quasi-elastic channels: $\nu_l + n \rightarrow l^- + p$ and $\bar{\nu}_l + p \rightarrow l^+ + n$. In the discussed energy region the condition $Q^2 \ll M_W^2$ holds and it is enough to consider processes in the effective Fermi theory approximation.

The matrix element contains leptonic part which is exactly known and the hadronic one, which cannot be calculated from first principles. The hadronic current contains four form-factors, functions of Q^2. Vector form-factors $F_{1,2}$ are determined (CVC) by their electromagnetic counterparts. The form-factor F_P can be expressed in terms of $F_{\rm A}$ (PCAC) [2]. In recent years an improvement to MC codes was introduced by a replacement of old-fashioned dipole form-factors with one of the available fits to experimental data [3]. $F_{\rm A}$ is considered to be in the dipole form with two parameters: $g_{\rm A}$ determined by the β decay and the axial mass $M_{\rm A}$ which is not exactly known. The value of $M_{\rm A}$ determines the shape of $d\sigma/dQ^2$ and the overall quasi-elastic cross section [4].

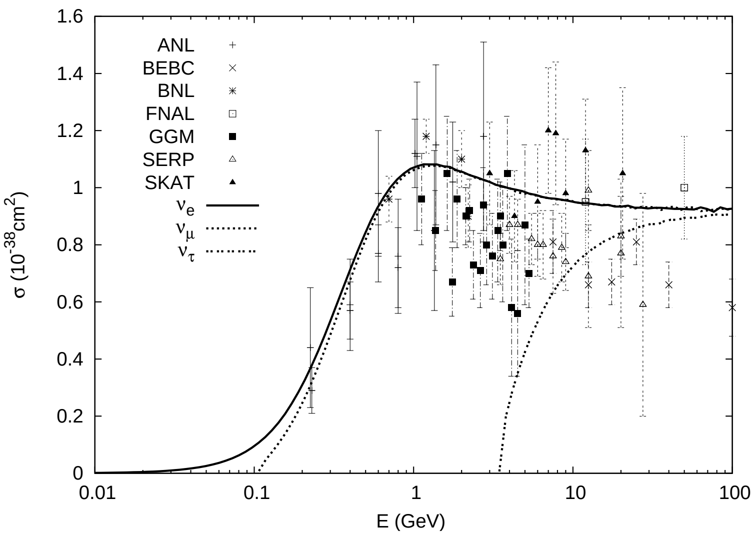

Fig. 2. Quasi-elastic cross section for ν_e, ν_μ and ν_τ. Experimental points refer to ν_μ scattering.

3. Single pion production

There are three CC SPP channels for neutrino reactions: $\nu_l + n \rightarrow l^- + p + \pi^0$, $\nu_l + n \rightarrow l^- + n + \pi^+$, $\nu_l + p \rightarrow l^- + p + \pi^+$, and another three for anti-neutrino reactions. The characteristic feature of SPP reactions is the appearance of the Δ resonance in the differential cross section $d\sigma/dW$.

There are several theoretical models of SPP. Almost all Monte Carlo generators use the Rein–Sehgal (RS) model [5]. It includes contributions from 18 resonances of mass $M_{\rm res} < 2\,{\rm GeV}$ treated in the coherent way. The non-resonant contribution is then added in the incoherent way in order to get an agreement with available data. An alternative model has been developed recently by the Dortmund group [6]. It is based on experimental data on electromagnetic helicity amplitudes. The model contains the following resonances: $P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$ and $S_{11}(1535)$. It includes m_l^2 (m_l is the charged lepton mass) terms which reduce the $d\sigma/dQ^2$ by $\sim$ 5–10% at low Q^2 and which are absent in the original RS model. A similar model with few resonances was constructed many years ago by Fogli–Nardulli [7].

A common difficulty of SPP models is related to the issue of description of the non-resonant background. The most systematic approach is the one adopted in the Sato–Lee model [8]. It is based on the quark model with the pion cloud effects taken into account. It predicts the non-resonant contribution to ν-neutron SPP channels on the level of $\sim$ 25% and to ν-proton SPP on the level of $\sim$ 5%. Alternative effective descriptions of non-resonant

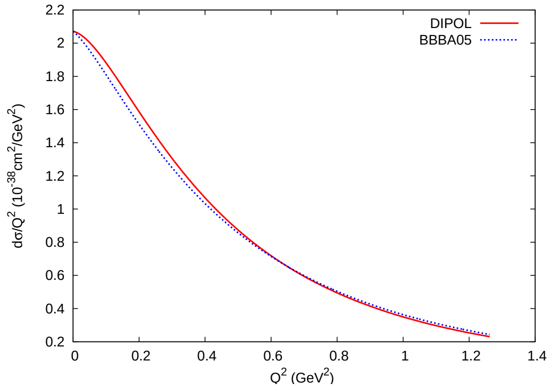

Fig. 3. Modification of the shape of $d\sigma/dQ^2$ due to non-dipole electromagnetic form factors. For both curves $M_{\mathrm{A}} = 1.03\,\mathrm{GeV}$.

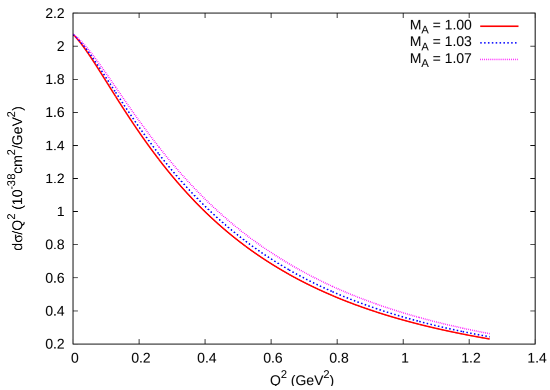

Fig. 4. Modification of the shape of $d\sigma/dQ^2$ due to different choices of the axial mass $M_{\mathrm{A}} = 1,\ 1.03,\ 1.07\,\mathrm{GeV}$. It is seen that the choice of smaller M_{A} reduces the overall cross section in the different way than the substitution of dipole form-factors by BBBA05 ones (see the Fig. 3).

background are introduced in some MC codes. The idea is to simulate the background by a fraction of the DIS cross section in the resonance kinematical domain *i.e.* for $W < W_{\mathrm{cut}}$ [9, 10]. Another theoretical possibility to deal with the non-resonant background is suggested by the hypothetical two-component quark–hadron duality [11]. If the hypothesis is true, the background is given by the sea quark contribution to DIS structure functions. One can also try to model the non-resonant background by assuming that its dependence on the invariant mass is like in electron scattering *i.e.* $\sim \sqrt{W - W_{\mathrm{thr}}}\sum a_j(Q^2)(W - W_{\mathrm{thr}})^j$, where W_{thr} is the threshold for the pion production [12].

The lack of precise experimental data makes it impossible to select a preferred model of SPP.

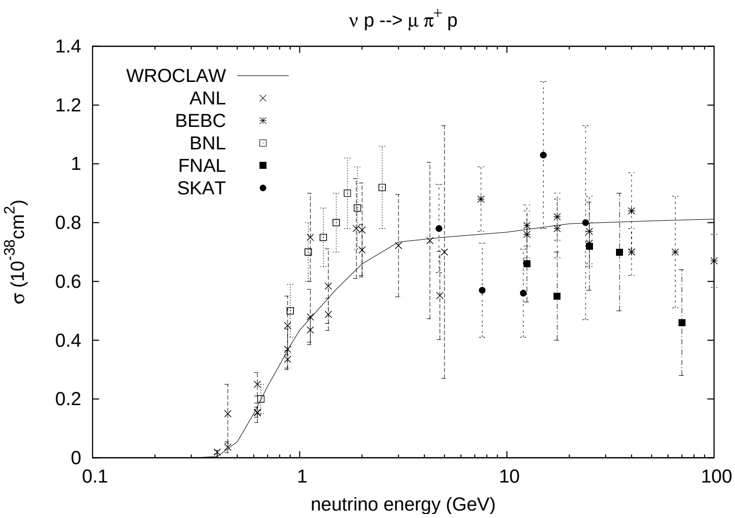

Fig. 5. Cross section for $\nu_\mu p \to \mu^- p \pi^+$ as predicted by the WROCLAW MC generator.

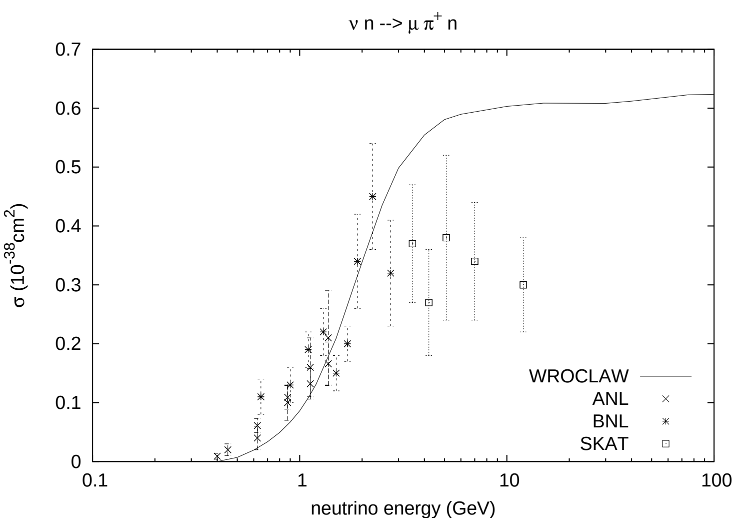

Fig. 6. Cross section for $\nu_\mu n \to \mu^- n \pi^+$ as predicted by the WROCLAW MC generator.

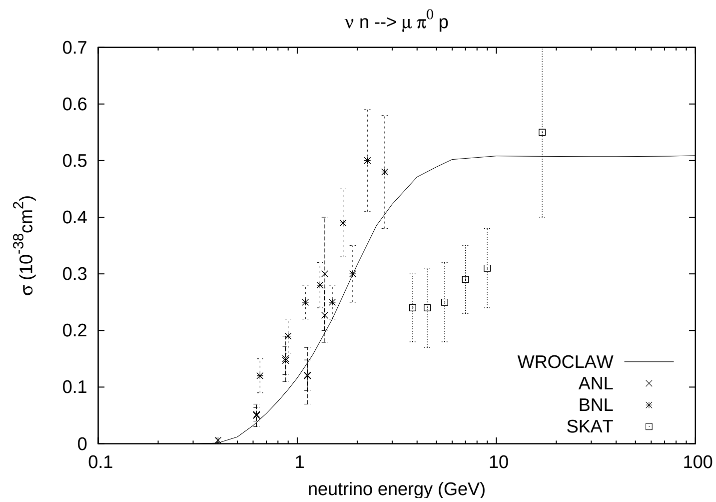

Fig. 7. Cross section for $\nu_\mu n \to \mu^- p \pi^0$ as predicted by the WROCLAW MC generator.

4. More inelastic channels

The DIS formalism describes the inclusive νN cross section in the scaling limit. The justification of the theory comes from the perturbative QCD. In the few GeV neutrino energy region an important contribution to the cross section comes from the small Q^2 region, where the theory behind the DIS formalism is not valid. Thus the first problem is to get a correct form of the structure functions. The standard procedure is to express $F_{4,5}$ in terms of $F_{1,2}$ [13] and then to express F_1 in terms of F_2 and $R \equiv (\sigma_\mathrm{L}/\sigma_\mathrm{T})$. In the scaling limit remaining $F_{2,3}$ are given as combinations of PDF's (parton distribution functions). In the region we focus on, the target mass and twist corrections must be taken into account [14]. The choice which is adopted in most MC codes is to apply structure functions with corrections modelled in analogy to the electron scattering case. Corrections which are available in the literature are applied to LO GRV98 PDF's [15]. Their form is closely related to the issue of quark–hadron duality: the DIS structure functions describe on average the electron scattering data in the resonance region [16]. It is an open problem whether quark–hadron duality should hold also in νN scattering. There has been recently a lot of investigation in this field [17]. The theoretical analysis of the duality has been done in the framework of SU(6) quark model of resonances [18]. It is not clear whether arguments valid for the vector part of the hadronic current should hold true also for its axial counterpart. Another problem is to understand what happens in the kinematical region $Q^2 < 0.5\,\mathrm{GeV}^2$, where one does not expect the quark–hadron duality to be present. In electron scattering it is known that $F_2^{eN} \to Q^2$ and $F_L^{eN} \to Q^4$, but for neutrino structure functions the presence of the axial current, which is not conserved makes the situation more complicated [19].

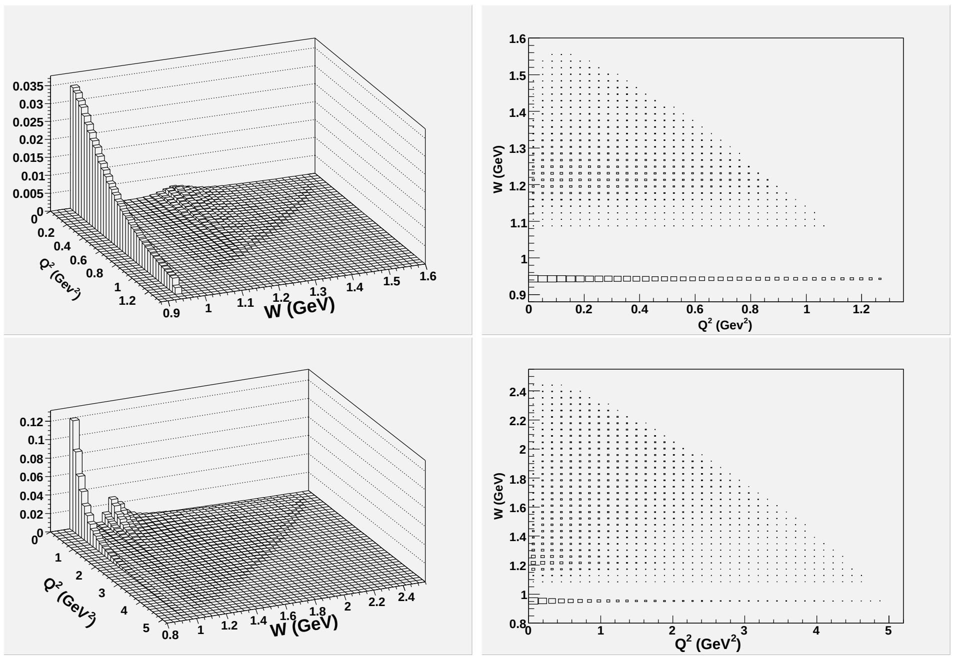

Fig. 8. Distribution of events in W and Q^2 for $\nu_\mu N$ scattering at energies $E_\nu = 1\,\mathrm{GeV}$ (top) and $E_\nu = 3\,\mathrm{GeV}$ (bottom) as predicted by the WROCLAW MC generator. The quasi-elastic contribution is seen as the peak at $W = M$ (nucleon's mass). The Δ excitation region is also clearly seen.

Once the inclusive cross section is calculated one has to evaluate contributions from exclusive channels. One possibility is to use the KNO scaling which provides average multiplicities of particles in the final state [10]. The only remaining problem is then to redistribute to the particles energy and momentum transfer. Another strategy is to use the LUND fragmentation and hadronization routines [20, 21].

In MC implementation of either scheme it is necessary to decide on several important points. Where should be a boundary between DIS and resonance (SPP) contributions? The very definition of the RS model suggests that one should define $W_{\mathrm{cut}} = 2\,\mathrm{eV}$. Some authors argue that the RS model underestimates the cross section at higher W and the better choice is $W_{\mathrm{cut}} \sim 1.7\,\mathrm{eV}$ [19]. This is the choice adopted by the authors of the NEUGEN/GENIE MC code [10]. The comprehensive analysis of all the available date led other authors to the conclusion that one should take

$W_{\rm cut} \sim 1.5\,{\rm GeV}$ [22]. This is approximately the choice implemented in the WROCLAW MC generator [21] where only Δ resonance contribution is included.

5. Nuclear effects — generalities

The treatment of nuclear effects depends on the neutrino energy. In the few GeV energy region one can rely on the picture in which neutrino interacts with individual nucleons inside nucleus (impulse approximation (IA)). It is not completely clear starting from which energies IA is the correct approach. Some authors argue that the Fermi gas model (the simplest mean field theory realization of IA) works well for electron neutrino energies $E_\nu \geq 200\,{\rm MeV}$ [23]. Other authors are more conservative and argue that the IA picture makes sense for momentum transfers $q \geq 400\,{\rm MeV}$, which translates into higher neutrino energies [24]. For example, for neutrino energy $E_\nu = 0.8\,{\rm GeV}$ the contribution to the quasi-elastic cross section from the momentum transfers $q < 400\,{\rm MeV}$ is $\sim 20\%$.

The simplest realization of the IA is known as PWIA (plane wave impulse approximation): one assumes that the nucleon produced in the primary vertex leaves nucleus without further re-interactions. This is an obvious oversimplification and it is better to include FSI (final state interactions) effects. A possible systematic approach to deal with FSI is known as DWIA (distorted wave impulse approximation) [25]. In MC codes FSI effects are usually treated by means of inter-nuclear cascade modules [26]. The propagation of nucleons, pions and other particles inside nucleus is semiclassical. It is important to implement the concept of the formation zone. Many other theoretical schemes to deal with FSI has been developed as well [27].

6. Nuclear effects — some models

The advantage of the Fermi gas (FG) model is that it is easily applicable in MC routines. The basic FG model is defined by just two parameters: Fermi momentum and binding energy [28]. It is necessary to deal with the problem of how to calculate off-shell nucleon matrix elements and the de Forest prescription is the common way to handle it [29]. One has also to decide about the kinematics. Smith–Moniz approach is the simplest choice and the other is to take into account the recoil nucleus momentum [30]. An improvement to the above versions of the FG model is obtained in the framework of LDA (local density approximation): the Fermi momentum becomes a local quantity according to the density profile of the nucleus [31]. Further improvement is introduced by modifying the momentum distribution of the nucleons by adding the high momentum tail [30]. In the framework of the FG model the only FSI effect is Pauli blocking.

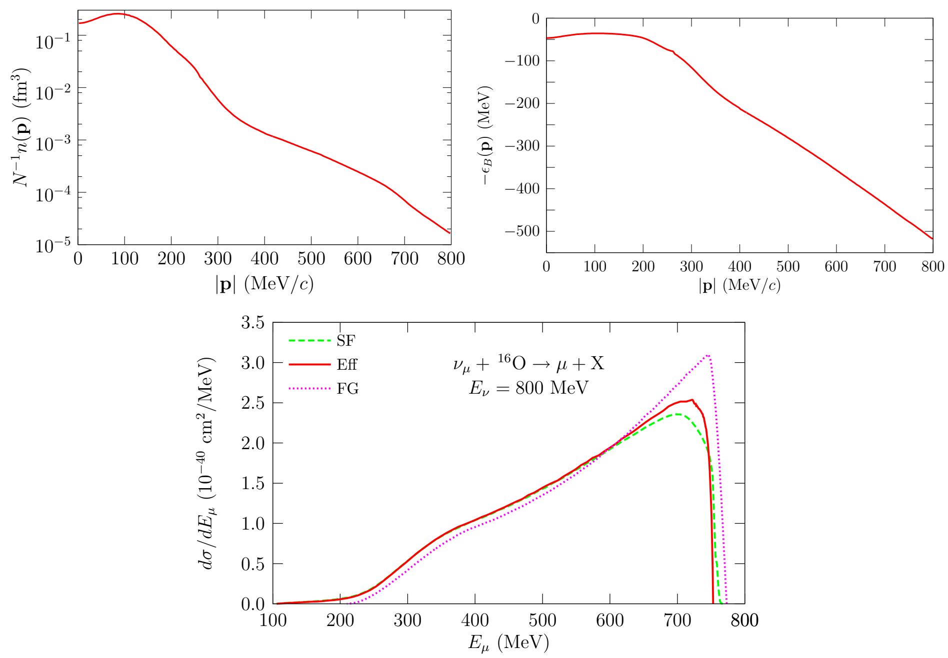

Fig. 9. The *effective* description [36] based on Benhar's oxygen spectral function [35]. The upper two plots show the momentum distribution (left-hand side) and the average momentum dependent binding energy (right-hand side). Below a comparison is shown between three modellings of nuclear effects: by Fermi gas (FG), spectral function (SF) and the *effective* one (Eff).

The spectral function (SF) approach represents an improvement with respect to FG model by providing a realistic probability distribution of momenta and binding energies of nucleons inside nuclei [32]. Theoretical models of SF are obtained by combining the mean field (shell model) and the correlated part. The second one is relevant at higher values of momenta and binding energies and comes from short range interactions due to correlated high momentum pairs of nucleons [33]. Recently the correlated part of the SF has been directly measured in electron experiments [34].

Reliable models of SF exist for lighter nuclei up to oxygen [35]. The mean field part of SF is clearly seen as probability distribution peaks at $E \sim -42\,\mathrm{MeV}$ ($1s$), $E \sim -19$ MeV ($1p_{3/2}$) and $E \sim -13\,\mathrm{MeV}$ ($1p_{1/2}$). The characteristic feature of the $1s$ level is that it is smeared out in E.

MC implementation of SF approach is straightforward. One can also use the *effective* approach in which the relevant information about the spectral function is contained in two functions: the probability distribution of nucleons momenta and the average momentum dependent binding energy [36].

In order to construct SF for heavier nuclei it is necessary to know the energy levels and spectroscopic factors. The correlated part of SF is universal and depends only on the nucleus size [32].

SF can be also used to model SPP in the resonance region. A new issue is the dependence of the Δ resonance width on the nuclear matter [37].

In the context of the DIS formalism there are specific methods to deal with nuclear effects to describe the shadowing, anti-shadowing *etc.* Recently a comprehensive model has been proposed to describe all the effects in the unique theoretical frame [38]. It includes also Fermi motion effects and in combining it with nuclear effects for quasi-elastic and SPP one has to be careful to avoid double counting.

A step beyond IA would be to include contributions from 2-body current. Is is believed they are necessary in order to explain the excess of the cross section in the DIP region between quasi-elastic and Δ excitation peaks [39]. The problem with 2-body currents is that the computations which must be performed are algebraically very involving [40]. Some authors tried to approximate the 2-body contribution to the neutrino cross section with the conclusion that it is very important: for $E_\nu = 700\,\mathrm{MeV}$ it is responsible for up to $\sim 25\%$ of the cross section in the kinematical region of energy transfer $\omega \in (80, 250)\,\mathrm{MeV}$ [41].

7. Final remarks

There has been efforts in the past to create a universal MC code to describe neutrino interactions [1]. So far each experiment uses its own MC focused on particular neutrino energy spectrum, target, detection techniques *etc.* The most promising ongoing project to construct a universal MC is that of GENIE [42].

All the theoretical considerations is this review were subject to big experimental uncertainty in νN cross sections. The good news are that the MINERvA experiment is under way [43]. It will enable us to settle a lot of unknowns in free nucleon and nuclei targets cross sections.

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