PLANCK SCALE REMNANTS IN RESUMMED QUANTUM GRAVITY* **

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We show that, in a new approach to quantum gravity in which its UV behavior is tamed by resummation of large IR effects, the final state of the Hawking radiation for an originally very massive black hole is a Planck scale remnant which is completely accessible to our Universe. This remnant would be expected to decay into n-body final states, leading to Planck scale cosmic rays.

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1. Introduction

Recently [1–4], we have introduced a new approach to the problem of quantum general relativity, resummed quantum gravity, in which we arrive at a UV finite representation of that theory. We start from the original approach of Feynman in Refs. [5, 6], in which he showed how one represents Einstein's theory as a sum of Feynman graphs by expanding in powers of $\kappa = \sqrt{8\pi G_{\rm N}}$ where $G_{\rm N}$ is Newton's constant. We improve on Feynman's results by resumming the large infrared (IR) parts of these graphs using the extension to non-Abelian IR algebras [7] of the methods of Ref. [8], which we have used successfully in Refs. [9] in the Abelian case in which they were derived. We have found that this resummation tames the bad UV behavior found by Feynman in Refs. [5, 6], so that the resulting resummed theory is UV finite. In what follows, we apply this new UV finite approach to quantum general relativity (QGR) to the problem of the final state of Hawking radiation [10] for an originally very massive black hole solution of Einstein's theory [11, 12].

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We emphasize that our theory is one of several efforts to solve the outstanding problem of quantum general relativity. The most successful effort theoretically to date is of course the popular superstring theory [13, 14]. There is also the loop quantum gravity approach [15] and the asymptotic safety approach [16] as realized phenomenologically in Refs. [17–20] using the exact renormalization group apparatus. We also mention the low energy effective Lagrangian approach [16] in which a low energy expansion of the theory is made [21,22] in complete analogy with the successful chiral perturbation theory low energy expansion in QCD [23]. An alternative approach to the IR regime for gravitational effects is given as well in Refs. [24]. As we have explained in Refs. [1–4], our approach in no way contradicts any of these other important efforts.

Specifically, since the new theory of QGR is still in its early stages of development and application, we start in the next section with a brief review of the Feynman approach upon which the new theory is based. This is followed in Section 3 with a presentation of the elements of the new theory. In Section 4, we then apply the new theory to the problem of the final state of Hawking radiation for an originally massive black hole solution of Einstein's theory and show that it leads to Planck scale remnants. Section 5 contains some concluding remarks.

2. Review of Feynman's approach to QGR

We specialize the Standard Model (SM) to its Higgs sector with just the Higgs and its gravitational interactions, as this will allow us to represent Feynman's approach to QGR. Any generalization to the spinning particles in the SM can then be carried-out as needed by known methods [1–4]. We take the mass of the Higgs to be 120 GeV for definiteness, in view of the LEP data [25]. The relevant Lagrangian, already considered by Feynman in Refs. [5, 6], is then

$$\begin{aligned}\mathcal{L}(x) &= -\frac{\sqrt{-g}}{2\kappa^2}R + \frac{\sqrt{-g}}{2}\left(g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - m_o^2\varphi^2\right)\\ &= \frac{1}{2}\left\{h^{\mu\nu,\lambda}\bar{h}_{\mu\nu,\lambda} - 2\eta^{\mu\mu'}\eta^{\lambda\lambda'}\bar{h}_{\mu_\lambda,\lambda'}\eta^{\sigma\sigma'}\bar{h}_{\mu'\sigma,\sigma'}\right\}\\ &\quad + \frac{1}{2}\left\{\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2\right\} - \kappa h^{\mu\nu}\left[\overline{\varphi_{,\mu}\varphi_{,\nu}} + \frac{1}{2}m_o^2\varphi^2\eta_{\mu\nu}\right]\\ &\quad - \kappa^2\left[\frac{1}{2}h_{\lambda\rho}\bar{h}^{\rho\lambda}(\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2) - 2\eta_{\rho\rho'}h^{\mu\rho}\bar{h}^{\rho'\nu}\varphi_{,\mu}\varphi_{,\nu}\right] + \ldots\,, \end{aligned} \tag{1}$$

where $\varphi_{,\mu} \equiv \partial_\mu\varphi$ and we have the metric $g_{\mu\nu}(x) = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}(x)$ with $\eta_{\mu\nu} = \mathrm{diag}\{1, -1, -1, -1\}$ and R is the curvature scalar. We define $\bar{y}_{\mu\nu} \equiv \frac{1}{2}\left(y_{\mu\nu} + y_{\nu\mu} - \eta_{\mu\nu}{y_\rho}^\rho\right)$ for any tensor $y_{\mu\nu}$. The Feynman rules for this theory were already worked-out by Feynman [5,6]. Here we use his gauge, $\partial^\mu \bar{h}_{\nu\mu}=0$. On this view, quantum gravity is just another quantum field theory where the metric now has quantum fluctuations as well.

3. Resummed QGR

We apply our extension in Ref. [7] of the methods of Ref. [8] to quantum general relativity, so that for the two point function for φ we get [1]

$$i\Delta'_F(k)|_{\rm resummed} = \frac{ie^{B''_g(k)}}{k^2 - m^2 - \Sigma'_s + i\varepsilon}\,, \tag{2}$$

for $(\Delta = k^2 - m^2)$

$$\begin{aligned} B''_g(k) &= -2i\kappa^2 k^4 \frac{\int d^4\ell}{16\pi^4} \frac{1}{\ell^2 - \lambda^2 + i\varepsilon} \frac{1}{(\ell^2 + 2\ell k + \Delta + i\varepsilon)^2} \\ &= \frac{\kappa^2 |k^2|}{8\pi^2} \ln\left(\frac{m^2}{m^2 + |k^2|}\right)\,, \end{aligned} \tag{3}$$

where the latter form holds for the UV regime, so that (2) falls faster than any power of $|k^2|$. An analogous result [1] holds for $m = 0$. We also note that, as Σ'_s starts in $\mathcal{O}(\kappa^2)$, we may drop it in calculating one-loop effects. It follows that when the respective analogs of (2) are used, one-loop corrections are finite. In fact, it can be shown that the use of our resummed propagators renders all quantum gravity loops UV finite [1]. We have called this representation of the quantum theory of general relativity resummed quantum gravity (RQG).

4. Black holes and Hawking radiation to Planck scale remnants

The finiteness of the loop corrections in resummed quantum gravity allows us to address any number of outstanding problems in quantum general relativity. Among those is the final state of the Hawking radiation for an originally very massive black hole solution of Einstein's theory. We now present our analysis of this problem.

Specifically, we start by calculating the effects of the one-loop corrections to the graviton propagator in Figs. 1 and 2. When the graphs Figs. 1 and 2 are computed in our resummed quantum gravity theory as presented in Refs. [1–3], the coefficient $c_{2,\rm eff}$ in Eq. (12) of Ref. [3], which describes the attendant effect on the denominator of the graviton propagator, becomes here, summing over the SM particles in the presence of the recently measured small cosmological constant [26], which implies the gravitational infrared cut-off of $m_g \cong 3.1 \times 10^{-33}$ eV

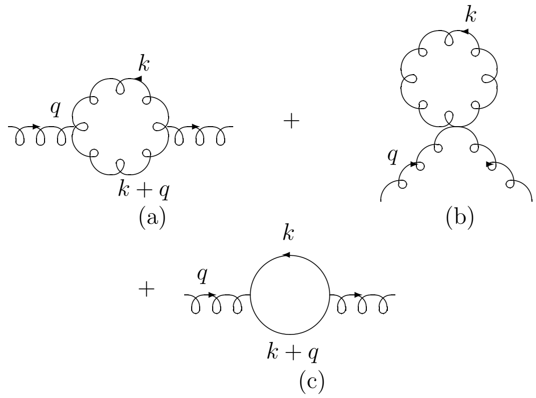

Fig. 1. The graviton ((a), (b)) and its ghost ((c)) one-loop contributions to the graviton propagator. q is the 4-momentum of the graviton.

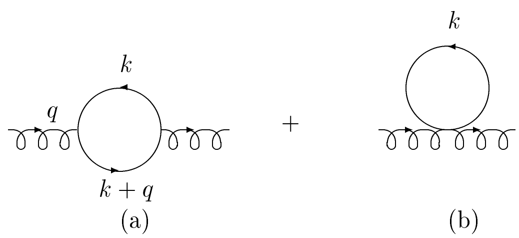

Fig. 2. The scalar one-loop contribution to the graviton propagator. q is the 4-momentum of the graviton.

$$c_{2,\mathrm{eff}} = \sum_j n_j I(\lambda_{\mathrm{c}}(j)) \,, \tag{4}$$

where we define [3] n_j as the effective number of degrees of freedom for particle j and the integral I is given by

$$I(\lambda_{\mathrm{c}}) \cong \int_0^\infty dx\ x^3 (1+x)^{-4-\lambda_{\mathrm{c}} x} \,, \tag{5}$$

with the further definition $\lambda_{\mathrm{c}}(j) = (2m_j^2)/(\pi M_{\mathrm{Pl}}^2)$, where the value of m_j is the rest mass of particle j when that is nonzero. When the rest mass of particle j is zero, the value of m_j turns-out to be [27] $\sqrt{2}$ times the gravitational infrared cut-off mass [26]. We further note that, from the exact one-loop analysis of Ref. [28], it also follows that the value of n_j for the graviton and its attendant ghost is 42. For $\lambda_{\mathrm{c}} \to 0$, we have found the approximate representation

$$I(\lambda_{\rm c}) \cong \ln\frac{1}{\lambda_{\rm c}} - \ln\ln\frac{1}{\lambda_{\rm c}} - \frac{\ln\ln\frac{1}{\lambda_{\rm c}}}{\ln\frac{1}{\lambda_{\rm c}} - \ln\ln\frac{1}{\lambda_{\rm c}}} - \frac{11}{6}\,. \tag{6}$$

In this way, we obtain from the standard Fourier transform of the respective graviton propagator the improved Newton potential

$$\Phi_{\rm N}(r) = -\frac{G_{\rm N}M}{r}(1-e^{-ar})\,, \tag{7}$$

where now, with

$$c_{2,\rm eff} \cong 2.56\times 10^4\,, \tag{8}$$

and, from Eq. (8) in Ref. [3],

$$a \cong \left(\frac{360\pi M_{\rm Pl}^2}{c_{2,\rm eff}}\right)^{\frac{1}{2}}\,, \tag{9}$$

we have that

$$a \cong 0.210\, M_{\rm Pl}. \tag{10}$$

What this means is that, when we make contact with the analysis of Ref. [18] for the final state of the Hawking radiation for an originally very massive black hole, we can have a smooth transition from the effective running Newton's constant $G(r)$ found in Ref. [18]

$$G(r) = \frac{G_{\rm N}\, r^3}{r^3 + \tilde{\omega}G_{\rm N}\left[r+\gamma G_{\rm N}M\right]}\,, \tag{11}$$

for a central body of mass M where γ is a phenomenological parameter [18] satisfying $0 \le \gamma \le 9/2$ and $\tilde{\omega} = 118/15\pi$, to our result in (7) via the matching at the outermost solution, $r_>$, of the equation

$$G(r) = G_{\rm N}(1-e^{-ar})\,. \tag{12}$$

For $r < r_>$, in the metric class

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2 d\Omega^2\,, \tag{13}$$

the lapse function is then taken to be

$$f(r) = 1 - \frac{2G_{\rm eff}(r)M}{r}\,, \tag{14}$$

with $G_{\rm eff}(r) = G_{\rm N}(1-e^{-ar})$ whereas for $r > r_>$ we set $G_{\rm eff}(r) = G(r)$. It the follows [4] that for the self-consistent value $\gamma = 0$ and $0.2 = \Omega \equiv$

$\tilde{\omega}/G_N M^2 = \tilde{\omega} M_{Pl}^2/M^2$ for definiteness we find that the inner horizon found in Ref. [18] moves to negative values of r and that the outer horizon moves to $r = 0$, so that the entire mass of the originally very massive black hole radiates away until a Planck scale remnant of mass $M'_{cr} = 2.38\ M_{Pl}$ is left[1], which then is completely accessible to our Universe. It would be expected to decay into n-body final states, $n = 2, 3, \ldots$, leading in general to Planck scale cosmic rays. The data in Ref. [30, 31] are not inconsistent with this conclusion, which also agrees with recent results by Hawking [32].

5. Conclusions

We conclude that resummed quantum gravity allows us to address many questions that have hitherto been either intractable or very cumbersome in rigorous quantum field theory. In our discussion, we have argued that the final state of the Hawking radiation from an originally very massive black hole is a Planck scale remnant that is entirely accessible to our Universe and that would be expected to decay into Planck scale cosmic rays. We encourage experimentalists to search for such.

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