COULD CP BREAKING MAJORANA PHASES BE MEASURED VIA NEUTRINOLESS DOUBLE BETA DECAY?*

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Studies of neutrinoless double beta decay can lead us to discovery of the CP symmetry breaking in lepton sector with Majorana neutrinos. In the article the necessary conditions for finding this phenomenon are obtained and discussed.

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Neutrinoless double beta decay ($(\beta\beta)_{0\nu}$) gives us possibility for studying the fundamental properties of neutrinos beyond the standard electroweak theory [1]. Studies of $(\beta\beta)_{0\nu}$ play a crucial role by probing some yet unresolved questions in neutrino physics: the Majorana nature of neutrinos, the neutrino mass spectrum, the absolute ν-mass scale, the Majorana CP phases. I will focus on the last problem.

For the massive neutrinos, the weak neutrino eigenstates (ν_α) are related to mass eigenstates (ν_i) by a transformation:

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}|\nu_i\rangle \,, \tag{1}$$

where $U_{\alpha i}$ is a unitary mixing matrix. It can be parameterised in a standard way:

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$$U_{\alpha i} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{2}$$

where c_{ij} and s_{ij} are cosines and sines of the θ_{ij} $(ij = 12, 13, 23)$ angles.

CP symmetry can be violated by three phases: one Dirac phase δ and two Majorana phases, α_1 and α_2. The latter ones appear only for Majorana neutrinos. Generally, the Dirac phase can be seen in neutrino oscillations, but in practice it is very difficult. CP breaking signal coming from this phase is very small or even vanishing due to fact that $\sin\theta_{13}$ and $e^{\pm i\delta}$ always appear in a combination and from the present fits [2–4] it follows that $\sin^2\theta_{13} < 0.05$ for 99.7% C.L. As far as Majorana phases are considered they do not affect neutrino oscillations.

Another way for discovering the CP violation is searching for the double beta decay $(\beta\beta)_{0\nu}$ [5]. It is a nuclear process which changes the nuclear charge Z by two units while leaving the atomic mass A unchanged without neutrino emission. It is allowed when neutrino and antineutrino are identical particles (Fig. 1).

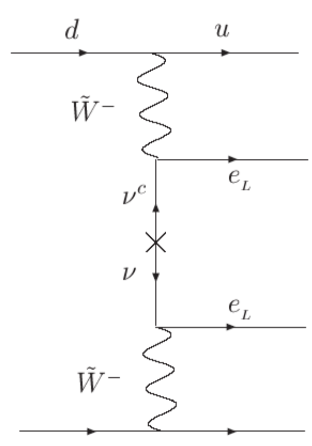

Fig. 1. The Feynman diagram for neutrinoless double beta decay.

If neutrino exchange process mainly governs $(\beta\beta)_{0\nu}$ process it's decay half-life time is given by the expression:

$$\left[T_{1/2}^{0\nu}(A,Z)\right]^{-1} = |\langle m_\nu \rangle|^2 |M^{0\nu}(A,Z)|^2 G^{0\nu}(E_0,Z)\,, \tag{3}$$

where

$$\langle m_\nu \rangle = \left| \sum_{i=1}^{3} U_{ei}^2 m_i \right| \tag{4}$$

is the effective Majorana mass, $M^{0\nu}(A,Z)$ — the nuclear matrix element (NME), and $G^{0\nu}(E_0,Z)$ — phase-space factor. Having measured $T_{1/2}^{0\nu}(A,Z)$ we can find $\langle m_\nu \rangle$, provided that we know NME ($G^{0\nu}(E_0,Z)$ is directly calculable). Unfortunately, calculation of the NME is a very complicated nuclear problem since many intermediate nuclear states must be taken into account causing that different calculations of the same NME differ by factor 2–3 or even more. Recently the new calculation, where the observed $(\beta\beta)_{2\nu}$ decay has been used to fix relevant parameters, has shown the great stability of the final results [6]. Then it seems that this problem was solved, opening the way to gain valuable information about neutrinos from $(\beta\beta)_{0\nu}$ studies.

The possible precision of the future experiments will give a chance to look for CP violation only for higher neutrino masses ($m_1 \gtrsim 0.1$ eV), where the mass spectrum starts to be degenerated $m_1 \approx m_2 \approx m_3 = m_\nu$. In this case the effective neutrino mass m_β measured in tritium beta decay is just equal to neutrino masses [7] $m_\beta = \left[\sum_{i=1}^{3} |U_{ei}|^2 m_i^2\right]^{1/2} = m_\nu$. We can combine both measurements to find values of CP violating phases.

For Majorana neutrinos CP symmetry holds if $\alpha_i, \delta \in \{0, \pm\frac{\pi}{2}, \pm\pi\}$. Taking into account this values, four conserving CP values of $\langle m_\nu \rangle$ can be obtained:

$$\begin{aligned}
\langle m_\nu \rangle_{(1)} &= m_\beta, \\
\langle m_\nu \rangle_{(2)} &= m_\beta \cos 2\theta_{13}, \\
\langle m_\nu \rangle_{(3)} &= m_\beta \left(\cos^2\theta_{13} |\cos 2\theta_{12}| + \sin^2\theta_{13}\right), \\
\langle m_\nu \rangle_{(4)} &= m_\beta \left(\cos^2\theta_{13} |\cos 2\theta_{12}| - \sin^2\theta_{13}\right).
\end{aligned}$$

In all cases, the relation between $\langle m_\nu \rangle$ and m_β is linear: $\langle m_\nu \rangle_{(i)} = c_i m_\beta$.

Let us assume that θ_{ij} mixing angles are known with definite precision:

$$\sin^2\theta_{ij} \in \left((\sin^2\theta_{ij})_{\min}, (\sin^2\theta_{ij})_{\max}\right)$$

with central value $(\sin^2\theta_{ij})_{bf}$. For each c_i ($i = 2,3,4$) we can calculate the maximal and minimal values $c_i^{\max}, c_i^{\min}$ (see Fig. 2.). If we denote a future

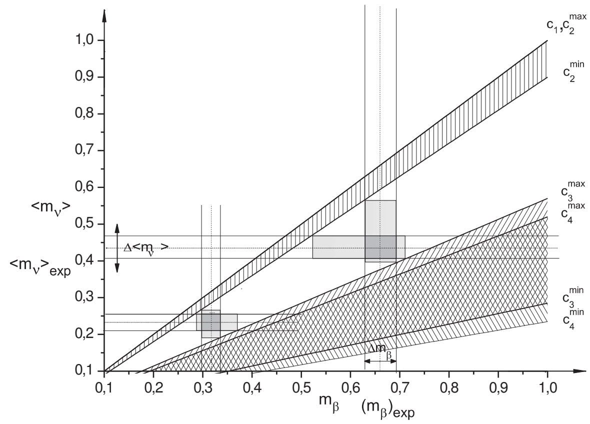

Fig. 2. A localisation of the ($c_i^{\min}$ $c_i^{\max}$) regions for the present θ_{13} and θ_{12} angles precision.

experiments precision of $\langle m_\nu\rangle$ and m_β as Δm_β and $\Delta\langle m_\nu\rangle$, localisation of the rectangle $R = (\Delta m_\beta, \Delta\langle m_\nu\rangle)$ between the lines $c_1 = 1$ and $c_4^{\min}$ will decide about CP symmetry breaking. If the rectangle R is fully located between two lines with the $c_3^{\max}$ and $c_2^{\min}$ slopes then CP symmetry is broken. This gives us the set of necessary conditions for CP symmetry breaking:

$$\begin{aligned} \Delta m_\beta &< \langle m_\nu\rangle C - \Delta\langle m_\nu\rangle D, \\ \Delta\langle m_\nu\rangle &< (m_\beta)A - (\Delta m_\beta)B, \\ c_3^{\max}\left((m_\beta)_{\exp} + \frac{\Delta m_\beta}{2}\right) &< \left(\langle m_\nu\rangle_{\exp} - \frac{\Delta\langle m_\nu\rangle}{2}\right), \\ \left(\langle m_\nu\rangle_{\exp} + \frac{\Delta\langle m_\nu\rangle}{2}\right) &< \left((m_\beta)_{\exp} - \frac{\Delta m_\beta}{2}\right) c_2^{\min}, \end{aligned} \tag{5}$$

where

$$A = c_2^{\min} - c_3^{\max}\ ,\ B = \frac{c_2^{\min} + c_3^{\max}}{2}\,,$$
$$C = \frac{A}{c_2^{\min} c_3^{\max}}\ ,\ D = \frac{B}{c_2^{\min} c_3^{\max}}\,.$$

Now we can parameterise the relative error which measures the uncertainty coming from theoretical calculations of nuclear matrix elements and experimental measurements of $(\beta\beta)_{0\nu}$ decay lifetime by $2x$, and similarly by $2y$ the relative error of the effective mass *e.g.* from tritium beta decay:

$$\Delta\langle m_\nu\rangle = 2x\langle m_\nu\rangle, \qquad \Delta m_\beta = 2y\, m_\beta\,, \tag{6}$$

Then, taking into account all conditions we find that both x and y must satisfy the same inequality:

$$x,\, y \leq \frac{1-\cos 2\theta_{12\ \min} - 3\sin^2\theta_{13\ \max} + \sin^2\theta_{13\ \min}\cos 2\theta_{12\ \min}}{1+\cos 2\theta_{12\ \min} - \sin^2\theta_{13\ \max} - \sin^2\theta_{13\ \min}\cos 2\theta_{12\ \min}}\,. \tag{7}$$

Looking at the formula we can see that the best circumstances to find CP violation arise for $\sin^2\theta_{13} \to 0$ and $\sin^2\theta_{12} \to \frac{1}{2}$, what makes the opposite situation than in case of finding Dirac phase [8].

From the same inequalities, for given relative errors x and Δm_β, we can also find the lower limit for the m_β and $\langle m_\nu \rangle$ effective masses for which measurements are still possible

$$\langle m_\nu \rangle > \frac{\Delta m_\beta}{C - 2xD}$$

and

$$m_\beta > \frac{\Delta m_\beta}{A}\left(B + \frac{2x}{C-2xD}\right).$$

Using θ_{12} and θ_{13} mixing angles recently determined [9] we obtain $x < 0.2$. Now, we can check it for the isotope of germanium ^{76}Ge where evidence for the $(\beta\beta)_{0\nu}$ decay is claimed to have been obtained [10]. Even assuming that the precision of this measurement is much better than it is:

$$x_T = \frac{\Delta T(^{76}\mathrm{Ge})}{2\langle T(^{76}\mathrm{Ge})\rangle} \leq 0.3\,,$$

and taking the new method of calculation of the NME into account, we get $x \sim 0.24$, which is still above the present necessary precision ($x < 0.2$). More careful analysis, taking into account the present precision of the mixing angle determination can give regions of relative errors $\frac{\Delta m_\nu}{m_\nu}$ and $\frac{\Delta m_\beta}{m_\beta}$ for which CP violation could be seen with various C.L. (Fig. 3).

Let us make some assumptions about future. We can expect that during the next years the precision of experiments will be strongly improved and the best values of mixing angles will not change [11, 12]:

$$\sin^2\theta_{12} \approx 0.28 \pm 0.01\,, \qquad \sin^2\theta_{13} = 0.005 \pm 0.0001\,. \tag{8}$$

Additionally, weak lensing of galaxies by large scale structure together with CMB data will measure the sum of neutrino masses $\sum = m_1 + m_2 + m_3$ to an uncertainty of 0.04 eV, so we can expect that each individual mass is known with the precision $\Delta m_\beta = 0.015$ eV [13].

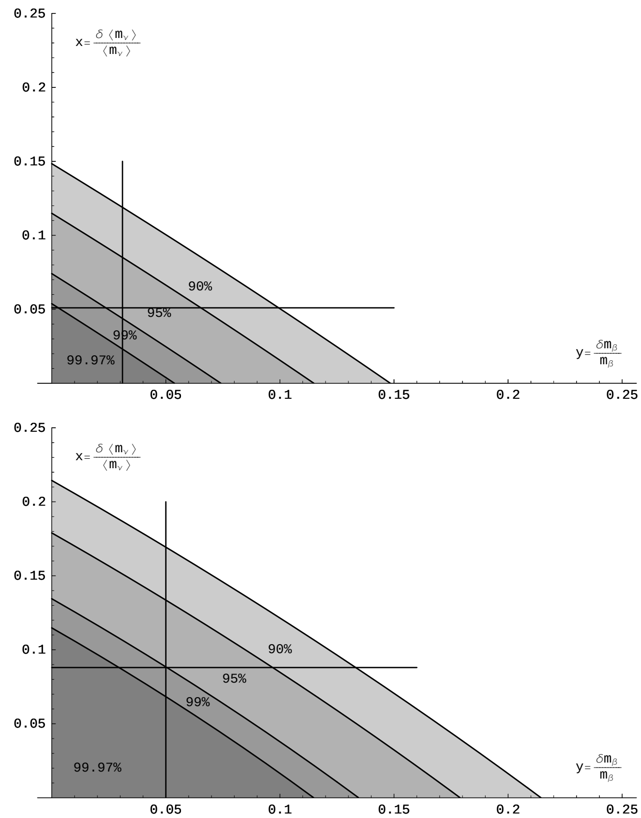

Fig. 3. Regions of relative errors $\frac{\Delta m_\nu}{m_\nu}$ and $\frac{\Delta m_\beta}{m_\beta}$ for which CP violation could be seen at present (up) and in the future (down).

If this conditions are met, the required precision of Δm_β and $\Delta\langle m_\nu\rangle$ will be x, $y < 0.36$. This precision will be obtained if relative experimental error for $T(^{76}\mathrm{Ge})$ is

$$x_T = \frac{\Delta T(^{76}\mathrm{Ge})}{2\langle T(^{76}\mathrm{Ge})\rangle} \leq 0.5\,, \tag{9}$$

which can be accomplished.

From the presented estimations it follows that measurement of CP violation for Majorana neutrinos in neutrinoless double beta decay could be possible for almost degenerate spectrum of their masses ($m_\beta > 0.1$ eV), but only under several conditions. First, neutrinoless double beta decay lifetime T should be measured with precision better than 10%, nuclear matrix elements of decaying isotopes must be calculated with much better precision, and there should be independent information about a full mechanism of the

$(\beta\beta)_{0\nu}$ decay. We need oscillation mixing angles to be measured with better precision *e.g.* $\Delta(\sin\theta_{13} \approx 0.01)$ and $\Delta(\sin\theta_{12} \approx 0.1)$ and absolute neutrino masses m_β should be found with precision $\Delta m_\beta \approx 0.02$ eV with the central value in the range $m_\beta > 0.15$ eV.

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