MASS-FLAVOUR TRANSITIONS OF SUPERNOVA NEUTRINO STATES IN THE TERRESTRIAL MATTER*

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(Received May 8, 2006)

Neutrinos coming from the distant astrophysical objects reach the Earth in incoherent mass states. Simple approximations for transitions between mass and flavour states in the Earth are given.

PACS numbers: 13.15.+g, 14.60.Lm, 14.60.Pq

1. Introduction

Supernova are very strong sources of neutrinos. Almost 99 % of the Supernova's binding energy is carried away by neutrinos. All types of neutrinos are produced during the burst. Theoretical models [1] describe some important properties for them, for instance, their energy spectrum has been confirmed by an observation of the energy spectrum of neutrinos connected with explosion of Supernova SN1987A. Each neutrino with a given flavour is produced in the Supernova core as a coherent superposition of mass states. Due to convection and diffusion they are further transported to the outer region from which they are emitted.

Oscillations between different neutrino flavour states are possible only if there is an overlapping of various mass states. We can describe the coherence between these states using so called coherence length [2]:

$$L^{\mathrm{coh}} = \frac{4\sqrt{2}E^2}{\delta m_{ij}^2}\,\sigma_x\,, \tag{1.1}$$

where E is a neutrino energy, σ_x is a half-size of the wave packet at a production point, $\delta m_{ij}^2 = |m_i^2 - m_j^2|$ and $m_{i(j)}$ is a neutrino mass. It can be said that L^{coh} is the distance beyond which neutrinos do not oscillate, no overlapping

* Presented at the Cracow Epiphany Conference on Neutrinos and Dark Matter, Cracow, Poland, 5–8 January 2006.

between different mass states occurs. The wave packet size σ_x for Supernova neutrinos is estimated to be $10^{-15} \sim 10^{-14}$ [m] for the Supernova core and 10^{-10} [m] for the neutrino-sphere. The coherence length is $L^{\rm coh} \simeq 10^{-4}$ [m] and $L^{\rm coh} \simeq 10^{-1}$ [m], respectively [3]. In practice the coherence length is much smaller than a distance between Supernova and the detection point, it means that neutrinos reach the Earth as separated eigenmass states. When Supernova neutrino hit the Earth and travel to a detector through the terrestrial matter by not a very large distance $L \lesssim 1000$ [km], the matter density is approximately uniform with $\rho = 2.5 \left[\mathrm{g/cm^3}\right]$.

In this case it is easy to find full analytical solutions for this problem, however, the results are quite large at size and not too convenient for further qualitative analysis. It is then desirable to look for some efficient approximations, they are presented in the next section.

2. Approximation

To get the amplitudes $A_{i\to\alpha}$ from which the probabilities are calculated, the eigenproblem for the effective Hamiltonian H in the terrestrial matter in the effective mass base ($\delta_{\overline{n}\,\overline{m}} = \langle \nu_{\overline{n}} | \nu_{\overline{m}} \rangle$) must be solved:

$$\mathcal{H}\, |\nu_{\overline{m}}\rangle = \frac{\lambda^2_{\overline{m}}}{2E}\, |\nu_{\overline{m}}\rangle\,. \tag{2.1}$$

The effective Hamiltonian takes the form:

$$\mathcal{H} = \frac{\delta m^2_{31}}{2E} \left(U M_0 U^+ + \varLambda V\right), \tag{2.2}$$

where M_0 is a diagonal matrix:

$$M_0 = \frac{\Delta M}{\delta m^2_{31}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2.3}$$

U is a standard mixing matrix, V has only one non zero element $V_{11} = 1$ and $\varLambda$ is defined by the equation:

$$\varLambda = \frac{2E\sqrt{2} G_{\rm F} N_e}{\delta m^2_{31}} = \frac{1.54 \times 10^{-4}\ Y_e}{\delta m^2_{31}} \rho \ \left[\frac{\rm g}{\rm cm^3}\right] E\,[\mathrm{MeV}]\,. \tag{2.4}$$

In Eq. (2.4) the numerator (in the literature denoted by $A_{\rm CC}$) describes the interaction of the charged current of electron neutrinos with the terrestrial matter. N_e is an electron number, $G_{\rm F}$ is the Fermi constant and $Y_e \sim 1/2$.

If $W_{\alpha\overline{m}} = \langle \nu_\alpha | \nu_{\overline{m}} \rangle$ is the matrix which diagonalises the Hamiltonian (2.2) in the flavour base, then each element of the amplitude receives the form:

$$A_{i\to\alpha} = \sum_{\overline{m}} W_{\alpha\overline{m}} \widetilde{W}^*_{i\overline{m}} e^{-i\frac{\lambda^2_{\overline{m}}}{2E}L}\,, \tag{2.5}$$

where

$$\widetilde{W}^*_{i\overline{m}} = \sum_\gamma U_{\gamma i} W^*_{\gamma\overline{m}}\,. \tag{2.6}$$

In a standard neutrino oscillations' model there are defined two small factors useful in perturbation calculations:

$$\alpha = \frac{\delta m^2_{31}}{\delta m^2_{21}} \simeq 0.028\,, \qquad \sin^2 2\theta_{13} < 0.05\,. \tag{2.7}$$

In the case of Supernova neutrinos oscillating in the terrestrial matter an additional small factor Λ is presented. This term is proportional to the density ρ (for terrestrial matter: $2 < \rho < 11\ \left[\mathrm{g/cm^3}\right]$) and to the neutrino energy E. As one can see (Fig. 1), Λ is the smallest of all these factors and is the best parameter to use as a perturbation parameter. Let us take the first term in (2.2) as a non-perturbated Hamiltonian and the second one, which depends on the matrix V as the first order correction, we can make a non-degenerated perturbative calculation resulting in determination of eigenvalues and eigenvectors. In this way the terms $W_{i\alpha}$ and $\widetilde{W}^*_{i\overline{m}}$ can be found. What follows, the probabilities $P_{i\to\alpha}$ can be determined and, finally, the result can be expanded in Λ

$$P_{i\alpha} = C_0 + C_1\Lambda + \mathcal{O}(\Lambda)\,. \tag{2.8}$$

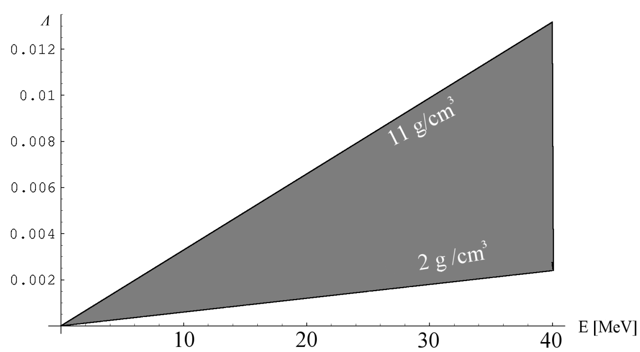

Fig. 1. A range of variability of Λ as a function of the neutrino energy and the matter density.

The coefficient C_0 reproduces an appropriate element of the mixing matrix U

$$C_0 = |U_{i\alpha}|^2\,. \tag{2.9}$$

The second coefficient C_1 can be further expanded as series of $\sin(2\theta_{13})$ (up to the linear term). Finally, the probability oscillation formula for the mass-flavour transition of Supernova neutrinos induced by the terrestrial matter can be cast into the following form:

$$P_{i\alpha} = |U_{i\alpha}|^2 + \Lambda\,(\gamma_0^{i\alpha} + \gamma_1^{i\alpha}\sin 2\theta_{13})\,. \tag{2.10}$$

3. Results and conclusions

The elements $\gamma_0^{i\alpha}$ and $\gamma_1^{i\alpha}$ which are presented in Eq. (2.10) are:

$$P_{1e}:\quad \gamma_0^{1e} = \frac{1}{2\alpha}\sin^2(2\theta_{12})(-1+\cos[\alpha\Delta])\,, \tag{3.1}$$

$$P_{2e}:\quad \gamma_0^{2e} = -\gamma_0^{1e}\,, \tag{3.2}$$

$$P_{1\mu}:\quad \gamma_0^{1\mu} = -\gamma_0^{1e}\cos^2(2\theta_{23})\,, \tag{3.3}$$

$$\begin{aligned}\gamma_1^{1\mu} &= \frac{1}{4\alpha}\sin(2\theta_{12})\sin(2\theta_{23})\big(-\alpha D_1 + D_4\\ &\quad + \cos[\delta]\left(2\cos^2(\theta_{12}) + d_1 + (-1+2\sin^2(\theta_{12}))\cos[\alpha\Delta]\right)\big)\,,\end{aligned}$$

$$P_{2\mu}:\quad \gamma_0^{2\mu} = -\gamma_0^{1\mu}\,, \tag{3.4}$$

$$\begin{aligned}\gamma_1^{2\mu} &= \frac{1}{4\alpha d_1}\sin(2\theta_{12})\sin(2\theta_{23})\big(-\alpha D_2 - d_1 D_4\\ &\quad + \cos[\delta]\big(\cos(2\theta_{12}) + 2\alpha\sin^2(\theta_{12}) + \cos(2\theta_{12})d_1\cos[\alpha\Delta]\big)\big)\,,\end{aligned}$$

$$P_{3\mu}:\quad \gamma_1^{3\mu} = \frac{1}{4d_1}(\sin(2\theta_{12})\sin(2\theta_{23})(d_1 D_1 + D_2 - \alpha\cos[\delta]))\,, \tag{3.5}$$

$$P_{1\tau}:\quad \gamma_0^{1\tau} = -\frac{1}{2\alpha}\sin^2(2\theta_{12})\sin^2(\theta_{23})\,, \tag{3.6}$$

$$\begin{aligned}\gamma_1^{1\tau} &= \frac{1}{4\alpha}\sin(2\theta_{12})\sin(2\theta_{23})\big(-\alpha D_1 + D_4\\ &\quad + \cos[\delta]\big(2\cos^2(\theta_{12}) + d_1 + (-1+2\sin^2(\theta_{12}))\cos[\alpha\Delta]\big)\big)\,,\end{aligned}$$

$$P_{2\tau}:\quad \gamma_0^{2\tau} = \gamma_0^{1e}\sin^2(2\theta_{12})\sin^2(\theta_{23})\,, \tag{3.7}$$

$$\begin{aligned}\gamma_1^{2\tau} &= \frac{1}{4d_1\alpha}\sin(2\theta_{12})\sin(2\theta_{23})(\alpha D_2 + d_1 D_4\\ &\quad + \cos[\delta](-1-2d_1\sin^2(\theta_{12}) - \cos(2\theta_{12})d_1\cos[\alpha\Delta]))\,,\end{aligned}$$

$$P_{3\tau}:\quad \gamma_1^{3\tau} = \frac{1}{4d_1}(\sin(2\theta_{12})\sin(2_{23})(-d_1 D_1 - D_2 + \alpha\cos[\delta]))\,, \tag{3.8}$$

where

$$D_1 = \cos(\delta)\cos(\varDelta) + \sin(\delta)\sin(\varDelta)\,, \tag{3.9}$$

$$D_2 = \cos(\delta)\cos(d_1\varDelta) - \sin(\delta)\sin(d_1\varDelta)\,, \tag{3.10}$$

$$D_3 = -1 + \cos(\alpha\varDelta)\,, \tag{3.11}$$

$$D_4 = \sin(\delta)\sin(\alpha\varDelta)\,, \tag{3.12}$$

$$d_1 = -1 + \alpha\,, \tag{3.13}$$

$$\varDelta = 2.54\,\frac{\delta m_{31}^2\,L}{2E}\,. \tag{3.14}$$

Approximations which have been made result in a disappearance of the $\gamma_0^{i\alpha}$ and $\gamma_1^{i\alpha}$ coefficients for P_{3e}. It means that this probability is effectively equivalent to oscillations in vacuum. The terms which contain functions D_1, D_2 and D_4 are responsible for the CP violation effects and are presented only in $\gamma_1^{i\alpha}$, they are suppressed at least by the first power of $\sin(2\theta_{13})$. It is also characteristic that probabilities $P_{3\mu}$ and $P_{3\tau}$ have only these CP terms. It can be easily checked that in our approximation the total probabilities are conserved:

$$\sum_{i=1}^{3} P_{i\alpha} = 1\,, \tag{3.15}$$

$$\sum_{\alpha=e,\mu,\tau} P_{i\alpha} = 1\,. \tag{3.16}$$

Each probability depends on L and E. As an example, in Fig. 2 the probability $P_{1\to e}(E)$ as a function of the typical Supernova neutrino energies is given. Comparing obtained analytical expressions with the exact probability $P_{i\alpha}^{\mathrm{exact}}$ (calculated numerically) we can see that approximations are better and better with decreasing E and L. Relative errors:

$$\varDelta P(E,L) = \frac{P_{i\alpha}(E,L) - P_{i\alpha}^{\mathrm{exact}}(E,L)}{P_{i\alpha}(E,L)}\,, \tag{3.17}$$

are below 2% in the whole range of energies.

In conclusion, approximated oscillation probability formulas for neutrinos coming from astrophysical sources have been presented. They are compact and suitable for analytical analyses at a very good confidence level. Results presented in this paper are only a preface to some detailed studies of the problem. In the future work the whole process of production, propagation and detection of Supernova neutrinos [5] must be considered in details. The approximations given here can be helpful in further qualitative

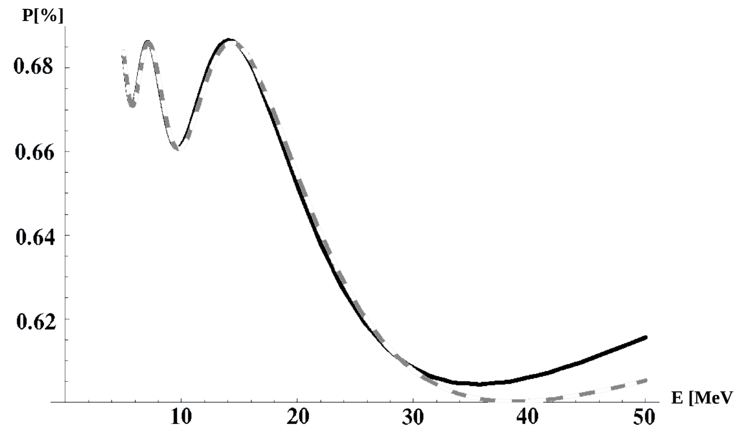

Fig. 2. The approximated oscillation probability $P_{1\rightarrow e}(E)$ (dashed line), the exact oscillation probability (solid line). $L = 1000\,[\mathrm{km}]$, $\rho = 2.5\,[\mathrm{g/cm}^3]$.

and quantitative investigations of this issue. Certainly, it will be also interesting to find another reliable approximations which would be reliable in deeper layers of the Earth ($L > 1000\,[\mathrm{km}]$).

We would like to thank J. Gluza and M. Zrałek for discussions. The work was supported in part by the Polish State Committee for Scientific Research (KBN) project 1P03B04926.

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