NEUTRINO MASSES FROM
R-PARITY NON-CONSERVING LOOPS*

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We present new formulae for the neutrino masses generated by R-parity violating interactions within minimal supersymmetric standard model. The importance of inclusion of CP phases in the neutrino mass matrix is discussed in detail.

1. Introduction

The existence of physics beyond the Standard Model (SM) has been suspected for a long time, but only quite recently this hypothesis gained a strong experimental evidence. The observation of oscillations of the solar, atmospheric, and reactor types of neutrinos [1–4] confirmed the non-zero mass of these particles, thus shedding light on new physics.

The study of neutrino properties, both experimental and theoretical, is of primary importance for the development of non-standard physics. Most effects predicted by various models beyond the SM are extremely weak and therefore very difficult to observe in experiments. The possibility of collecting precise neutrino data is the best chance by now to get an insight into this range of physics.

In the present paper we investigate the model of Majorana neutrino masses generated by particle–sparticle loops within supersymmetric standard model without R-parity [5–7]. In particular, we discuss the influence of CP Dirac and Majorana phases on the phenomenological neutrino mass matrix elements used in the calculations.

2. Neutrino masses from trilinear R-parity violating interactions

Let us briefly recall the basics of the minimal supersymmetric standard model (MSSM) [8]. The model is described by the superpotential $W + W^{\not R}$, where the so-called R-parity conserving part of the superpotential has the form

$$W = \varepsilon_{ab} \Big[(\boldsymbol{Y}_E)_{ij} L_i^a H_1^b \bar{E}_j + (\boldsymbol{Y}_D)_{ij} Q_i^{ax} H_1^b \bar{D}_{jx} + (\boldsymbol{Y}_U)_{ij} Q_i^{ax} H_2^b \bar{U}_{jx} + \mu H_1^a H_2^b \Big] ,$$

(1)

while its R-parity violating part reads

$$W^{\not R} = \varepsilon_{ab} \left[\frac{1}{2} \lambda_{ijk} L_i^a L_j^b \bar{E}_k + \lambda'_{ijk} L_i^a Q_j^{xb} \bar{D}_{kx} + \kappa^i L_i^a H_2^b \right] + \frac{1}{2} \varepsilon_{xyz} \lambda''_{ijk} \bar{U}_i^x \bar{D}_j^y \bar{D}_k^z .$$

(2)

Our notation is as follows: $\boldsymbol{Y}$'s are 3×3 Yukawa matrices, L and Q stand for lepton and quark left-handed SU(2) doublet superfields while $\bar{E}$, $\bar{U}$ and $\bar{D}$ denote the right-handed lepton, up-quark and down-quark SU(2) singlet superfields, respectively. H_1 and H_2 mean two Higgs doublet superfields. We have introduced color (x, y, z), generation (i, j, k), and the SU(2) spinor indices (a, b).

The R-parity is an accidental symmetry present in the MSSM, and is defined as $R = (-1)^{3B+L+2S}$, where B, L, and S are the baryon, lepton, and spin numbers, respectively. The introduction of R-parity violation $W^{\not R}$ implies the existence of lepton or baryon number violating processes, like the unobserved proton decay and neutrinoless double beta decay ($0\nu2\beta$). In order to get rid of too rapid proton decay and to allow for lepton number violating processes it is customary to set $\lambda'' = 0$.

The R-parity violating interactions imply the existence of Feynman diagrams depicted in Fig. 1, which generate the one-loop Majorana neutrino mass terms. The amplitudes of these processes give the following shape of the mass matrices. From the quark–squark contribution we have:

$$\mathcal{M}_{ii'}^q = \frac{3}{16\pi^2} \sum_{jkla} \left\{ \lambda'_{ijk} \lambda'_{i'kl} \left(V_{ja} V_{la} v_{ak}^q m_{d^a} \right) + \lambda'_{ijk} \lambda'_{i'lj} \left(V_{ka} V_{la} v_{aj}^q m_{d^a} \right) \right\} ,$$

(3)

where the loop integral is:

$$v_{jk}^q = \frac{1}{2}\,\sin(2\theta^k)\left(\frac{\log x_2^{jk}}{(1 - x_2^{jk})} - (x_2 \to x_1)\right).\qquad(4)$$

Here m_{d^j} is j-th generation down quark mass, θ^k is the squark mixing angle between the k-th squark mass eigenstates $M_{\tilde{d}_{1,2}^k}$, and $x_{1,2}^{jk} = m_{d^j}^2/M_{\tilde{d}_{1,2}^k}^2$. The factor 3 comes from summation over three colors of quarks. Eq. (3) includes the possible quark mixing through the CKM matrix V.
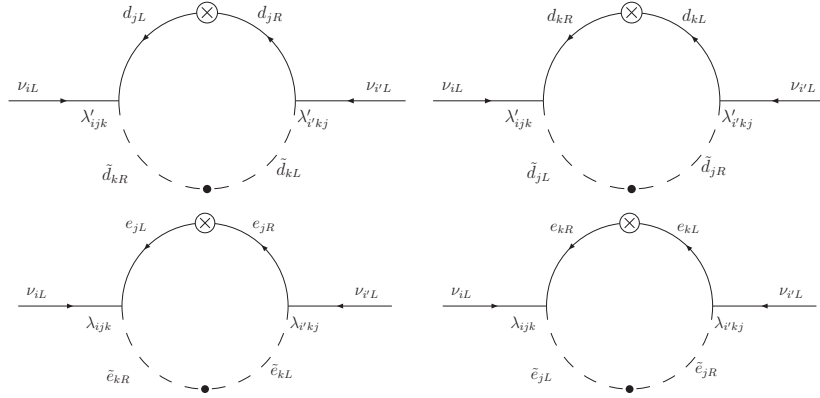

Fig. 1. Feynman diagrams leading to Majorana neutrino masses.

For loops containing lepton–slepton pairs we do not include the weak effect of mixing of leptons. The resulting neutrino mass matrix reads:

$$\mathcal{M}_{ii'}^\ell = \frac{1}{16\pi^2}\sum_{jk}\lambda_{ijk}\lambda_{i'kj}(v_{jk}^\ell m_{e^j} + v_{kj}^\ell m_{e^k}),\qquad(5)$$

with the loop integral v^ℓ having analogous form to v^q with quarks and squarks replaced by leptons and sleptons, respectively.

3. Phenomenological mass matrices and CP phases

The presented above formulae Eqs. (3)–(5) contain superparticle masses and trilinear coupling constants λ and λ'. The masses of superparticles in MSSM may be generated using the renormalisation group equations (RGE), with some unification conditions assumed at high energies (see [6] and references therein). It is important to note, that we have no information about the λ and λ' couplings neither at $m_{\rm Planck}$ nor at the electroweak scale m_Z and therefore we cannot use the RGE procedure for them (although the required RG equations are known). It follows that, after obtaining the numerical forms of $\mathcal{M}^q$ and $\mathcal{M}^\ell$ from the experimental data, we can put constraints on

the non-standard couplings λ and λ'. This idea has already been discussed in literature in various forms [5]. All of them, however, were based on many simplifications, like the approximate treatment of squark (slepton) mixing, negligence of quark mixing and very simplified treatment of the MSSM mass spectrum.

A problem directly related to the discussed mass formulae is the determination of the phenomenological neutrino mass matrices from the experimental data. These can be evaluated using the well-known relation $\mathcal{M}^{\mathrm{ph}} = U \cdot \mathrm{diag}(m_1, m_2, m_3) \cdot U^T$, m_i being the neutrino mass eigenvalues. The standard parameterisation of the PMNS matrix in terms of the three mixing angles is:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}, \quad (6)$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$, and θ_{ij} is the mixing angle between the neutrino flavour eigenstates labelled by indices i and j. The recent global analysis of neutrino oscillations [9] yields the best fit values: $\sin\theta_{12} = 0.55$, $\sin\theta_{23} = 0.71$ and $\sin\theta_{13} = 0$.

In the case of Majorana neutrinos the matrix (6) has to be multiplied by $\mathrm{diag}(1, e^{i\alpha_{21}}, e^{i\alpha_{31}})$. The Majorana phases α_{21} and α_{31}, and the Dirac phase δ are undetermined. They can take arbitrary values and for simplicity are often neglected. This simplification may lead to significant errors. Let us investigate the influence of all three phases on the elements of $\mathcal{M}^{\mathrm{ph}}$ in the case of normal hierarchy of neutrino masses. In such a case we have $m_1 \ll m_2 \ll m_3$, and all the mass eigenvalues may be expressed in terms of the lightest m_1: $m_2 = \sqrt{\Delta m_{21}^2 + m_1^2}$ and $m_3 = \sqrt{\Delta m_{31}^2 + m_1^2}$, where $\Delta m_{21}^2 = 7.1 \times 10^{-5}$ eV^2, and $\Delta m_{31}^2 = 2.0 \times 10^{-3}$ eV^2 [9]. The results are presented in Fig. 2. A similar analysis for the case of inverted hierarchy is shown in Fig. 3.

One sees that the influence of the possible CP phases is much greater than the uncertainty in the experimental values of the neutrino mixing angles. This proves the importance of careful treatment of these phases in theoretical calculations. At this point one remark is in order. We have found several regions of $m_{1(3)}$ for which there exist combinations of phases which give certain elements in the neutrino mass matrix equal to zero. This is an interesting observation, especially when related to the $\mathcal{M}_{ee}$ element, which is governing the $0\nu2\beta$ decay rate. It turns out that for some sets of parameters this decay may be suppressed even for three massive Majorana neutrinos. For example, our calculations suggest that $\mathcal{M}_{ee} = 0$ for normal hierarchy and $5.72 \times 10^{-5} < m_1 < 7.94 \times 10^{-3}$ eV.
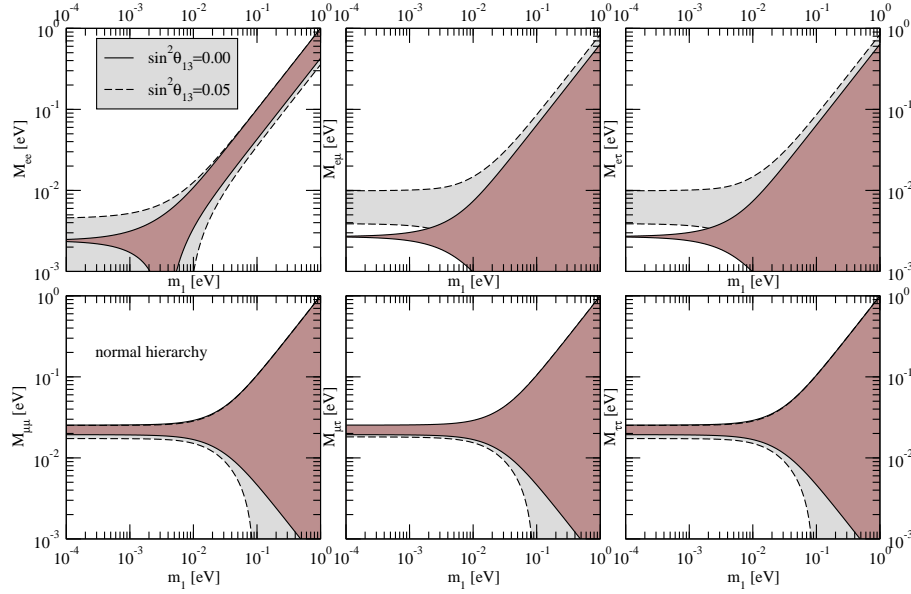
Fig. 2. Allowed values of the neutrino mass matrix elements as functions of m_1 in the case of normal neutrino mass hierarchy. The best fit values of mixing angles were used with the exception of $\sin^2\theta_{13}$ for which we consider two separate cases.
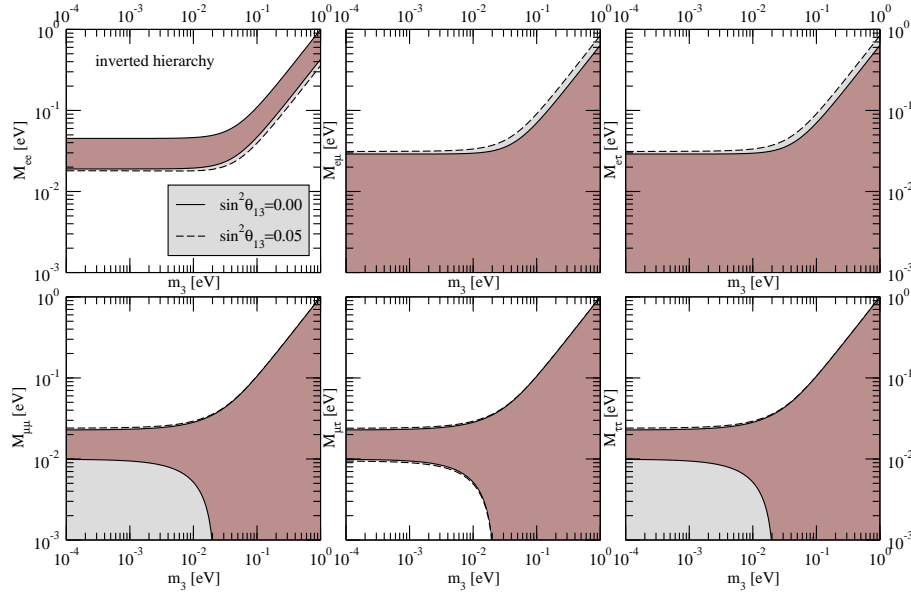
Fig. 3. Same as Fig. 2 but assuming the inverted hierarchy of neutrino masses.

As a consequence for the trilinear R-parity violating mechanism described in the previous section, these combinations of neutrino parameters imply either $\lambda = \lambda' = 0$, or the existence of other mechanism which suppresses the amplitudes of the diagrams from Fig. 1. A more detailed study on this topic is surely needed.

4. Conclusions

We have discussed the possibility of generating small Majorana neutrino masses within the framework of R-parity violating minimal supersymmetric standard model. The 1-loop level contributions are given by particle–sparticle loops. We have calculated the amplitudes of such processes taking into account quark, squark and slepton mixing in an exact manner. We have also outlined the procedure for finding constraints on the trilinear nonstandard coupling constants λ and λ'. The numerical results will be published in a full-length paper [7]. We have also discussed in detail the importance of CP phases present in the phenomenological neutrino mass matrix, which can have an important impact on the R-parity violating MSSM model as well as on the predictions for exotic nuclear processes.

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