SPIN AMPLITUDE FORMALISMS FOR MASSIVE PARTICLES IN THE DRELL–YAN PROCESS*

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In this contribution calculations of leading and next-to-leading order (hard photon radiation) matrix element for Drell–Yan process ($qq \to Z \to ll$) are presented. Two different spin amplitude formalisms are used in the calculations. Results are compared numerically and cross-checked against the published calculations based on the Dirac-trace methods.

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1. Introduction

Higher order calculation of cross section (which can be boiled down to calculation of matrix element (ME) squared $|\mathcal{M}|^2$) are crucial both for precise measurements of Standard Model parameters (W mass, the Weinberg angle, and other) and for searches for "new physics". Apart from multi-loop contributions, the higher-order tree amplitudes play an essential role in such calculations. The evaluation of tree-level Feynman diagrams by "standard algebraic techniques", although straight-forward, becomes impractical when both the number of external lines and the number of diagrams involved become large. This lead theorists to invent spin amplitude (SA) formalisms, through which ME can be calculated and evaluated numerically very efficiency, even for complicated processes. In the standard method after writing down Feynman diagrams corresponding to a given amplitude $\mathcal{M}$ one usually proceeds to derive an analytic expression for $\sum |\mathcal{M}|^2$, with an appropriate spin and/or color sum or average. The result, which is usually a function of Minkowski dot products of the particle four-momenta, is then evaluated numerically at given phase-space points. On the other hand, the SA provides a compact expression for the matrix elements in terms of the

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so-called basic bricks. These bricks are defined for each spin formalism separately. Subsequently, the ME is calculated numerically at given phase-space points. The result being simply a complex number needs to be squared in order to obtain $|\mathcal{M}|^2$. The spin amplitude formalism not only facilitates calculations of matrix elements for multiparticle processes but also allows to study polarization effects. Moreover, within this formalism one can easily incorporate "new-physics" phenomena, such as extra intermediate bosons (*e.g.* Z'-boson, Kaluza–Klein towers in extra-dimension scenarios), anomalous couplings, *etc.* In this paper we compare results of calculations based on two different spin-amplitude formalisms for massive particles. The first one proposed by Hagiwara and Zeppenfeld (HZ) [1] is based on the Weyl spinor representation and provides expression for spin amplitudes in terms of the so-called spinorial string functions. The second method extends the CALKUL helicity-amplitude formalism to the massive-fermion case and provides analytical results for spin amplitudes in terms of the so-called spinor inner products and was developed by Kleiss and Stirling [2]. Both methods are widely used for tree-level calculation and exponentiations, *e.g.* HZ method was used in Monte Carlo generator WINHAC [3], the KS method, on the other hand was exploited in [4]. In this note we present analytical formulae we have obtained using the HZ method, whereas in the case of KS method we used expressions from Ref. [4].

2. Hagiwara–Zeppenfeld formalism

In this approach, spinors are expressed in the Weyl basis, the vector–boson polarizations in the Cartesian basis, and the spin amplitudes are evaluated numerically for arbitrary four–momenta and masses of fermions and bosons. This evaluation amounts, in practice, to multiplying 2×2 c-number matrices by 2-dimensional c-number vectors. The basic brick of this method is the spinorial string function:

$$S(p_i,a_1,\ldots,a_n,p_j)^{\alpha}_{\lambda_i,\lambda_j} = \chi^{\dagger}_{\lambda_i}(p_i)[a_1,\ldots,a_n]^{\alpha}\chi_{\lambda_j}(p_j), \tag{1}$$

where

$$\begin{aligned}\chi_+(p) &= \frac{1}{2|\vec{p}|(|\vec{p}|+p_z)}\begin{bmatrix}|\vec{p}|+p_z\\ p_x+ip_y\end{bmatrix},\\ \chi_-(p) &= \frac{1}{2|\vec{p}|(|\vec{p}|+p_z)}\begin{bmatrix}-p_x+ip_y\\ |\vec{p}|+p_z\end{bmatrix}\end{aligned} \tag{2}$$

are the two-component Pauli spinors corresponding to an external fermion with four-momentum p. The internal part of the above string function:

$$[a_1,a_2,\ldots,a_n]^{\alpha} = (\not{a}_1)_{\alpha}(\not{a}_2)_{-\alpha}\ldots(\not{a}_n)_{(-1)^{n+1}\alpha} \tag{3}$$

is the product of 2×2 c-number matrices, where

$$(\not{a})_{\pm} = \begin{bmatrix} a^0 \mp a^3 & \mp(a^1 - ia^2) \\ \mp(a^1 + ia^2) & a^0 \pm a^3 \end{bmatrix}, \tag{4}$$

with $a = (a^0, a^1, a^2, a^3)$ being the four-vector in the Minkowski space. In this section we present analytical calculation which we performed using the HZ formalism[1]. Results of the calculations are compared numerically with other methods.

2.1. Born level calculations

The Born-level Feynman diagram for single-Z production in quark–quark collisions

$$q(p_1, \sigma_1) + \bar{q}(p_2, \sigma_2) \longrightarrow Z(Q, \lambda),$$

is depicted in Fig. 1 (first diagram on the r.h.s.) where $(p_i,\, \sigma_i)$ denotes the four-momentum and helicity $(\sigma_i = \pm 1)$ of the corresponding quark, while $(Q,\, \lambda)$ is the four-momentum and polarization of the Z-boson $(\lambda = 1, 2, 3)$. The spin amplitudes for this process, in the convention of [1], read

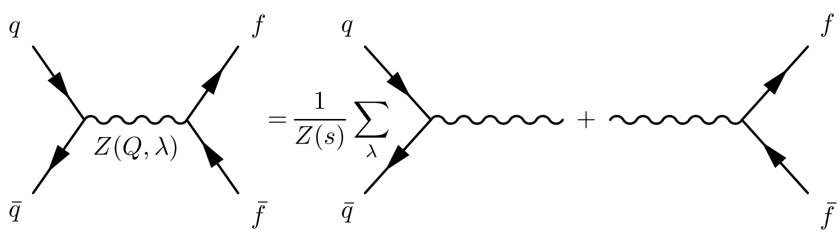

Fig. 1. The Born-level Feynman diagram.

$$\begin{aligned} \mathcal{M}_P^{(0)}(\sigma_1, \sigma_2; \lambda) \;=\; & ie\Big[c_{\rm L}\omega_{-\sigma_1}(p_1)\sigma_2\omega_{\sigma_2}(p_2)S(p_2, \epsilon_Z^*(Q,\lambda), p_1)^{-}_{-\sigma_2,\sigma_1} \\ & -c_{\rm R}\omega_{\sigma_1}(p_1)\tau_2\omega_{-\sigma_2}(p_2)S(p_1, \epsilon_Z^*(Q,\lambda), p_2)^{+}_{-\sigma_2,\sigma_1}\Big], \end{aligned} \tag{5}$$

where e is the positron electric charge, $c_{\rm L}$ and $c_{\rm R}$ are coupling constants for left and right handled fermions, respectively, $\omega_{\pm}(p) = \sqrt{p^0 \pm |\vec{p}|}$, $\epsilon_Z(Q, \lambda)$ is the Z-boson polarization vector ($*$ denotes the c-number conjugation); and $S(\ldots)$ is the spinorial string function. The spin amplitudes for the Born-level Z-boson decay:

$$Z(Q, \lambda) \longrightarrow f(q_1, \tau_1) + \bar{f}(q_2, \tau_2)\,,$$

[1] Only the final formulae and the numerical results are shown. Detailed calculations were presented in [5].

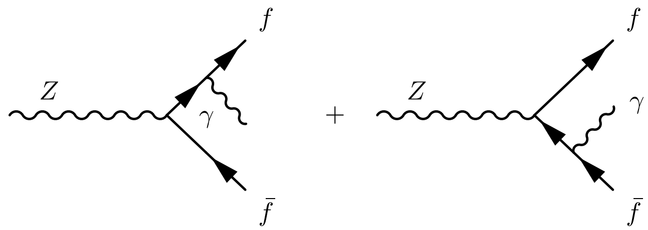

Fig. 2. The Feynman diagrams for Z-boson decay including single real-photon radiation.

shown diagrammatically in Fig. 1 (second diagram on the r.h.s.), are given by

$$\mathcal{M}_D^{(0)}(\lambda;\tau_1,\tau_2) \;=\; ie\Big[c_{\rm L}\omega_{-\tau_1}(q_1)\tau_2\omega_{\tau_2}(q_2)S\big(q_1,\epsilon_Z(Q,\lambda),q_2\big)^-_{\tau_1,-\tau_2} \\ -c_{\rm R}\omega_{\tau_1}(q_1)\tau_2\omega_{-\tau_2}(q_2)S\big(q_1,\epsilon_Z(Q,\lambda),q_2\big)^+_{\tau_1,-\tau_2}\Big], \quad (6)$$

where $\tau_{1,2}$ denote the helicities of the final-state fermions. Then, the Born-level matrix element for the single-Z production and decay is given by the coherent sum of the above spin amplitudes over the Z-boson polarizations multiplied by the Breit–Wigner function corresponding to the Z propagator:

$$\mathcal{M}_0(\sigma_1,\sigma_2;\tau_1,\tau_2) = \frac{1}{Q^2-M_Z^2+iM_Z\mathit{\Gamma}_Z}\sum_\lambda \mathcal{M}_P^{(0)}(\sigma_1,\sigma_2;\lambda)\mathcal{M}_D^{(0)}(\lambda;\tau_1,\tau_2)\,, \quad (7)$$

where M_Z and $\mathit{\Gamma}_Z$ are mass and width of the Z-boson.

2.2. Real hard-photon radiation

In this subsection we present the scattering amplitudes for single hard-photon radiation in fermionic Z-boson decays using the spin-amplitude formalism of Ref. [1] and the notation introduced in the previous subsections. For the process

$$Z(Q,\lambda) \longrightarrow f(q_1,\tau_1) + \bar{f}(q_2,\tau_2) + \gamma(k,\kappa), \quad (8)$$

in this case we need only to calculate amplitude

$$
\begin{aligned}
M_D^{(1)}(\lambda;\tau_1,\tau_2,\kappa) &= \frac{-ie^2 Q_f}{2} \\
&\times\Bigg[c_{\mathrm{L}}\omega_{-\tau_1}(q_1)\tau_2\omega_{\tau_2}(q_2)\Bigg\{\left(\frac{2q_2\cdot\varepsilon^*}{q_2\cdot k}-\frac{2q_1\cdot\varepsilon^*}{q_1\cdot k}\right)S(q_1,\varepsilon_Z,q_2)^-_{\tau_1,-\tau_2} \\
&+\frac{1}{q_2\cdot k}S(q_1,\varepsilon_Z,k,\varepsilon^*,q_2)^-_{\tau_1,-\tau_2}-\frac{1}{q_1\cdot k}S\big(q_1,\varepsilon^*,k,\varepsilon_Z,q_2\big)^-_{\tau_1,-\tau_2}\Bigg\} \\
&-c_{\mathrm{R}}\omega_{\tau_1}(q_1)\tau_2\omega_{-\tau_2}(q_2)\Bigg\{\left(\frac{2q_2\cdot\varepsilon^*}{q_2\cdot k}-\frac{2q_1\cdot\varepsilon^*}{q_1\cdot k}\right)S(q_1,\varepsilon_Z,q_2)^+_{\tau_1,-\tau_2} \\
&+\frac{1}{q_2\cdot k}S(q_1,\varepsilon_Z,k,\varepsilon^*,q_2)^+_{\tau_1,-\tau_2}-\frac{1}{q_1\cdot k}S\big(q_1,\varepsilon^*,k,\varepsilon_Z,q_2\big)^+_{\tau_1,-\tau_2}\Bigg\}\Bigg], \quad (9)
\end{aligned}
$$

where Q_f is the electric charges (in units of the positron charge) of the fermion $\varepsilon = \varepsilon(k,\kappa)$ is the polarization vector of the photon with four-momentum k (because the photon is massless, $\kappa = 1,2$) and $\varepsilon_Z = \varepsilon(Q,\lambda)$ is the polarization vector of the Z-boson. The QED gauge invariance for these amplitudes means that $\mathcal{M}_D^{(1)}(\varepsilon\to k)=0$. We have checked both analytically and numerically that after the replacement $\varepsilon\to k$ in Eq. (9) the values of the spin amplitudes are consistent with zero (in the numerical case within the double-precision accuracy). We have also checked the soft-photon ($k^0 \ll Q^0$) approximation:

$$
\mathcal{M}_D^{(1)}(q_1,q_2,k)\approx\mathcal{M}_0(q_1,q_2)\left(\frac{q_2\cdot\varepsilon^*}{q_2\cdot k}-\frac{q_1\cdot\varepsilon^*}{q_1\cdot k}\right). \quad (10)
$$

Then, the matrix element for single-Z production and radiative Z decay:

$$
q(p_1,\sigma_1)+\bar{q}(p_2,\sigma_2)\longrightarrow Z(Q,\lambda)\longrightarrow f(q_1,\tau_1)+\bar{f}(q_2,\tau_2)+\gamma(k,\kappa), \quad (11)
$$

can be obtained through substitution in Eq. (7) for $\mathcal{M}_D^{(0)}(\lambda;\tau_1,\tau_2)$ by the amplitude $\mathcal{M}_D^{(1)}(\lambda;\tau_1,\tau_2)$

2.3. Numerical results

Fig. 3 shows a cross-check of spin amplitudes against analytical (trace) calculations at Born-level. In order to compare analytical (trace) calculation with the one based on HZ and KS methods for the hard photon radiation, randomly selected phase-space points have been used. Cross-checks for a few very different phase-space-points are summarized in Table I where ξ is an angle between momentum of photon and fermion, k^0 is energy of radiated photon, $\mathcal{M}^2_{\mathrm{trace}}$ is squared matrix element calculated using trace method [6] and δ_{HZ} and δ_{KS} are defined as follows: $\delta_x = \frac{|\mathcal{M}_x^2-\mathcal{M}^2_{\mathrm{trace}}|}{\mathcal{M}^2_{\mathrm{trace}}}$. These results confirm agreement with astonishing accuracy.

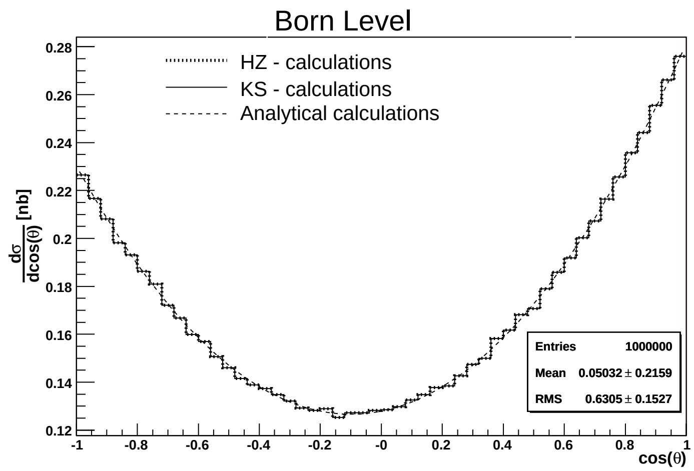

Fig. 3. The histograms of the Born-level calculations in case of massive fermions.

TABLE I

The comparison of the NLO ME calculation based on the HZ and KS methods for a few very different phase-space-points.

$\cos\xi$	k^0 [GeV]	$\mathcal{M}^2_{\rm trace}$	$\delta_{\rm HZ}$	$\delta_{\rm KS}$
-0.511788	25.6668	0.000328551442	1.955977×10^{-10}	1.955939×10^{-10}
-0.875777	0.17290	55.76984701199	1.607094×10^{-10}	1.607068×10^{-10}
0.355232	14.0743	0.002236807849	1.751157×10^{-10}	1.751977×10^{-10}
-0.101052	37.3339	0.000165045975	2.088114×10^{-10}	2.092016×10^{-10}
0.751291	11.2580	0.010689805713	1.430983×10^{-10}	1.431504×10^{-10}
-0.615211	0.78082	0.954606033791	1.669959×10^{-10}	1.669928×10^{-10}
0.765231	4.82919	0.057749194232	1.381154×10^{-10}	1.381294×10^{-10}
0.993571	18.2526	0.047609412950	1.618049×10^{-10}	1.625703×10^{-10}
-0.999340	41.5709	0.006670198811	1.478458×10^{-10}	1.547418×10^{-10}
-0.862227	9.89213	0.011222199139	1.672257×10^{-10}	1.671238×10^{-10}

3. Outlook

The presented matrix elements in the (HZ) Hagiwara–Zeppenfeld formalism for single-photon radiation will be incorporated in the Yennie–Frautschi–Suura exclusive exponentiation framework for multiphoton radiation effects in the Drell–Yan process. In addition to the QED corrections, also the electroweak and QCD effects will be included in the future. The above calculation will be a basis for constructing a Monte Carlo event generator for the Drell–Yan process at the current and future high-energy physics colliders, such as the Tevatron and the LHC.

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