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THEORY AND PHENOMENOLOGY OF NUCLEAR DEEP INELASTIC SCATTERING* **

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I will discuss the results of recent phenomenological study of unpolarized nuclear structure functions for a wide range of nuclei. As a basis of our phenomenology we develop a model which addresses a number of different mechanisms of nuclear scattering including corrections due to Fermi motion, binding, off-shell modification of the bound nucleon structure functions, nuclear shadowing, nuclear pion excess. The application of this approach to charged-current neutrino-nuclear scattering is also reviewed.

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1. Introduction and motivations

The lepton Deep Inelastic Scattering (DIS) remains to be the primary source of experimental information on the distribution of quark and gluon fields in the nucleon and nuclei. Nuclei serve a dual purpose in the DIS studies. On the one hand the study of nuclei at small space-time scales is interesting by itself and it can provide valuable insights into the origin of nuclear force and properties of hadrons in nuclear medium. On the other hand the nuclear data often serve as the source of information on hadrons otherwise not directly accessible (*e.g.*, extraction of the neutron structure function which is usually obtained from deuterium and proton data).

Significant nuclear effects were experimentally observed in charged-lepton DIS experiments (for a review see [1–3]), which indicated that the nuclear environment plays an important role even at energies and momenta much larger than those involved in typical nuclear ground state processes. The understanding of nuclear effects is therefore directly related to the interpretation of high-energy experiments with nuclei from hadron colliders to fixed

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target experiments. This is particularly relevant for neutrino processes, in which weak interaction with matter requires the use of heavy nuclear targets in order to collect a significant number of interactions, and a reliable treatment of nuclear effects is important and in some cases crucial for reducing systematic uncertainty.

In this paper we briefly review the results of recent studies of charged-lepton nuclear DIS of Ref. [4]. The applications to neutrino scattering are also discussed.

2. Nuclear structure functions

We recall that nuclear effects in DIS are experimentally studied through the measurement of the ratio $\mathcal{R}_2$ of the structure function F_2 of two nuclei (usually a complex nucleus to deuterium). One can separate a few regions of characteristic nuclear effects observed for variety of targets: depletion of nuclear structure functions at small Bjorken x ($x < 0.05$) known as shadowing region; a small enhancement of nuclear structure functions for $0.1 < x < 0.3$ (antishadowing); depletion with a minimum around $x = 0.6 \div 0.7$ followed by a rise at large x (known as "EMC effect" after the name of the experiment which discovered it, see *e.g.* [1,2]). It is interesting to note that a clear Q^2 dependence has been reported only in the shadowing region, while for $0.1 < x < 0.6$ $\mathcal{R}_2$ is almost Q^2 independent. However, the data available on the Q^2 dependence of nuclear effects are still scarce.

Many different models have been proposed to explain the basic features of data (see the reviews [1–3] and the references cited therein, see also [4]). However, consistent and quantitative description of nuclear effects in DIS in a wide kinematical region of x and Q^2 and for a wide range of nuclei is clearly needed. In this paper we review the results of the analysis of Ref. [4] which was aimed to develop a quantitative model of nuclear DIS applicable in the analysis of existing data and in the interpretation of future experiments.

The theoretical background of the analysis of Ref. [4] involves the treatment of a number of mechanisms which appear to be different for small and large Bjorken x as viewed from the laboratory system. The physics scale of this separation comes from comparison of a characteristic DIS time $1/Mx$ (or longitudinal correlation length) in the laboratory system to an average distance between bound nucleons. At large $x > 0.1$, the small DIS time justifies the use of the incoherent scattering approximation (impulse approximation). Important nuclear corrections in this region are due to nuclear binding and momentum distribution (Fermi motion).

It must be emphasized that the effects beyond the impulse approximation are important. It was realized long ago that the nuclear pion correction to the structure functions is important to balance a "missing" nuclear light-cone momentum in impulse approximation. Furthermore, at small $x \ll 0.1$ the DIS time becomes large on the nuclear scale and in this region one has to take into account the multiple nuclear interactions of virtual hadronic states the intermediate boson fluctuates to, the mechanism which leads to nuclear shadowing effect (for a review see Ref. [2]).

Summarizing, for the nuclear structure function (to be specific we discuss F_2) we can write

$$F_2^A = F_2^{p/A} + F_2^{n/A} + \delta F_2^{\pi/A} + \delta_{\mathrm{coh}} F_2^A \,, \tag{1}$$

where $F_2^{p/A}$ ($F_2^{n/A}$) are the incoherent contributions from the bound protons (neutrons) corrected for Fermi motion, nuclear binding and off-shell effects (impulse approximation). The term $\delta F_2^{\pi/A}$ is a correction associated with scattering off nuclear pion (meson) field. The last term in (1) is a correction due to coherent interaction of intermediate virtual boson with nuclear target.

The terms $F_a^{p/A}$ and $F_a^{n/A}$ are given in terms of the proton and the neutron structure function averaged with the nuclear spectral function and corresponding kinematical factors. In particular, for $F_2^{p/A}$ we have[1]

$$F_2^{p/A}(x, Q^2) = \int \mathrm{d}\varepsilon \mathrm{d}^3\boldsymbol{k}\, \mathcal{P}^p(\varepsilon, \boldsymbol{k}) \big(1 + k_z/M\big) F_2^p(x', Q^2, k^2) \,, \tag{2}$$

where $k = (M + \varepsilon, \boldsymbol{k})$ and $x' = x/(1 + (\varepsilon + k_z)/M)$ are the four-momentum of bound proton and its Bjorken variable and $\mathcal{P}^p(\varepsilon, \boldsymbol{k})$ is the proton spectral function which describes the distribution of bound protons over momentum and energy. Similar expression holds for the neutron term.

The analysis of Ref. [4], which is based on realistic nuclear spectral function, indicates that, although the Fermi motion and binding corrections provide a correct trend of observed behavior of the ratios $\mathcal{R}_2 = F_2^A/F_2^D$ (EMC ratio), the quantitative description of data is missing in impulse approximation. In a quantitative treatment it is important to go beyond this approximation and take into account the modification of the nucleon the nucleon structure functions in nuclear environment. In Ref. [4] this effect is controlled by off-shell correction, *i.e.* the k^2 dependence of the nucleon structure functions in Eq. (2). Since characteristic energies and momenta of bound nucleons are small compared to the nucleon mass the off-shell effect can be treated as a linear correction in $k^2 - M^2$ to the structure function of the on-shell nucleon

$$F_2(x, Q^2, k^2) = F_2(x, Q^2) \left[1 + \delta f_2(x, Q^2) \frac{k^2 - M^2}{M^2}\right] . \tag{3}$$

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[1] We note that Eq. (2) is written for the kinematics of the Bjorken limit assuming that momentum transfer is along z axis $q = (q_0, 0_\perp, -|\boldsymbol{q}|)$. For the derivation and more general expressions valid at finite Q see [4].

The function $\delta f_2(x, Q^2)$ describes the relative off-shell effect. In Ref. [4] this function was studied phenomenologically. In particular, δf_2 was assumed to be independent of Q^2 and parametrized as

$$\delta f_2 = C_N(x - x_1)(x - x_0)(h - x), \tag{4}$$

where $0 < x_1 < x_0 < 1$ and $h > 1$. These parameters were fixed from data as will be discussed below.

It should be also noted that because of binding the nucleons do not carry all of the light-cone momentum of the nucleus and the momentum balance equation is violated in the impulse approximation thus indicating the presence of non-nucleon degrees of freedom in nuclei which carry the missing momentum. A natural way to solve this problem is to explicitly consider the scattering off the pion field in nuclei that gives rise to the corresponding correction to the structure functions. In Ref. [4] this correction was considered in the convolution approximation

$$\delta F_2^{\pi/A} = \int_x \mathrm{d}y f_{\pi/A}(y) F_2^{\pi}\left(\frac{x}{y}, Q^2\right), \tag{5}$$

where $f_{\pi/A}(y)$ is the distribution of nuclear pion excess in a nucleus and F_2^{π} is the pion structure function. The distribution function $f_{\pi/A}(y)$ was calculated in [4] using the constraints from equations of motion for interacting pion-nucleon system. In fact, by using the light-cone momentum balance equation we effectively constrain the contribution from all mesonic fields responsible for nuclear binding.

In the small-x region the coherent effects in DIS are relevant (for a review see Ref. [2]). These effects are associated with the fluctuations of intermediate virtual boson into quark-gluon (or hadronic) states. At small x an average time of life of such fluctuation is significantly larger than the average distance between bound nucleons. For this reason the virtual hadronic states undergo multiple nuclear interactions while traversing a nucleus that causes nuclear shadowing effect. The rate of this effect depends on the scattering amplitude of the virtual hadronic states off the nucleon. In our approach we model the interaction of virtual hadronic states with the nucleon as scattering of a single state with effective cross section $\bar{\sigma}$. In this approximation the relative nuclear correction to the structure function is determined by the corresponding correction to effective cross section $\bar{\sigma}$. The nucleon effective cross section $\bar{\sigma}$ is treated phenomenologically in [4] and parametrized as

$$\bar{\sigma} = \sigma_1 + \frac{\sigma_0 - \sigma_1}{1 + \frac{Q^2}{Q_0^2}}, \tag{6}$$

where the parameters σ_0 and σ_1 describe low-Q and high-Q limits while the scale Q_0 controls transition region. The nuclear effective cross section was calculated using the Glauber–Gribov multiple scattering theory.

We used the outlined approach in the analysis of data on the ratios $\mathcal{R}_2(A/B) = F_2^A/F_2^B$ of structure functions of two nuclei. The general goal was to develop a quantitative model of nuclear structure functions which, from one side, would include the major mechanisms of nuclear scattering and, from the other side, would describe the existing data with acceptable accuracy. We analyze data on $\mathcal{R}_2$ for a variety of targets from D to Pb for a wide kinematical region (for more detail see Table 1 of Ref. [4]). From preliminary analysis of data we observed strong correlations between some of the model parameters. In order to reduce the number of free parameters we used additional constraints. In particular, the parameters h and x_0 turned out to be fully correlated and related as $h = 1 + x_0$. The parameter x_1 is strongly correlated with C_N and Q_0.

We performed several fits with different fixed values of x_1 and found that $x_1 = 0.05$ corresponds to the lowest χ^2 and provides a good cancellation between shadowing and off-shell correction to the normalization of nuclear valence quark distribution. From preliminary fits the best fit value of σ_1 was consistent with zero and we fixed $\sigma_1 = 0$ in the final fit. The parameter σ_0 was fixed to 27 mb (averaged meson-nucleon total cross section in the vector meson dominance model [2, 5]) in order to reproduce the photoproduction limit. The remaining parameters C_N, x_0 and Q_0 were adjusted to reproduce data. The global fit to all data results in $C_N = 8.1 \pm 0.3 \pm 0.5$, $x_0 = 0.448 \pm 0.005 \pm 0.007$, $Q_0^2 = 1.43 \pm 0.06 \pm 0.2$ GeV^2 with χ^2/d.o.f. $= 459/556$ (the last error is the estimate of systematic/theoretical uncertainty) [4].

Note that these parameters are common for all nuclei. This approach leads to a very good agreement with data for many different nuclei as indicated by the value of χ^2. It was possible to reproduce the observed x, Q^2, and A dependencies of nuclear structure functions (for more detail see [4]). In order to test the model we performed a number of fits to different sub-sets of nuclei in the region from ^{4}He to ^{208}Pb. The results are compatible within the uncertainties with the result of the global fit thus indicating an excellent consistency between the model and the data for all nuclei.

Although the off-shell function (4) was defined for F_2 and extracted using data for F_2, an idea of universal off-shell correction δf common to all structure functions/parton distributions was tested in [4]. In particular, this hypothesis was favored by cancellation between shadowing and off-shell corrections to nuclear valence quark number.

The off-shell function $\delta f(x)$ is positive at $x > x_0$. Since $k^2 < M^2$ for the bound nucleon, the off-shell correction leads to suppression of the valence quark distribution in the bound nucleon at large x. It was argued in [4] that this observation indicates the increase of the nucleon valence core radius (or confinement scale) $r_{\rm c}$ in nuclear environment. In order to give a quantitative interpretation we note that the relative change of $r_{\rm c}$ for an off-shell nucleon in the vicinity of the mass shell is proportional to its virtuality $\delta r_{\rm c}/r_{\rm c} = \lambda(M^2 - k^2)/(2M^2)$. In order to evaluate the parameter λ we consider a simple model in which the valence quark distribution is governed by a single scale $r_{\rm c}$ and calculate $\delta f(x)$ in this model [4]. We found that the value of x_0 and the slope of the phenomenological off-shell function $\delta f_2(x)$ at $x > 0.25$ are reasonably reproduced if $\lambda \simeq 1$. The averaging with the spectral function results in about 10% increase in $r_{\rm c}$ for a bound nucleon in ^{56}Fe.

3. Neutrino–nucleus inelastic interactions

The developed model of nuclear structure functions have been applied to calculate nuclear effects in (anti)neutrino DIS interactions [4, 7, 8]. The important difference with the charged-lepton (CL) scattering, which is driven by electromagnetic interaction, is the presence of both the vector current (VC) and the axial current (AC) contributions in neutrino interactions. The VC–AC interference gives rise to P-odd and C-odd terms in the cross section which are described by the structure function F_3. This term has different sign for ν and $\bar{\nu}$ interactions and gives rise to ν–$\bar{\nu}$ asymmetry in the cross sections.

In contrast to the electromagnetic and vector currents the axial current is not conserved. While this effect can be neglected at high $Q^2 \gg 1$ GeV^2, at low Q^2 the AC contribution plays an important role and even dominates the (anti)neutrino cross sections through the longitudinal structure function $F_{\rm L}$. The latter is determined by the divergence of the current (Adler theorem [6]). At low momentum transfer the AC divergence is dominated by the pion field (PCAC relation) that allows us to calculate the leading term at low Q^2: $F_{\rm L}^{\rm PCAC} = f_\pi \sigma_\pi(s, Q^2)/\pi$, where $f_\pi = 0.93\, m_\pi$ is the pion decay constant and σ_π is the total pion-nucleon (nucleus) cross section with the center-of-mass energy squared $s = M^2 + Q^2(1/x - 1)$. It is important to note that the PCAC term leads to the rising ratio $R = F_{\rm L}/F_{\rm T}$ at low Q^2 in contrast to the CL case, in which R is vanishing at low Q^2.

In Refs. [7,8] a model of neutrino inelastic scattering was developed which interpolates between high and low Q^2 regions taking into account the CVC as well as PCAC. The approach was applied to calculate the (anti)neutrino charged-current differential cross sections for a number of nuclear targets and results were compared to NuTeV, CHORUS and NOMAD data. As a result, a good agreement between the data and our calculations was observed for all examined nuclei that provides a good test of the model of Ref. [4].

It should be noted that the data points at low x bins, which typically have low Q^2, are also reproduced by calculation. In this region the cross sections are dominated by the PCAC term.

We also remark that the studies of neutrino DIS sum rules for nuclear targets, such as the Gross–Llewellyn Smith (GLS) [9] and the Adler [10] sum rule, are of particular importance since they reflect the symmetries of interaction. At high Q^2 the GLS sum rule is related to the normalization of the valence quark distribution while the Adler sum rule is closely related to CVC and isospin symmetry[2]. More details on this subject will be given in Ref. [8].

4. Summary and perspectives

A detailed phenomenological analysis of unpolarized nuclear structure functions for a wide kinematical region of x and Q^2 was performed and a general approach was developed which, on one side, includes the main nuclear corrections and, on the other side, provides a good description of data on nuclear structure functions. We take into account the QCD treatment of the nucleon structure functions and address a number of nuclear effects including nuclear shadowing, Fermi motion and nuclear binding, nuclear pions and off-shell corrections to bound nucleon structure functions.

Our analysis suggested that data cannot be quantitatively explained in impulse approximation even at large x, by applying "standard" Fermi motion and nuclear binding corrections. This motivated us to address the off-shell effect in bound nucleon structure functions. The off-shell effect is related to the modification of the nucleon structure in nuclear environment. This relation was discussed in terms of a simple model in which the off-shell effect at large x was linked to the modification of the bound nucleon core radius (confinement scale). We found that the off-shell correction derived from our analysis favors the increase in the nucleon core radius in nuclear environment. The effective scattering amplitude which determines the magnitude of nuclear shadowing effect was also addressed phenomenologically.

The approach was applied to calculate the structure functions and differential cross sections of charged-current neutrino-nuclear scattering. It should be emphasized that neutrino DIS provides information complementary to that of the charged-lepton scattering and, therefore, the completion of new high-statistics measurements would have a large impact on our understanding of quark and gluon distributions in nuclei.

[2] We note that the analysis of Ref. [4] implicitly used the requirement of the valence quark normalization which was an important constraint on the off-shell effect.

We refer in this context to MINERνA experiment, which is dedicated to the direct studies of nuclear effects in neutrino structure functions [11] (see also [15]). The study of nuclear data from recent neutrino experiments is also useful in the context of nuclear effects. The cross-section data from NuTeV [13] and CHORUS [12] also provide information on nuclear effects in the iron and lead targets. The NOMAD experiment [14] collected large neutrino samples on ^{12}C, ^{27}Al and ^{56}Fe targets allowing a study of nuclear effects from ^{27}Al/^{12}C and ^{56}Fe/^{12}C ratios.

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