RELATIVISTIC APPROACH TO QUASI-ELASTIC NEUTRINO–NUCLEUS SCATTERING*

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A relativistic distorted-wave impulse-approximation model is applied to quasi-elastic neutrino–nucleus scattering. Neutral-current and charge-current cross sections are evaluated. The strange quark contribution to nucleon form factors is calculated in view of the possibility of its determination. Particular attention is paid to the effect of final state interactions. Their influence on the determination of the strange form factors is investigated.

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1. Introduction

Neutrino physics has gained in recent years a large interest both from the experimental and the theoretical point of view. General review papers about neutrino–nucleus reactions can be found in Refs. [1, 2]. Both weak neutral-current (NC) and charged-current (CC) scattering have stimulated detailed analyses in the intermediate-energy region [3–14], using a variety of methods including Fermi gas (FG), random phase approximation (RPA) and shell model calculations. The effects of final state interactions (FSI) were investigated in Ref. [15, 16] within the relativistic FG model or the RPA. The nuclear structure effects on the determination of strangeness contribution in NC neutrino–nucleus scattering were studied in Refs. [9, 17], and in Ref. [18] in the framework of a relativistic plane wave impulse approximation (RPWIA). The effects of FSI on the ratio of proton-to-neutron cross sections in NC scattering were discussed in Refs. [9, 19–21].

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In this contribution we are mainly interested in the effects of FSI on the determination of the strange content of the nucleon form factors in the frame of a fully relativistic model.

2. The formalism of quasi-elastic neutrino scattering

The ν-nucleus cross section of the process where a nucleon is emitted can be written as a contraction between the lepton and the hadron tensor, *i.e.*

$$d\sigma = \frac{G_{\mathrm{F}}^2}{2}\, 2\pi\, L^{\mu\nu}\, W_{\mu\nu}\, \frac{d^3k}{(2\pi)^3}\, \frac{d^3p_{\mathrm{N}}}{(2\pi)^3}\,, \tag{1}$$

where G_{F} is the Fermi constant, $k^\mu = (\varepsilon, \boldsymbol{k})$ the four-momentum of the final lepton, and $\boldsymbol{p}_{\mathrm{N}}$ is the momentum of the emitted nucleon. For charged-current processes G_{F}^2 has to be multiplied by $\cos^2\vartheta_{\mathrm{C}} \simeq 0.9749$, where ϑ_{C} is the Cabibbo angle.

The lepton tensor $L^{\mu\nu}$ separates into a symmetrical and an antisymmetrical component [22–24]. The hadron tensor is given by a bilinear product of the transition matrix elements of the nuclear weak-current operator J^μ between the initial state $|\Psi_0\rangle$ of the nucleus, of energy E_0, and the final states of energy E_{f}, that are given by the product of a discrete (or continuum) state $|n\rangle$ of the residual nucleus and a scattering state $\chi^{(-)}_{\boldsymbol{p}_{\mathrm{N}}}$ of the emitted nucleon, with momentum $\boldsymbol{p}_{\mathrm{N}}$. One has

$$\begin{aligned} W^{\mu\nu}(\omega, q) \;=\; & \sum_n \left\langle n; \chi^{(-)}_{\boldsymbol{p}_{\mathrm{N}}} \mid J^\mu(\boldsymbol{q}) \mid \Psi_0 \right\rangle \left\langle \Psi_0 \mid J^{\nu\dagger}(\boldsymbol{q}) \mid n; \chi^{(-)}_{\boldsymbol{p}_{\mathrm{N}}} \right\rangle \\ & \times \delta(E_0 + \omega - E_{\mathrm{f}})\,, \end{aligned} \tag{2}$$

where the sum runs over all the states of the residual nucleus. Using impulse approximation, the transition amplitude can be written as

$$\left\langle n; \chi^{(-)}_{\boldsymbol{p}_{\mathrm{N}}} \mid J^\mu(\boldsymbol{q}) \mid \Psi_0 \right\rangle = \left\langle \chi^{(-)}_{\boldsymbol{p}_{\mathrm{N}}} \mid j^\mu(\boldsymbol{q}) \mid \varphi_n \right\rangle, \tag{3}$$

where $\varphi_n = \langle n | \Psi_0 \rangle$ describes the overlap between the initial nuclear state and the final state of the residual nucleus, corresponding to one hole in the ground state of the target. The single-particle current operator related to the weak current is

$$j^\mu = \left[F_1^{\mathrm{V}} \gamma^\mu + i \frac{\kappa}{2M} F_2^{\mathrm{V}} \sigma^{\mu\nu} q_\nu - G_{\mathrm{A}} \gamma^\mu \gamma^5 + F_{\mathrm{P}} q^\mu \gamma^5 \right] O_\tau\,, \tag{4}$$

where $O_\tau = \tau_\pm$ are the isospin operators for CC reactions, while for NC scattering $O_\tau = 1$, and $q^\mu = (\omega, \boldsymbol{q})$ with $Q^2 = |\boldsymbol{q}|^2 - \omega^2$ is the four-momentum transfer. G_{A} is the axial form factor and F_{P} the pseudoscalar form factor.

The weak isovector form factors, $F_1^{\rm V}$ and $F_2^{\rm V}$, are related to the corresponding electromagnetic form factors by the conservation of the vector current plus, for NC reactions, a possible isoscalar strange quark contribution F_i^s, *i.e.*,

$$\begin{aligned} F_i^{{\rm V},p(n)} &= \left(\frac{1}{2} - 2\sin^2\theta_{\rm W}\right) F_i^{p(n)} - \frac{1}{2}F_i^{n(p)} - \frac{1}{2}F_i^s\,, && \text{(NC)} \\ F_i^{\rm V} &= F_i^p - F_i^n\,, && \text{(CC)} \end{aligned} \tag{5}$$

where $\theta_{\rm W}$ is the Weinberg angle. The electromagnetic form factors are taken from Ref. [25] and the strange form factors are taken as [1]

$$F_1^s(Q^2) = \frac{(\rho^s + \mu^s)\tau}{(1+\tau)(1+Q^2/M_{\rm V}^2)^2}\,, \quad F_2^s(Q^2) = \frac{(\mu^s - \tau\rho^s)}{(1+\tau)(1+Q^2/M_{\rm V}^2)^2}\,, \tag{6}$$

where $\tau = Q^2/(4M^2)$ and $M_{\rm V} = 0.843\,{\rm GeV}$.

The axial form factor is expressed as [26]

$$\begin{aligned} G_{\rm A} &= \frac{1}{2}\left(\tau_3 g_{\rm A} - g_{\rm A}^s\right) G\,, && \text{(NC)} \\ G_{\rm A} &= g_{\rm A} G\,, && \text{(CC)} \end{aligned} \tag{7}$$

where $g_{\rm A} \simeq 1.26$, $g_{\rm A}^s$ describes possible strange quark contributions, $G = (1+Q^2/M_{\rm A}^2)^{-2}$, and $\tau_3 = +1(-1)$ for proton (neutron) knockout. The axial mass has been taken as $M_{\rm A} = (1.026 \pm 0.021)\,{\rm GeV}$ [27]. The pseudoscalar form factor contributes only to CC scattering and it is almost negligible.

The single differential cross section for the quasi-elastic $\nu(\bar{\nu})$-nucleus scattering with respect to the outgoing nucleon kinetic energy T_N is obtained after integrating over the energy and the angle of the final lepton and over the solid angle of the final nucleon. In the calculation of the transition amplitudes the single-particle overlap functions φ_n are taken as the Dirac–Hartree solutions of a relativistic Lagrangian, containing scalar and vector potentials, obtained in the framework of the relativistic mean field theory [28]. The relativistic single-particle scattering wave function is written as in Refs. [29] in terms of its upper component, following the direct Pauli reduction scheme and solving a Schrödinger-like equation containing equivalent central and spin-orbit potentials, written in terms of the relativistic scalar and vector potentials [30, 31].

We treat the quasi-elastic neutrino scattering as a process where the cross section is obtained from the sum of all the integrated exclusive one-nucleon knockout channels even if the outgoing nucleon can be re-scattered in a detected channel, thus simulating the kinematics of a quasi-elastic reaction.

The relevance of these contributions to the experimental cross section depends on kinematics and should not be too large in the situations considered here. An alternative treatment of FSI in neutrino reactions can be found in Ref. [32].

3. Results

Results are presented for NC and CC neutrino and antineutrino scattering from ^{12}C in an energy range up to 1000 MeV, where the quasi-elastic one-nucleon knockout is expected to be the most important contribution. We have used the same relativistic bound state wave functions and optical potentials as in Refs. [29, 33], where the relativistic distorted wave impulse approximation (RDWIA) was able to fairly reproduce $(e, e'p)$, (γ, p), and (e, e') data. The relativistic bound state wave functions have been obtained from Ref. [28], where relativistic Hartree–Bogoliubov equations are solved in the context of a relativistic mean field theory. The scattering states are computed by means of the energy-dependent and A-dependent EDAD1 complex phenomenological optical potential of Ref. [34], which is fitted to proton elastic scattering data on several nuclei in an energy range up to 1040 MeV. The initial states φ_n are single-particle one-hole states in the target with unitary spectral strength. The sum in Eq. (2) runs over all the occupied states in the shell model.

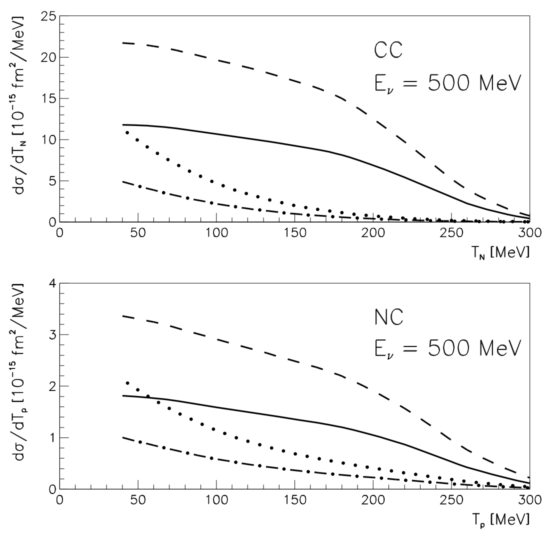

Fig. 1. Differential cross sections of the CC (upper panel) and NC (lower panel) $\nu(\bar{\nu})$ quasi-elastic scattering on ^{12}C as a function of the outgoing nucleon kinetic energy. Solid and dashed lines are the results in RDWIA and RPWIA, respectively, for an incident neutrino. Dot-dashed and dotted lines are the results in RDWIA and RPWIA, respectively, for an incident antineutrino.

In order to study the effects of FSI, in Fig. 1 results of the CC and NC $\nu(\bar{\nu})$-nucleus cross section in RPWIA and RDWIA are compared. FSI effects are large and reduce the cross sections of $\simeq 50\%$. This reduction is in agreement with the one found in electromagnetic one-nucleon knockout reactions. The results are consistent with those of Ref. [21].

The effects of a non-zero strange quark contribution to axial and vector form factors on the NC cross sections are shown in Fig. 2 for proton emission. We have chosen typical values for the strangeness parameters to show up their effect on the results, being aware that the value of g_{A}^{s} is correlated to the value of the axial mass M_{A} and that the values of μ^{s} and ρ^{s} are also highly correlated (see, *e.g.*, Ref. [19]). We have used $g_{\mathrm{A}}^{s} = -0.10$, $\mu^{s} = -0.50$, and $\rho^{s} = +2$. The results with $g_{\mathrm{A}}^{s} = -0.10$ are enhanced in the case of proton knockout (and reduced in the case of neutron knockout) by $\simeq 10\%$ with respect to those with $g_{\mathrm{A}}^{s} = 0$. The effect of μ^{s} is large and comparable with that of g_{A}^{s}, whereas the contribution of ρ^{s} is very small.

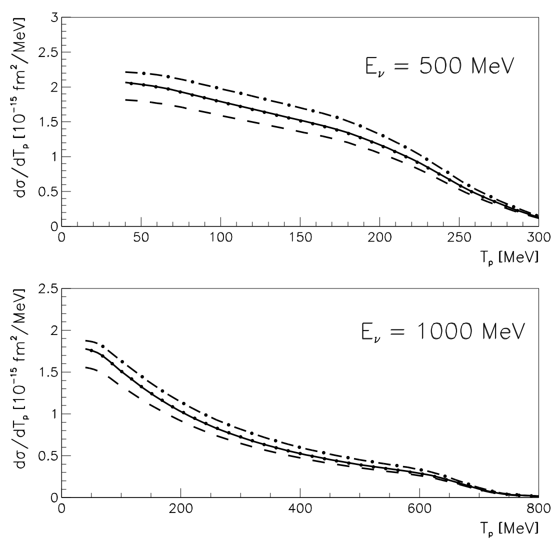

Fig. 2. Differential cross section of the NC ν quasi-elastic scattering on ^{12}C as a function of the outgoing proton kinetic energy. Dashed lines are the results with no strangeness contribution, solid lines with $g_{\mathrm{A}}^{s} = -0.10$, dot-dashed lines with $g_{\mathrm{A}}^{s} = -0.10$ and $\mu^{s} = -0.50$, dotted lines with $g_{\mathrm{A}}^{s} = -0.10$ and $\rho^{s} = +2$.

The role of the strangeness contribution can also be studied in the ratio of proton-to-neutron (p/n) NC cross sections [9, 19, 20]. This ratio is very sensitive to the strange quark contribution as g_{A} and g_{A}^{s} interfere with one sign in the numerator and with the opposite sign in the denominator (see Eq. (7)). Moreover, it is expected to be less sensitive to distortion effects.

The p/n ratio calculated in RDWIA for an incident neutrino is displayed in Fig. 3 (upper panel) as a function of T_{N}. The RPWIA results are also shown in the figure and are almost coincident with RDWIA ones. The p/n ratio is enhanced by a factor $\simeq$ 20–30% when g_{A}^s is included and by $\simeq$ 50% when both g_{A}^s and μ^s are included. A minor effect is produced by ρ^s, which gives only a slight reduction. Precise measurements of the p/n ratio appear, however, problematic due to the difficulties associated with neutron detection. This is the reason why the most attractive quantity to extract experimental information about the strangeness content seems the ratio of the neutral-to-charged (NC/CC) cross sections. In fact, although sensitive to the strange quark effects only in the numerator, the NC/CC ratio is simply related to the number of events with an outgoing proton and a missing mass with respect to the events with an outgoing proton in coincidence with a muon. Our RDWIA results for the NC/CC ratio are presented in Fig. 3 (lower panel) as a function of the energy of the emitted proton. The inclusion of a strangeness contribution produces a somewhat constant enhancement of the results with respect to the no strangeness case. The simultaneous inclusion of g_{A}^s and μ^s gives an enhancement that is about a factor of 2 larger than the one corresponding to the case with only g_{A}^s included. The effect of ρ^s is very small.

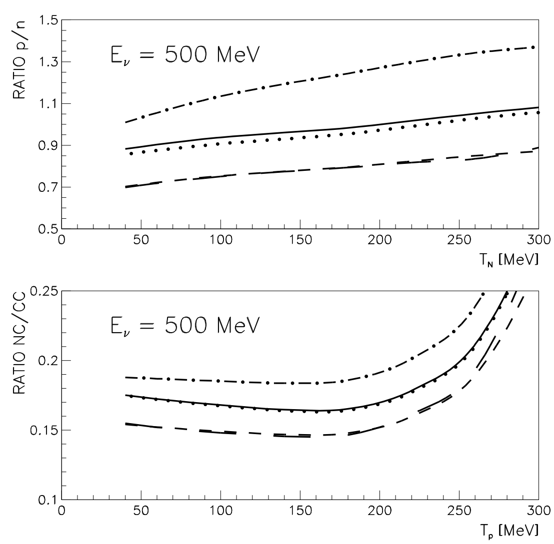

Fig. 3. Ratio of proton-to-neutron NC cross sections (upper panel) of the ν quasi-elastic scattering on ^{12}C and ratio of neutral-to-charged current cross sections (lower panel) of the ν quasi-elastic scattering on ^{12}C as a function of the nucleon energy. Long dashed lines are the RPWIA result without strangeness contribution. The other lines as in Fig. 2.

In conclusion, we find that the FSI are large both in CC and in NC reactions, but they are almost negligible in the ratio between proton- and neutron-emission cross sections or in the ratio between NC and CC cross sections. The presence of strangeness in the axial form factor with $g_{\rm A}^s = -0.10$ gives a 10–15% increase (decrease) in proton (neutron) knockout. The contribution of $\mu^s = -0.5$ in the vector form factor gives a further effect in the same direction. On the contrary the effect of ρ^s is much smaller.

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